Theoretische Teilchenphysik II

V: Prof. Kirill Melnikov, U: Dr. Ming-Ming Long, U: Dr. Andrey Pikelner

Exercise Sheet 13 WS-2023 Due date: 05.02.24

Vacuum stability in Higgs-Yukawa theories(100 Points)

The goal of the current exercise is to analyze the form of the scalar potential of a Standard-Model-like theory after taking into account radiative corrections. For very large values of a scalar field it can be approximated with the tree-level expression and modified (running) coupling constant that depends on the field itself. Shape of the potential is determined by the running scalar field self-coupling and our first step is to derive renormalization group equations in such a theory and determine evolution of the self-coupling.

Exercise 13.1: *(60 points)* Consider the heory of a scalar Higgs-like field ϕ interacting with single fermion field ψ

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}\left(\partial_\mu\phi\right)^2 - m_\psi^2\bar{\psi}\psi - y\bar{\psi}\psi\phi - \lambda\phi^4. \tag{1}$$

(a) *(10 points)* Starting with the Lagrangian given in Eq. (1) in terms of bare parameters construct it's renormalized version with counterterms. Summarize all needed Feynman rules for calculation of loop corrections in this theory.

(b) *(10 points)* Identify all Green functions needed for one-loop calculation of beta-functions of all coupling constants present in the Lagrangian. Draw all diagrams needed for one-loop renormalization of the theory. Since we are interested in divergent diagrams only, calculate degree of divergencies of all needed diagrams.

(c) *(10 points)* Explain which counterterms and corresponding renormalization constants are needed for renormalization of the theory at one-loop. Combine diagrams from the previous item into groups needed for each renormalization constant calculation.

(d) *(20 points)* Working in dimensional regularization calculate divergencies of all needed one-loop diagrams and determine renormalization constants.

(e) *(10 points)* From the set of renormalization constants derive beta-functions for all coupling constants of the theory and check that it has the following form

$$\beta_\lambda = \frac{\partial a_\lambda(\mu)}{\partial \log\mu} = 72a_\lambda^2 + 8a_\lambda a_y - 2a_y, \qquad \beta_y = \frac{\partial a_y(\mu)}{\partial \log\mu} = 10a_y, \tag{2}$$

where $a_\lambda = \lambda(\mu)/(4\pi)^2$ and $a_y = y(\mu)^2/(4\pi)^2$. In contrast to examples studied in lectures now we have a coupled system of equations.

It turns out that the scalar potential of the theory changes because of radiative corrections; these changes can be accommodated by writing $V(\phi) = \lambda\phi^4$ as $V(\phi) = \lambda_{\text{eff}}(\phi)\,\phi^4$, where $\lambda_{\text{eff}}(\mu) = \lambda(\mu)$ is the running coupling constant. Different dependencies of λ on ϕ may change the behavior of $V(\phi)$ in a dramatic fashion by e.g. forcing it to develop a minimum at $\phi \neq 0$.

Exercise 13.2: *(15 points)* Solve equations in Eq. (2) numerically for values of μ between from $\mu = 100\text{GeV}$ and the Planck scale $\mu = M_{\text{pl}} \sim 10^{18}\text{GeV}$. Explain why the solutions of the system in Eq. (2) depend only on the single ratio of two coupling constants $x = a_\lambda/a_y$ at $\mu = 100\ GeV$. Take $a_y = 1$ and for several different values of x plot $\lambda(\mu)$ as a function of μ. Show that behavior at large values of μ is very sensitive to initial values of the coupling constants.

Exercise 13.3: *(25 points)* In more complicated theories such as the Standard Model, it is possible that the scalar potential develops a new minimum at finite values of ϕ. There is a narrow region of input parameters when such a situation to become possible. Our goal is to study this high-sensitivity phenomenon with a system of renormalization group equations that are similar to equations in the full SM

$$\mu\frac{\partial a_\lambda}{\partial\mu} = (24a_\lambda^2 + 12a_\lambda a_t - 6a_t^2) + (312a_\lambda^3 + 144a_\lambda^2 a_t - 80a_\lambda a_s a_t + 3a_\lambda a_t^2 + 32a_s a_t^2 - 30a_t^3), \tag{3a}$$

$$\mu\frac{\partial a_t}{\partial\mu} = (9a_t^2 - 16a_s a_t) - (24a_t^3 - 12a_\lambda^2 a_t + 216a_s^2 a_t + 24a_\lambda a_t^2 - 72a_s a_t^2), \tag{3b}$$

$$\mu\frac{\partial a_s}{\partial\mu} = -14a_s^2 - (52a_s^3 + 4a_s^2 a_t), \tag{3c}$$

where $a_\lambda = \lambda/(16\pi^2), a_t = y_t^2/(16\pi^2), a_s = g_s^2/(16\pi^2)$ are the appropriately normalized self-coupling of the Higgs field, the top quark Yukawa coupling and the strong coupling constant.

Solve RG equation (3) numerically and plot $\lambda(\mu)$ in the range between $\mu = M_t$ and Planck scale $\mu = M_{\rm pl} \sim 10^{18}\mathrm{GeV}$. As input conditions use

$$\lambda(M_t) = 0.136493, \quad y_t(M_t) = 0.976427, \quad g_s(M_t) = 1.164602, \tag{4}$$

where vlaue of y_t corresponds to the top quark mass $M_t = 170\mathrm{GeV}$. Make similar plots for $y_t = 1.14874$ and $y_t = 1.20617$ corresponding to $M_t = 200\mathrm{GeV}$ and $M_t = 210\mathrm{GeV}$ respectively. Notice that a relatively small change in input parameters leads to significant changes in the high-energy behavior of the Standard Model potential.

In complicated theories like the Standard Model, different effects become competing, and it becomes crucial to reduce errors in input parameters used for analysis and include higher-order corrections to evolution equations (3), for more details see[1, 2].

[1] F. Bezrukov, M. Y. Kalmykov, B. A. Kniehl and M. Shaposhnikov, JHEP **10** (2012), 140 doi:10.1007/JHEP10(2012)140 [arXiv:1205.2893 [hep-ph]].

[2] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia, JHEP **08** (2012), 098 doi:10.1007/JHEP08(2012)098 [arXiv:1205.6497 [hep-ph]].