Theoretische Teilchenphysik II

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Exercise Sheet 1 | WS-2023 | Due date: 30.10.23

The goal of this exercise is to refresh basic topics that have been discussed in TTP1 lectures last semester. Consult the corresponding script if you have problems doing the exercises.

Higgs decay to photons (40 Points)

Exercise 1.1: *(40 points)* Although the Higgs boson is an electrically-neutral particle, it can couple to photons in a peculiar way. The Lagrangian that describes such interaction reads

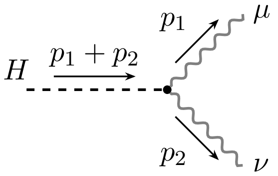
$$\mathcal{L} = \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{g}{4v}F_{\mu\nu}F_{\mu\nu}H, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, A_μ is the electromagnetic vector potential and H is the Higgs field.

(a) *(5 points)* Explain which terms in the Lagrangian (1) correspond to a free Lagrangian and which can be treated as interaction terms. Determine the mass dimension of the coupling g.

(b) *(10 points)* Use the Lagrangian (1) to derive Feynman rules required to calculate Higgs boson decay amplitude to two photons. Compare your result for the interaction vertex with Eq. (2).

(c) *(15 points)* Use Feynman rules to derive the $H \to \gamma\gamma$ amplitude, the conjugated amplitude and the amplitude squared and summed over polarizations for the process $H \to \gamma\gamma$.



$$\sim i\frac{g_{AAH}}{v}\left(g_{\mu\nu}p_1 \cdot p_2 - p_{1,\mu}p_{2,\nu}\right) \tag{2}$$

Hint: momentum dependence in Feynman rule comes from field derivatives in the Lagrangian interaction term.

(d) *(10 points)* Use the result for the amplitude squared to calculate the decay width of the Higgs boson to two photons.

Higgs production at a muon collider (60 Points)

Exercise 1.2: *(60 points)* A primary source of information about particle physics comes from high-energy colliders where outcomes of collisions of two particles A_1 and A_2 are analyzed. Until now, electron-positron, proton-(anti)-proton, proton-proton and electron-proton colliders were built. There are reasons to believe that a muon collider would be an interesting option for the future. One of the reasons for that is the possibility to produce the Higgs boson in a simple way that will be discussed in this exercise.

(a) *(10 points)* Construct a Lagrangian describing the interaction of the neutral scalar Higgs particle with two different types of charged fermions $\mu^\pm$ and $\tau^\pm$. The Lagrangian should be a) simple, b) Lorentz invariant, c) involve operators of mass-dimension four only, d) be parity-even.
Hint: remind yourself about the Yukawa Lagrangian discussed in TTP1 lectures.

(b) *(10 points)* Summarise (i.e. draw pictures and write down) Feynman rules needed for calculations in this theory.

(c) *(25 points)* Working in the leading order of perturbation theory draw relevant diagrams describing the process $\mu^+\mu^- \to \tau^+\tau^-$ with the intermediate Higgs boson. Write expressions for the amplitude, construct the complex-conjugated amplitude and determine the amplitude squared summed over polarizations of all fermions. Neglect masses of fermions during the calculation.

(d) *(10 points)* Use the expressions for the squared amplitude to calculate total cross section of the process $\mu^+\mu^- \to \tau^+\tau^-$.

(e) *(5 points)* The result that you obtain depends on the collider energy and the Higgs boson mass. If the goal is to study Higgs bosons (i.e. maximize the number of Higgs bosons produced in the collisions), which energy of the muon collider should be chosen?