Theoretische Teilchenphysik II

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Exercise Sheet 2 WS-2023 Due date: 06.11.23

Feynman diagrams and dimensional analysis (100 Points)

Exercise 2.1: *(40 points)* In the TTP1 lectures we discuss several renormalizable quantum firld theories such as the self-interacting scalar theories in $d = 4$ and $d = 6$, quantum electrodynamics and the Yukawa theory.

Summarize Feynman rules for propagators and vertices in these four theories needed to write expression for loop diagrams.

Calculate mass dimensions of the coupling constants in each of the four theories, namely

(a) *(10 points)* in the theory of the scalar field with the interaction $\lambda\varphi^4$ in $d = 4$;

(b) *(10 points)* in the scalar theory with the interaction term $g\varphi^3$ in $d = 6$;

(c) *(10 points)* in QED in $d = 4$ where the interaction term is $g\bar{\psi}\gamma_\mu\psi\, A^\mu$;

(d) *(10 points)* in the Yukawa theory with the interaction term $g_Y\bar{\psi}\gamma_5\psi\phi$ in $d = 4$.

Exercise 2.2: *(40 points)* Consider the following loop diagrams.

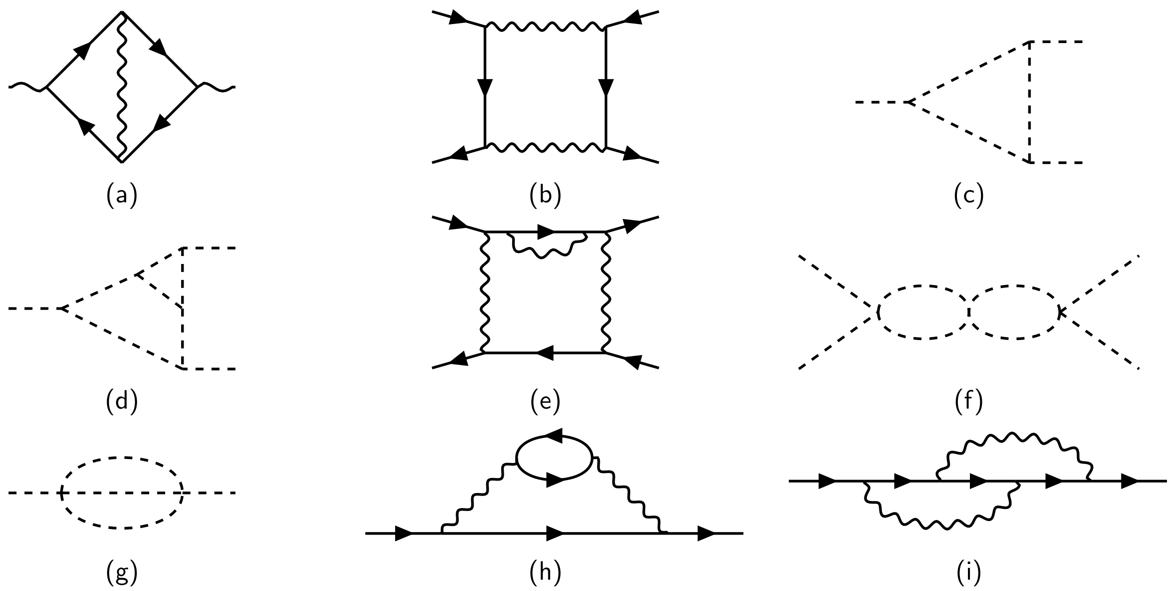

Figure 1: Multi-loop diagrams

These diagrams are built out of scalar (dashed) lines, wavy (photon) lines and solid oriented lines (fermions).

(a) *(10 points)* From the type of lines and vertices guess underlying field theory for each diagram.

(b) *(15 points)* Using power counting arguments calculate superficial degree of divergence for each diagram.

(c) *(15 points)* Multiloop diagrams with negative degree of superficial divergence can be still divergent due to presence of sub-divergences. For each diagram try to find all divergent sub-diagram. To do so identify candidate sub-diagrams built from subset of vertices and lines, and apply same power-counting rules to identify their degrees of divergence.

Exercise 2.3: *(20 points)* In this course, our preferred regularisation scheme is dimensional regularisation. To apply it to the calculation of divergent integrals, we need to perform an analytic continuation of various mathematical quantities to d dimensions. One of such quantities is the solid angle in the d-dimensional space whose dependence on d was given in the lecture. The goal of this exercise is to derive that formulas.

Consider auxiliary integral

$$\int e^{-k^2} \mathrm{d}^d k, \tag{1}$$

where $k^2 = \sum\limits_{i=1}^{d} k_i^2$. First, calculate the integral in Eq. (1) using Cartesian coordinates where this integral is gievn by a product of d one-dimensional Gaussian integrals. Second, introduce spherical coordinates with d-dimensional solid angle Ω_d and perform the radial integration. Equating two results, find Ω_d. Recall the definition of the Gamma-function

$$\Gamma(x) = \int\limits_0^\infty \mathrm{d}t\, t^{x-1} e^{-t}. \tag{2}$$