Theoretische Teilchenphysik II

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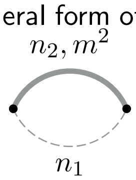
Exercise Sheet 3 WS-2023 Due date: 13.11.23

Divergencies in scalar theories (100 Points)

Exercise 3.1: *(30 points)* Consider general form of one-loop tadpole diagram with one massless and one massive line:

$$T_{0m}(n_1, n_2) =$$



$$= \int \frac{d^d k}{[k^2]^{n_1}\,[k^2+m^2]^{n_2}}. \tag{1}$$

(a) *(20 points)* Using Feynman parametrisation and result for tadpole integral with single massive line derived in lectures, calculate diagram in dimensional regularisation for arbitrary powers of propagators n_1 and n_2.

(b) *(10 points)* Explain *where* do divergences of integrals $T_{0m}(1,1)$ and $T_{0m}(0,2)$ come from. Calculate integrals and expand obtained result in small $\varepsilon = 2 - d/2$ to get divergent part explicitly. Show that integral $T_{0m}(2,1)$ for small values of loop momenta has infra-red (IR) divergence and calculate it. Expand the obtained result and extract the divergent part.

Exercise 3.2: *(20 points)* Consider scalar theories with n-field self-interaction vertex described by the Lagrangian

$$\mathcal{L}_n = \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi + \frac{\lambda_n}{n!}\varphi^n \tag{2}$$

The dimensionality of space-time is assumed to be arbitrary and is denoted by d.

(a) *(10 points)* Draw several one particle irreducible(1PI) loop diagrams for the theory with Lagrangian $\mathcal{L}_n$ for $n = 3, 4, 5, 6$ and verify, that for each of them the following relation between the number of loops L, the number of propagators in a diagram E and the number of vertices V,

$$L = E - V + 1, \tag{3}$$

is fulfilled. Check explicitly, by drawing diagrams and counting vertice and propagator lines(edges), that for 1PI diagrams with n external legs, in the theory described by the Lagrangian $\mathcal{L}_n$ in Eq. (2) the following relation

$$2E = n(V-1), \tag{4}$$

holds.

(b) *(10 points)* Derive a relation between the space-time dimension d and the exponent n in (2) for which the constant λ_n has zero mass dimension. Draw example diagrams for $n = 5$ and check result obtained with general expression. What is appropriate space-time dimension for $n \to \infty$?

Exercise 3.3: *(50 points)* In this exercise we will apply renormalization techniques for scalar theories developed in the lectures to φ^3 theory in $d = 6 - 2\varepsilon$. The Lagrangian reads

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi - \frac{m^2}{2}\varphi^2 + \frac{\lambda}{3!}\varphi^3. \tag{5}$$

(a) *(10 points)* Draw all one-loop one-particle irreducible divergent Green's functions in this theory. Write integral representation for one-loop diagrams that contribute to these Green's functions assuming that all external momenta are non-zero and arbitrary.

(b) *(10 points)* From the obtained integral representations show that UV divergencies of diagrams do not change if we put all external momenta to zero. For each diagram find the superficial degree of divergence.

(c) *(10 points)* Calcualte the divergent part of the one-loop diagram that contributes to the vertex renormalization. Use the limit of zero external momenta. For calculation of remaining massive tadpole integrals use results from previous exercise (1).

(d) *(20 points)* Self-energy diagram in this theory is quadratically divergent (i.e. the superficial divergence is two). Show that the dependence of divergences on the external momentum q and the mass m can be described by the following formula $Aq^2 + Bm^2$ where A, B are two ϵ-dependent constants.

To calculate divergent contributions to A and B required for the field and mass renormalizations, we need to differentiate the self-energy diagram in q^2 and m^2, respectively.

Check explicitly that after the differentiation, resulting expressions are only logarithmically divergent. Put all external momenta to zero and calculate divergences in the same way as in the point (c).