Au 9.

$$(A|b) \sim \left(\begin{array}{ccc|c} 3 & 4 & 2 & \alpha \\ 1 & 1 & \alpha & 2 \\ 2 & 3 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & \alpha & 2 \\ 2 & 3 & -1 & 1 \\ 3 & 4 & 2 & \alpha \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & \alpha & 2 \\ 0 & 1 & -1-2\alpha & -3 \\ 0 & 1 & 2-3\alpha & \alpha-6 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & \alpha & 2 \\ 0 & 1 & -1-2\alpha & -3 \\ 0 & 0 & 3-\alpha & \alpha-3 \end{array}\right) \overset{\alpha\neq 3}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & \alpha & 2 \\ 0 & 1 & -1-2\alpha & -3 \\ 0 & 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2+\alpha \\ 0 & 1 & 0 & -4-2\alpha \\ 0 & 0 & 1 & -1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6+3\alpha \\ 0 & 1 & 0 & -4-2\alpha \\ 0 & 0 & 1 & -1 \end{array}\right) \Rightarrow \alpha \neq 3 \quad x = \begin{pmatrix} -1 \\ 6+3\alpha \\ -4-2\alpha \end{pmatrix}$$

$\mathrm{Bild}(A) = \mathbb{R}^3 \Rightarrow$

$\mathrm{Kern}(A) = \Theta$

$$\alpha = 3: \left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & -7 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \dim(\mathrm{Bild}(A)) = 2$$

$$\Downarrow$$

$$x_3^0 = 0,\ x_2^0 = -3,\ x_1^0 = 5$$

$$\vec{x} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} + \mathrm{Lin}_{\mathbb{R}}\left(\begin{pmatrix} -10 \\ 7 \\ 1 \end{pmatrix}\right)$$

$$\mathrm{Bild}(A) = \mathrm{Lin}_{\mathbb{R}}\left(\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}\right) = \mathrm{Lin}_{\mathbb{R}}\left(\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}\right)$$

$$= \mathrm{Lin}_{\mathbb{R}}\left(\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}\right)$$

$$\mathrm{Kern}(A) = \mathrm{Lin}_{\mathbb{R}}\left(\begin{pmatrix} -10 \\ 7 \\ 1 \end{pmatrix}\right)$$

oder: $\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & -7 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$1x_2 - 7x_3 = -3 \Rightarrow x_2 = -3 + 7c$

$x_1 + x_2 + 3x_3 = 2 \Rightarrow x_1 = 2 - 3c + 3 - 7c = 5 - 10c$

$$\Downarrow$$

$$x = \begin{pmatrix} 5-10c \\ -3+7c \\ c \end{pmatrix} \quad c \in \mathbb{R}$$

Au 10.

(1) $a_0+a_1x+a_2x^2+a_3x^3 = b_0+b_2-b_3+x(b_1-2b_2+3b_3)+x^2(b_2-3b_3)+b_3x^3$

(2) $a_0+a_1x+a_2((x-1)+1)^2+a_3((x-1)+1)^3 = a_0+a_1x+a_2[(x-1)^2+2x-2+1]+a_3[(x-1)^3+3(x-1)^2+$
$+3(x-1)+1] = a_0-a_2-2a_3+x(a_1+2a_2+3a_3)+(x-1)^2(a_2+3a_3)+(x-1)^3 a_3 = b_0+b_1x+b_2(x-1)^2+b_3(x-1)^3$

(1) $\Rightarrow a_0 = b_0+b_2-b_3$
$a_1 = b_1-2b_2+3b_3$
$a_2 = b_2-3b_3$
$a_3 = b_3$

$\Downarrow$

$$\underbrace{\begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{B} b = a$$

(2) $\Rightarrow b_0 = a_0-a_2-2a_3$
$b_1 = a_1+2a_2+3a_3$
$b_2 = a_2+3a_3$
$b_3 = a_3$

$\Downarrow$

$$\underbrace{\begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{A} a = b$$

A, B müssen invertbar sein, weil sie Basiswechsel beschreiben

$$AB = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad BA = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow B = A^{-1}, A = B^{-1}$$

Au 11.

a, b, c lin. unabhängig $\iff (\alpha_1 a + \alpha_2 b + \alpha_3 c = 0 \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0)$

$$\begin{aligned} \beta_1 + 4\beta_2 + 4\beta_3 &= 0 \\ -2\beta_1 - \beta_2 + 13\beta_3 &= 0 \\ \beta_1 - \beta_2 - 11\beta_3 &= 0 \end{aligned} \iff \begin{pmatrix} 1 & 4 & 4 \\ -2 & -1 & 13 \\ 1 & -1 & -11 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & 4 \\ 0 & 7 & 21 \\ 0 & -5 & -15 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & 4 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 4 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \beta_3 = c,\ \beta_2 = -3c,\ \beta_1 = 8c$$ $\Rightarrow$ Vektoren sind nicht lin. unabhängig.

Au 12.

Die Frage ist, ob $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{C}$ existiert so, dass

(1) $\alpha_1 a + \alpha_2 b + \alpha_3 c = x$

(2) $\beta_1 a + \beta_2 b + \beta_3 c = w$

(1)
$$\begin{aligned} \alpha_1 + 7\alpha_2 + 3\alpha_3 &= 0 \\ \alpha_1 + (-8)\alpha_2 + (-2)\alpha_3 &= 5 \\ 2\alpha_1 + 7\alpha_2 + \alpha_3 &= -7 \end{aligned} \Leftrightarrow \left(\begin{array}{ccc|c} 1 & 7 & 3 & 0 \\ 1 & -8 & -2 & 5 \\ 2 & 7 & 1 & -7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 3 & 0 \\ 0 & -15 & -5 & 5 \\ 0 & -7 & -5 & -7 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 7 & 3 & 0 \\ 0 & -3 & -1 & 1 \\ 0 & 21 & 15 & 21 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 3 & 0 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & 8 & 28 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 0 & -\frac{21}{2} \\ 0 & -3 & 0 & \frac{9}{2} \\ 0 & 0 & 1 & \frac{7}{2} \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{3}{2} \\ 0 & 0 & 1 & \frac{7}{2} \end{array}\right) \Rightarrow \begin{aligned} \alpha_1 &= 0 \\ \alpha_2 &= -\frac{3}{2} \\ \alpha_3 &= \frac{7}{2} \end{aligned}$$

(2)
$$\left(\begin{array}{ccc|c} 1 & 7 & 3 & 2 \\ 1 & -8 & -2 & 4 \\ 2 & 7 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 3 & 2 \\ 0 & -15 & -5 & 2 \\ 0 & -7 & -5 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 3 & 2 \\ 0 & 3 & 1 & -\frac{2}{5} \\ 0 & -21 & -15 & -15 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & 7 & 3 & 2 \\ 0 & 3 & 1 & -\frac{2}{5} \\ 0 & 0 & -8 & \frac{89}{-5} \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 0 & -\frac{187}{40} \\ 0 & 3 & 0 & -\frac{105}{40} \\ 0 & 0 & 1 & \frac{89}{40} \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{29}{20} \\ 0 & 1 & 0 & -\frac{7}{8} \\ 0 & 0 & 1 & \frac{89}{40} \end{array}\right)$$

$$\Rightarrow \begin{aligned} \beta_1 &= \frac{29}{20} \\ \beta_2 &= -\frac{7}{8} \\ \beta_3 &= \frac{89}{40} \end{aligned}$$

Au 13)

1. $\left(\begin{array}{ccc|c} -1 & 1 & -5 & 0 \\ 2 & 1 & 8 & 1 \\ 1 & 2 & 3 & \alpha \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 1 & -5 & 0 \\ 0 & 3 & -2 & 1 \\ 0 & 3 & -2 & \alpha \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 1 & -5 & 0 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & 0 & \alpha-1 \end{array}\right)$

$\alpha = 1 \Rightarrow x \in Lin_{\mathbb{R}}(a, b, c)$

2. $\left(\begin{array}{ccc|c} -1 & -1 & 2 & 1 \\ 2 & 1 & 7 & 2 \\ 1 & 1 & \alpha & -4 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & -1 & 2 & 1 \\ 0 & -1 & 11 & 4 \\ 0 & 0 & 2-\alpha & -3 \end{array}\right) \Rightarrow \alpha - 2 \neq 0$

3. $\left(\begin{array}{ccc|c} 0 & -1 & 2 & 2 \\ 1 & 3 & 2 & -3 \\ 1 & 2 & -4 & \alpha \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & -3 \\ 0 & -1 & 2 & 2 \\ 0 & -1 & -6 & \alpha+3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & -3 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -8 & \alpha+5 \end{array}\right)$

$\Rightarrow \alpha \in \mathbb{R}$

4. $\left(\begin{array}{ccc|c} 2 & 3 & -1 & \alpha \\ 0 & 3 & 1 & 1 \\ 4 & 3 & -3 & 2\alpha+1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 3 & -1 & \alpha \\ 0 & 3 & 1 & 1 \\ 0 & -3 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 3 & -1 & \alpha \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right)$

$\Rightarrow \alpha \notin \mathbb{R}$ Lösung existiert nicht

Lösungen: 1. $\alpha_1 a + \alpha_2 b + \alpha_3 c = x \Rightarrow \alpha_3 = 0 \quad \alpha_2 = \frac{1}{3} \quad \alpha_1 = \frac{1}{3}$

2. $\alpha_1 a + \alpha_2 b + \alpha_3 c = x \Rightarrow \alpha_3 = \frac{-3}{2-\alpha}$; $\alpha_2 = -4 + \frac{-33}{2-\alpha}$; $\alpha_3 = -1 - \alpha_2 + 2\alpha_3$

3. $\alpha_1 a + \alpha_2 b + \alpha_3 c = x \Rightarrow \alpha_3 = -\frac{\alpha+5}{8}$; $\alpha_2 = -2 - \frac{\alpha+5}{4}$; $\alpha_1 = -3 + \frac{\alpha+5}{4} + 6 + \frac{3\alpha+15}{4}$

4. $\alpha_1 a + \alpha_2 b + \alpha_3 c = x \Rightarrow \alpha_1, \alpha_2, \alpha_3$ existieren nicht

Ан 14.

$$a_1 q_1 + a_2 q_2 + a_3 q_3 = 0 \wedge (a_1 \neq 0 \vee a_2 \neq 0 \vee a_3 \neq 0) \Leftarrow$$

$$\begin{pmatrix} 1 & 4 & -2 & 3 & 4 \\ 2 & 4 & -3 & -2 & 3 \\ \alpha & \beta & \gamma & 0 & 11 \end{pmatrix} \sim \begin{pmatrix} 2 & 8 & -4 & 6 & 8 \\ 6 & 12 & -9 & -6 & 9 \\ \alpha & \beta & \gamma & 0 & 11 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 4 & -2 & 3 & 4 \\ 8 & 20 & -13 & 0 & 17 \\ \alpha & \beta & \gamma & 0 & 11 \end{pmatrix} \Rightarrow \begin{matrix} 17\alpha = 88 \\ 17\beta = 220 \\ 17\gamma = -143 \end{matrix}$$