Theoretische Teilchenphysik I

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Exercise Sheet 12 SS-2023 Due date: 18.07.23

EM interaction of the Higgs boson (100 points)

In lectures and previous exercises we considered interaction of fermions with photons and Higgs boson. In absence of the direct interaction of Higgs with photons, in the exercises we will try to construct such interaction considering higher orders of perturbation theory.

Exercise 12.1: *(10 points)* Consider a quantum field theory with a charged fermion of mass m, a real scalar particle (the Higgs boson) with mass m_H and the electromagnetic field. The coupling of the Higgs boson to a fermion is of the Yukawa type. Write down a Lagrangian for such a theory and summarize the Feynman rules.

Exercise 12.2: *(35 points)* Theory constructed in the previous exercise does not contain an interaction vertex between the Higgs boson and photons. However, since fermion can couple to both the Higgs boson and the photons, they can induce such interaction. To see this

(a) *(5 points)* Draw diagrams for the process $H \to \gamma\gamma$ of the Higgs decay into two photons. Closed fermion loops are allowed; you are supposed to find the first order in perturbation theory where such an interaction can happen. You are suppose to find two diagrams; they are nearly identical, but with reverse direction of the "fermion flow". *Hint: Closed loops are allowed, consider only leading order diagrams.*

(b) *(10 points)* Write an expression for the amplitude of the process $H \to \gamma\gamma$ as an integral over the loop momentum.

(c) *(15 points)* In general, the amplitude for this process can be written in the following way

$$M = \epsilon_1^\mu \epsilon_2^\nu A_{\mu\nu}(p_1, p_2, m), \tag{1}$$

where $\epsilon_{1,2}$ and $p_{1,2}$ are the polarization vectors and and the momenta of the outgoing photons, respectively. Write down the most general form of the tensor $A_{\mu\nu}$ assuming that parity is conserved. Remember, that photon polarization vectors are transversal, i.e. $p_\mu \epsilon^\mu(p) = 0$.

Use diagrams constructed in the item (b) to show that

$$A_{\mu\nu} p_1^\mu = A_{\mu\nu} p_2^\nu = 0. \tag{2}$$

Impose these conditions on tensor $A_{\mu\nu}$ and show that it can be parametrized in terms of a single function (form factor) that depends on the Higgs boson mass.

(d) *(5 points)* Calculate fermion traces in any convinient way (by hand, with `FeynCalc`, `FORM`, etc.)

Exercise 12.3: *(45 points)* In the next step we focus on the expression for Feynman diagrams that contribute to $H \to \gamma\gamma$ amplitude.

(a) *(5 points)* Use traces calculated in point (d) and consider the limit of the integrand where all loop momenta components k_μ become very large. Separate all contributions that are potentially divergent at $k \to \infty$ from the contributions that are finite. Split the integral into potentially-divergent (S) and definitely-finite (R) parts

$$A_{\mu\nu}(p_1, p_2, m) = S_{\mu\nu}(p_1, p_2, m) + R_{\mu\nu}(p_1, p_2, m). \tag{3}$$

Hint: count dimension in units of mass loop momenta in numerator, denominator and integration measure.

(b) *(10 points)* Use Feynman parametrisation to combine denominators in expression for the amplitude. Since it is allowed to shift loop momentum by finite linear combinations of external momenta, find such a loop momentum shift $k \to l$, which transforms all terms of the amplitude into the form

$$\int \frac{d^4k}{[(k+p_1)^2 - m^2] \dots [(k+p_n)^2 - m^2]} \to \int \frac{d^4l}{[l^2 - \Delta]^n}, \tag{4}$$

where Δ is a function of introduced Feynman parameters and external momenta scalar products.

(c) *(10 points)* Consider amplitudes in the right hand side of Eq. (3) as functions of the fermion mass. To fully define the amplitude (i.e. to make it actually calculable without the need to worry about divergent contributions) we can subtract from the original amplitude with mass m fermion in the loop identical amplitude where the mass m is replaced with a very large mass Λ

$$A^{\rm reg}_{\mu\nu}(p_1, p_2, m, \Lambda) = [S_{\mu\nu}(p_1, p_2, m) - S_{\mu\nu}(p_1, p_2, \Lambda)] + [R_{\mu\nu}(p_1, p_2, m) - R_{\mu\nu}(p_1, p_2, \Lambda)]\,. \tag{5}$$

Show explicitly that the difference $S_{\mu\nu}(p_1, p_2, m) - S_{\mu\nu}(p_1, p_2, \Lambda)$ is well defined for large values of the loop momentum and is actually zero. Ignore all the terms that are suppressed by powers of $1/\Lambda$.
Hint: to calculate tensor integral, use averaging over external momenta. Work with expression for amplitude in the form (4) *after loop momentum shift.*

(d) *(10 points)* After cancellation of the singular part in the regularized amplitude $A^{\rm reg}$ remaining part is finite. Perform integration over Feynman parameters and calculate $[R_{\mu\nu}(p_1, p_2, m) - R_{\mu\nu}(p_1, p_2, \Lambda)]$ explicitly. Show that the obtained result is independent of Λ in the limit $\Lambda \to \infty$, but that $R_{\mu\nu}(p_1, p_2, \Lambda)$ does contribute to the final result.

(e) *(10 points)* Explain how your result would differ if you would not introduce a regulator and, instead, would average over directions of external momenta early on in the computation.

Exercise 12.4: *(10 points)* Calculate decay rate of the Higgs boson to two photons, by taking square of the finite amplitude $A^{\rm reg}_{\mu\nu}$ derived in previous exercise.