Theoretische Teilchenphysik I

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Exercise Sheet 10 | SS-2023 | Due date: 04.07.23

Trace of gamma matrices (40 points)

Exercise 10.1: *(25 points)* In this exercise we want to become familiar with some properties of Dirac γ matrices and their traces.

(a) *(5 points)* Compute the trace of γ^5.

(b) *(5 points)* Prove that the trace of an *odd* number of γ matrices vanishes. *Hint: use the properties of γ^5.*

(c) *(5 points)* Prove that

$$\text{Tr}\big(\gamma^\mu \gamma^5\big) = 0\ . \tag{1}$$

(d) *(10 points)* Prove that

$$\text{Tr}\big(\gamma^\mu \gamma^\nu \gamma^5\big) = 0\ , \qquad \text{Tr}\big(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5\big) = 4i\epsilon^{\mu\nu\rho\sigma}\ , \tag{2}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is a rank-4, antisymmetric tensor.

Exercise 10.2: *(15 points)* When the number of Dirac matrices is large, computing traces by hand becomes prohibitive. In the exercise folder you can find a simple example of computing traces with a `Mathematica` package called `FeynCalc`.

(a) *(5 points)* Use `FeynCalc` to compute $\text{Tr}\big(\gamma^\mu \gamma^\nu\big)$ and $\text{Tr}\big(\gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta\big)$ and compare the results with the expressions given in the lecture notes.

(b) *(5 points)* Use `FeynCalc` to compute $\text{Tr}\big(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\tau\big)$ and $\text{Tr}\big(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\tau \gamma^\eta\big)$. Express the result in terms of metric tensors.

(c) *(5 points)* Use `FeynCalc` to check the result in Eq. (12.32) in the lecture notes.

Scattering amplitudes and cross-sections (60 points)

Exercise 10.3: *(60 points)* Consider the process

$$\pi(k) \to e^-(q) + \bar{\nu}_e(p)\ , \tag{3}$$

where π is a scalar particle, called pion, e^- is an electron, and $\bar{\nu}_e$ is anti-neutrino. We assume the antineutrino to be massless, while the pion and the electron are massive. The interaction occurring between these particles is describe by the Lagrangian

$$\mathcal{L} = -g\,\bar{\psi}_e \gamma^\mu (a + b\,\gamma^5)\psi_\nu\,\partial_\mu \pi\ , \tag{4}$$

where g is the (real) coupling constant, and a and b are real constants.

(a) *(5 points)* Write the Feynman rule for the interaction vertex and draw the corresponding Feynman diagram.

(b) *(15 points)* Write the explicit expression of the scattering amplitude of the process in Eq. (3) (at leading order in g), and the corresponding conjugated amplitude.

(c) *(5 points)* Compute the squared amplitude and sum over the spin degree of freedom of the final-state partons. Compute the relevant traces and express the result in terms of scalar products of momenta k, q, p.

(d) *(10 points)* Write the expression of the phase-space measure that you need to compute the decay with of the pion, and express the scalar products of momenta k, q, p in terms of m_π and m_e.

(e) *(10 points)* Compute the decay width of the pion integrating the squared amplitude over the relevant phase space and introducing the correct flux factors. Express the result as a function of m_π and m_e.

(f) *(10 points)* Compute the lifetime ot the pion setting $a = b = 1$ and compare the result with the literature.

(g) *(5 points)* Would the process in Eq. (3) occur if $m_e = 0$? Prove your answer by only looking at the amplitude of the process and using the Dirac equation.