Theoretische Teilchenphysik I

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Exercise Sheet 11 | SS-2023 | Due date: 11.07.23

Higgs and Z-boson exchange in hydrogen atom (100 points)

In the lectures, electromagnetic interaction of an electron with positron and the nucleus was discussed. In this exercise, we consider the interaction of the electron with the proton in the hydrogen atom. We will assume that the proton is a point-like spin $1/2$ particle with the mass m_p and the charge $-e$.

Exercise 11.1: *(50 points)* Electromagnetic interactions between electrons and protons arise because of the photon exchange. As we know now, electron and proton can interact by exchanging additional particles, for example the Higgs boson. Higgs boson interaction with fermions is described by the Yukawa Lagrangian

$$\mathcal{L}_{\text{int}} = \frac{m_f}{v} H \bar{f} f, \tag{1}$$

where m_f is the fermion f mass and $v = 250\,\text{GeV}$ is a Higgs field vacuum expectation value. The mass of the Higgs boson is $m_H = 125\,\text{GeV}$.

(a) *(15 points)* Write down the amplitude for the electron-proton scattering due to the Higgs boson exchange at the leading order in the coupling constants $m_e/v, m_p/v$. Simplify the amplitude by considering the leading non-relativistic approximation, which means neglecting all higher-order terms in small external momentum expansion.

(b) *(15 points)* Use the relation between scattering amplitude and the interaction potential considered in lectures to find the interaction potential between electron and proton caused by the Higgs boson exchange. Show that in the position space, the potential is proportional to $\delta(\vec{r}_e - \vec{r}_p)$, where $\vec{r}_{e,p}$ are position vectors of the electron and the proton.

(c) *(10 points)* Calculate the Higgs-boson induced correction to n-th energy level of the hydrogen atom. Explain why only energies of S-states are affected by this interaction.

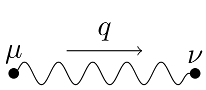
(d) *(5 points)* From the result for the nS energy level, calculate contribution to the energy difference between $2S$ and $1S$ levels induced by Higgs boson exchange. Compare the calculated number with the error of very precise measurement of $1S$ and $2S$ energy levels difference $2.067 \times 10^{-13}\,\text{eV}$. Find the Higgs boson mass value, which would give shift in the energy levels of the same order as the current experimental uncertainty.

(e) *(5 points)* Conversely, imagine that the Yukawa interaction of the Higgs boson with an electron contains an additional unknown factor κ_e. Use considerations as in the previous item to derive an upper bound on κ_e from the measured energy spectrum of the hydrogen atom.

Exercise 11.2: *(50 points)* In the exercise, we again consider QED interaction of the electron with the proton in the hydrogen atom but now with additional interaction caused by Z-boson exchange. The interaction of Z-boson with fermions is described by the following Lagrangian

$$\mathcal{L}_{\text{int}} = \sum_{i=e,p} Z_\mu \bar{f}_i (g_{i,V}\gamma_\mu + g_{i,A}\gamma_\mu\gamma_5) f_i, \tag{2}$$

where we as before consider proton as fundamental fermion. Axial coupling for proton is small compared to its vector coupling $g_{p,A} \ll g_{p,V}$. Mass of the Z-boson is $M_Z = 91\,\text{GeV}$ and we use the following Feynman rule for its propagator



$$= \frac{-i}{q^2 - M_Z^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right). \tag{3}$$

(a) *(10 points)* QED is symmetric under parity transformations. Explain why the Lagrangian (2) violates parity. What are the consequences of the presence of the parity-violating interactions on the spectrum of the hydrogen atom?

(b) *(20 points)* Write the amplitude for parity violating interaction of the electron with proton. Focus on terms proportional to $g_{e,A}g_{p,V}$. Perform all calculations in the first non-trivial order of the non-relativistic expansion. To simplify the final expression, take into account that a proton is much heavier than an electron $m_p \gg m_e$.

(c) *(20 points)* Use relation between scattering amplitudes and interaction potentials to find the parity-violating potential due to Z-exchange. Compare the result with the potential obtained in the previous exercise and explain why the found interaction potential is parity-violating.