VI Spontaneous Symmetry Breaking

2. GOLDSTONE theorem

For each spontaneously broken continuous symmetry there is a massless particle in the theory

Remark:

* example in 1.: 1 broken sym $\to$ η massless
* in nature: π meson $\approx$ GOLDSTONE bosons (broken sym. in QCD)

Proof: Consider N real-valued fields ϕ^a $a = 1, \dots, N$

(i)

$$\mathcal{L} = (\text{derivatives}) - V(\phi)$$

ϕ_0^a are constant fields which minimize V, i.e.

$$\frac{\partial}{\partial \phi^a} V \Big|_{\phi^a(x) = \phi_0^a} = 0$$

Expand V around minimum

$$\Rightarrow V(\phi) = V(\phi_0) + 0 + \frac{1}{2} \underbrace{\frac{\partial^2 V}{\partial \phi^a \partial \phi^b}\Big|_{\phi_0}}_{=: m_{ab}^2} (\phi^a - \phi_0^a)(\phi^b - \phi_0^b) + \dots$$

m^2_{ab} is symmetric matrix; eigenvalues (EVs) $\hat{=}$ {squares of} masses of fields

minimum $\Rightarrow$ EVs ≥ 0

To be shown: Each continuous symmetry of $\mathcal{L}$, which is not a symmetry of ϕ_0 leads to an eigenvalue 0. { $\to$ then the G-theorem is proven }

(ii) It is sufficient to consider an infinitesimal transformation

A general cont. transformation has the form

$$(*) \quad \phi^a \to \phi^a + \alpha \Delta^a(\phi^1, \dots, \phi^N)$$

α: infinitesimal parameter

$\mathcal{L}$ is invariant under $(*)$

$$\Rightarrow \quad V(\phi^a) = V(\phi^a + \alpha \Delta^a(\phi_1, \dots, \phi^N))$$

$$\Leftrightarrow \quad \Delta^a \cdot \frac{\partial}{\partial \phi^a} V(\phi) = 0 \qquad \{\text{sum over } a\}$$

(α is inf.)

apply $\frac{\partial}{\partial \phi^b}$; then $\phi = \phi_0$

$$(**) \quad \Rightarrow \quad \left.\frac{\partial \Delta^a}{\partial \phi^b}\right|_{\phi_0} \underbrace{\left.\frac{\partial V}{\partial \phi^a}\right|_{\phi_0}}_{=0 \text{ since } \phi_0 \text{ minimum}} + \Delta^a\Big|_{\phi_0} \left.\frac{\partial^2}{\partial \phi^a \partial \phi^b} V\right|_{\phi_0} = 0$$

(iii) There are 2 possibilities:

(1) ground state (i.e. ϕ_0) is invariant under symmetry

i.e: no spontaneous symmetry breaking

$\Rightarrow \Delta^a(\phi_0^1, \dots, \phi_N^0) = 0$

$\Rightarrow$ (**) is trivial

(2) Spontaneous symmetry breaking for $\Delta^a(\phi_0^1, \dots, \phi_0^N) \neq 0$

{for at least one a}

$\Rightarrow \Delta^a(\phi_0^1, \dots, \phi_0^N)$ is eigenvector with eigenvalue $\emptyset$ {end of proof}

since $m_{ab}^2 \cdot \Delta^a(\phi_0^1, \dots, \phi_0^N) = 0$ $\blacksquare$

3. The HIGGS model

(a) Extend GOLDSTONE model to $U(1)$ gauge symmetry {local symmetry}

vector field $A_\mu(x)$

scalar field $\phi(x)$

covariant derivative: $D_\mu \phi(x) = (\partial_\mu + ie A_\mu)\, \phi(x)$

$\Rightarrow$

$$\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - \mu^2 |\phi|^2 - \lambda |\phi|^4 - \frac{1}{4}(F^{\mu\nu})^2$$

$\triangleq$ abelian HIGGS model

$\mathcal{L}$ is invariant under

$$\phi \to \phi' = \phi\, e^{-ie\alpha(x)}$$
$$\phi^* \to \phi'^* = \phi^*\, e^{+ie\alpha(x)}$$
$$A_\mu \to A_\mu' = A_\mu + \partial_\mu \alpha(x)$$

{cf GOLDSTONE model} $\mu^2 > 0 \to$ no spontaneous symmetry breaking {sym. breaking not due to potential}

$\mu^2 < 0$: $\phi = \frac{1}{\sqrt{2}}\left(v + \sigma(x) + i\eta(x)\right)$; spontaneous symmetry breaking

$\longrightarrow$

$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{int}$ $\leftarrow$ cubic and quartic terms in $\{A_\mu, \sigma, \eta\}$ not shown explicitly

quadratic terms $\to$ $$\mathcal{L}_{kin} = \frac{1}{2}(\partial^\mu \sigma)^2 + \frac{1}{2}(\partial^\mu \eta)^2 - \frac{1}{2}(2\lambda v^2)\sigma^2$$ {$\leftarrow$ as for GOLDSTONE model}

$$- \frac{1}{4}(F^{\mu\nu})^2 + \underbrace{\frac{1}{2}(ev)^2 A^2}_{(*)} + \underbrace{ev A^\mu \partial_\mu \eta}_{(**)}$$

Origin of $(*)$:

$$(D^\mu\phi)^*(D_\mu\phi)\Big|_{v^2A^2} = \left(-ieA^\mu v/\sqrt{2}\right)\left(ieA_\mu v/\sqrt{2}\right) = \frac{1}{2}e^2v^2A^2$$

Origin of $(**)$:

$$(D^\mu\phi)^* D_\mu\phi\Big|_{A^\mu\partial_\mu\eta} = \left(-ieA^\mu\frac{v}{\sqrt{2}}\right)\left(\partial_\mu\frac{i\eta}{\sqrt{2}}\right) + \left(\partial_\mu\frac{-i\eta}{\sqrt{2}}\right)\left(ieA^\mu\frac{v}{\sqrt{2}}\right) = evA^\mu\partial_\mu\eta$$

No $A_\mu\partial^\mu\sigma$ term:

$$(D^\mu\phi)^* D_\mu\phi\Big|_{A_\mu\partial^\mu\sigma} = \left(-ieA^\mu\frac{v}{\sqrt{2}}\right)\left(\partial_\mu\frac{\sigma}{\sqrt{2}}\right) + \left(\partial_\mu\frac{\sigma}{\sqrt{2}}\right)\left(ieA^\mu\frac{v}{\sqrt{2}}\right) = 0$$

(b) Interpretation of $\mathcal{L}$:

σ: KG field ; mass $\sqrt{2\lambda v^2}$

η: KG field ; massless

A^μ: Vector field ; mass $|ev|$

Problems:

(i) $A^\mu \partial_\mu \eta$ term ?   {mu vector boson? massless scal. boson?}

(ii) degrees of freedom: $\mathcal{L}(\phi, A) : 4 \quad (\phi: 2, \text{m=0 } A: 2)$
$\mathcal{L}(\sigma, \eta, A) : 5 \quad (\sigma: 1, \eta: 1, \text{mv } A: 3)$ contradiction

Solution: $\mathcal{L}(\sigma, \eta, A)$ contains unphysical particle which can be eliminated : $\eta(x)$

Use gauge freedom $\Rightarrow \quad \phi = \frac{1}{\sqrt{2}}(v + \sigma(x))$ "unitary gauge"

$$\Rightarrow \quad \mathcal{L}_{kin} = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(2\lambda v^2)\sigma^2 - \frac{1}{4}(F^{\mu\nu})^2 + \underbrace{\frac{1}{2}(ev)^2 A^2}_{(**)}$$

$$\mathcal{L}_{int} = \underbrace{-\lambda v \sigma^3 - \frac{1}{4}\lambda \sigma^4}_{} + \underbrace{\frac{1}{2}e^2 A^2 (2v\sigma + \sigma^2)}_{(*)}$$

See GOLDSTONE model

$$(*) : \left.(D^\mu \phi)^* (D_\mu \phi)\right|_{A^2} = (-ieA^\mu)\frac{1}{\sqrt{2}}(v+\sigma)\,(ieA_\mu)\frac{1}{\sqrt{2}}(v+\sigma)$$

$$= \frac{1}{2}e^2 A^2 (v^2 + \sigma^2 + 2v\sigma)$$

no A: $\rightarrow (\partial_\mu \sigma)^2$
lin A: vanishes

mass term (**)

σ: $m \neq 0$ scalar field $\quad m_\sigma = \sqrt{2\lambda v^2}$ $\quad 1$

A^μ: $m \neq 0$ vector field $\quad m_A = |ev|$ $\quad 3$

$\Big\}$ degrees of freedom

Remarks:

* 1 complex scalar field + 1 massless vector field
  $\longrightarrow$ 1 real-valued scalar field + 1 massive vector field

* mass term for vector field without loss of gauge invariance
  "HIGGS mechanism" $\quad$ { sym. of $\mathcal{L}$ is not modified ! }
  $\sigma(x)$: HIGGS boson

* $\eta(x) \overset{\wedge}{=}$ GOLDSTONE boson in GOLDSTONE model
  eliminated
  $\overset{\wedge}{=}$ longitudinal degree of freedom of A_μ

* HIGGS mechanism also for non-abelian gauge theories (SM)

VII Decay Rates

1. 2-particle phase-space (brief repetition from/generalization of chapter V)

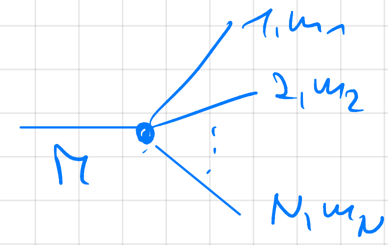
2. Higgsboson decays (more precisely: decays of scalar particle)

3. 3-particle phase-space

1. 2-particle phase space (again!)

(a) From chapter V:

$$d\Gamma = \frac{1}{2M} \underbrace{(2\pi)^4 \delta^{(4)}(P_N - P) \prod_{f=1}^{N} \frac{d^3 p_f}{(2\pi)^3\, 2 p_f^0}}_{= dPS^{(N)} \equiv d(LIPS)} |\mathcal{M}|^2$$



$P_N = p_1 + p_2 + \dots + p_N$

(b) $$dPS^{(2)} = \frac{1}{16\pi^2 M}\, d\Omega_1\, \frac{\sqrt{\lambda(M^2, m_1^2, m_2^2)}}{2M}$$

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$ KÄLLEN function

(i) $m_1 = m_2 = m$: $$dPS^{(2)} = \frac{1}{32\pi^2}\, d\Omega_1 \sqrt{1 - \frac{4m^2}{M^2}}$$ {see chap V}

(ii) $m_1 = m$; $m_2 = 0$ $\Rightarrow$ $\lambda = (M^2 - m^2)^2$

$$\Rightarrow dPS^{(2)} = \frac{1}{16\pi^2 M}\, d\Omega_1\, \frac{M^2 - m^2}{2M} = \frac{1}{32\pi^2}\, d\Omega_1 \left(1 - \frac{m^2}{M^2}\right)$$

(iii) no Ω_1-dependence in $|\mathcal{M}|^2$

(e.g. decaying particle has spin $\emptyset$)

$$\Rightarrow \int d\Omega_1 = 4\pi$$

(iv) identical particles (in final state) $\rightarrow$ factor $1/2$

(c) $dPS^{(3)}$: see section 3

section 2: applications : Higgs decays