Theoretical Particle Physics I

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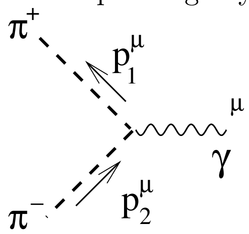
Exercise 1: Pion-Pair Production

For the description of the electromagnetic interaction of pions it has to be considered that they are composed of quarks and have an inner structure. This inner structure is described by the form factor $F_\pi(q^2)$ and the corresponding Feynman rule is given by



$$= ie(p_1 + p_2)^\mu F_\pi(q^2),$$

where $q = p_1 - p_2$ is the momentum of the photon.

a) Show that the matrix element for the production of a $\pi^+\pi^-$ pair in electron-positron collisions can be written as

$$\mathcal{M}_{fi}(e^+e^- \to \pi^+\pi^-) = -ie^2 \frac{F_\pi(s)}{s} \bar{v}(p_1)(\not{p}_3 - \not{p}_4)u(p_2),$$

where p_1, p_2 are the momenta of the incoming positron/electron, p_3, p_4 are the momenta of the outgoing π^+/π^- with $s = (p_1 + p_2)^2$.

b) Calculate the differential $\left(\frac{d\sigma}{d\Omega}|_{\text{cms}}\right)$ and the total cross section, where the electron mass is considered to be zero.

Exercise 2: Rosenbluth formula (10 points)

The electromagnetic interaction of protons can be described by the introduction of two form factors $F_1(q^2)$ and $F_2(q^2)$ as

$$\langle p'|j_\mu^{\text{elm}}|p\rangle = -ie\,\bar{u}(p')\left[\gamma_\mu\, F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2)\right]u(p)\,, \tag{1}$$

where m is the mass of the proton, p' and p are the momenta of the outgoing and incoming proton and $q = p' - p$ is the momentum of the photon.

a) Proof equation (1). Based on the properties of the electromagnetic current that it is a vector current and is conserved, construct the general tensor structure for j_μ^{elm}.
Hint: It might be useful to use the Gordon identity.

b) Consider the scattering of an electron, with energy $E \gg m_e$, and a proton at rest. Show that the differential cross section of the elastic scattering can be written as

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2} \frac{\left(F_1^2(q^2) - \frac{q^2}{4m^2}F_2^2(q^2)\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}(F_1(q^2) + F_2(q^2))^2 \sin^2\frac{\theta}{2}}{E^2 \sin^4\frac{\theta}{2}\left(1 + \frac{2E}{m}\sin^2\frac{\theta}{2}\right)}\,,$$

where θ is the scattering angle of the electron in the laboratory system.
