Auf 20)

Abbildung A linear $\iff A(\alpha x+y)=\alpha Ax+Ay$

1) $\phi(\alpha x+y)=\alpha x_1+y_1+2\alpha x_2+2y_2+3\alpha x_3+3y_3=\alpha(x_1+2x_2+3x_3)+y_1+2y_2+3y_3$

$=\alpha\phi(x)+\phi(y)$

linear ✓

$\underbrace{\begin{pmatrix}1 & 2 & 3\end{pmatrix}}_{\phi} \Rightarrow \operatorname{kern}\phi=\operatorname{Lin}_{\mathbb{R}}\left(\begin{pmatrix}3\\0\\-1\end{pmatrix},\begin{pmatrix}2\\-1\\0\end{pmatrix}\right)$

2) $\phi(\alpha x+y)=0=\alpha\cdot 0+0=\alpha\phi(x)+\phi(y)$

linear ✓

$\underbrace{\begin{pmatrix}0 & 0 & 0\end{pmatrix}}_{\phi} \Rightarrow \operatorname{kern}\phi=\mathbb{R}^3$

3) $\phi(\alpha x+y)=|\alpha x_1+y_1|\neq\alpha|x_1|+|y_1|$ z.B. $|-1+1|\neq 2$

nicht linear

4) $\phi(\alpha x+y)=\operatorname{Re}(\alpha x_1+y_1)=\operatorname{Re}(\alpha x_1)+\operatorname{Re}(y_1)=\operatorname{Re}\alpha\operatorname{Re}x_1-\operatorname{Im}\alpha\operatorname{Im}x_1+\operatorname{Re}y_1$

$\Rightarrow$ linear für $\alpha\in\mathbb{R}$, nicht linear für $\alpha\in(\mathbb{C}\setminus\mathbb{R})$

Auf 21)

1) $A(\alpha x+y)=\begin{pmatrix}\alpha x_1+y_1\\ \operatorname{Im}(\alpha(x_2+x_3)+y_2+y_3)\end{pmatrix}=Ay+\begin{pmatrix}\alpha x_1\\ \operatorname{Im}\ \alpha(x_2+x_3)\end{pmatrix}\neq Ay+\alpha\begin{pmatrix}x_1\\ \operatorname{Im}(x_2+x_3)\end{pmatrix}$

$\operatorname{Im}\alpha w=\operatorname{Im}\alpha\operatorname{Re}w+\operatorname{Re}\alpha\operatorname{Im}w$

nicht linear

2) $A(\alpha x+y)=\begin{pmatrix}2\alpha x_2+2y_2+i\alpha x_3+iy_3\\ \alpha x_1+\alpha x_2+\alpha x_3+y_1+y_2+y_3\end{pmatrix}=\alpha Ax+Ay$

linear

${}_{\mathcal{E}_2}A^{\mathcal{E}_3}=\begin{pmatrix}0&2&i\\1&1&1\end{pmatrix}\Rightarrow\operatorname{rang}A=2\qquad\ker A=\operatorname{Lin}_{\mathbb{C}}\left(\begin{pmatrix}2-i\\i\\-2\end{pmatrix}\right)$

3) $A(\alpha x+y)=\begin{pmatrix}\alpha x_2+y_2-2(\alpha x_3+y_3)^2\\ \alpha x_1+y_1\end{pmatrix}=Ay+\alpha Ax+\begin{pmatrix}-2\alpha x_3y_3-2\alpha^2x_3^2+2\alpha x_3^2\\0\end{pmatrix}$

nicht linear

4) $A(\alpha x+y)=\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}0\\0\end{pmatrix}=\alpha Ax+Ay$

${}_{\mathcal{E}_2}A^{\mathcal{E}_3}=\begin{pmatrix}0&0&0\\0&0&0\end{pmatrix}\Rightarrow\operatorname{rang}A=0\quad\ker A=\mathbb{C}^3$

Auf 22)

1) $Ax = \begin{pmatrix} 2x_2 + ix_3 \\ x_1 + x_2 + x_3 \end{pmatrix} = \begin{pmatrix} 0 & 2 & i \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $\qquad {}_{\mathcal{E}_2}A^{\mathcal{E}_3} = \begin{pmatrix} 0 & 2 & i \\ 1 & 1 & 1 \end{pmatrix}$

2) ${}_{\mathcal{E}_2}A^{\mathcal{X}} = {}_{\mathcal{E}_2}A^{\mathcal{E}_3}\,{}_{\mathcal{E}_3}U^{\mathcal{X}}$

Matrix von 1 $\qquad$ Basiswechsel in $\mathbb{C}^3$ von der Basis $\mathcal{X}$ zu der Standardbasis

$x^1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = e_1 + e_3 \qquad x^2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = e_1 - e_2 + e_3 \qquad x^3 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = -e_2 + e_3$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathcal{E}^3} = \underbrace{\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}}_{{}_{\mathcal{E}_3}U^{\mathcal{X}}} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{pmatrix}_{\mathcal{X}}$$

$${}_{\mathcal{E}_2}A^{\mathcal{X}} = \begin{pmatrix} 0 & 2 & i \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} i & -2+i & -2+i \\ 2 & 1 & 0 \end{pmatrix}$$

3) ${}_{\mathcal{Y}}A^{\mathcal{E}_3} = {}_{\mathcal{Y}}U^{\mathcal{E}_2}\,{}_{\mathcal{E}_2}A^{\mathcal{E}_3}$

$$\left(\begin{array}{cc|cc} i & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & -i & -i & 0 \\ 0 & 1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -i & i \\ 0 & 1 & 0 & 1 \end{array}\right)$$

$${}_{\mathcal{Y}}A^{\mathcal{E}_3} = \begin{pmatrix} -i & i \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2 & i \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} i & -i & 1+i \\ 1 & 1 & 1 \end{pmatrix}$$

4) ${}_{\mathcal{Y}}A^{\mathcal{X}} = {}_{\mathcal{Y}}U^{\mathcal{E}_2}\,{}_{\mathcal{E}_2}A^{\mathcal{E}_3}\,{}_{\mathcal{E}_3}U^{\mathcal{X}} = {}_{\mathcal{Y}}A^{\mathcal{E}_3}\,{}_{\mathcal{E}_3}U^{\mathcal{X}} = {}_{\mathcal{Y}}U^{\mathcal{E}_2}\,{}_{\mathcal{E}_2}A^{\mathcal{X}}$

$$\begin{pmatrix} i & -i & 1+i \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+2i & 1+3i & 2i+1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -i & i \\ 0 & 1 \end{pmatrix} \begin{pmatrix} i & -2+i & -2+i \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1+2i & 3i+1 & 1+2i \\ 2 & 1 & 0 \end{pmatrix}$$

Auf 23)

$p_1(x) = -1+x \qquad l_1(x) = 1$

$p_2(x) = 1-x+x^2 \qquad l_2(x) = x$

$p_3(x) = x-x^2 \qquad l_3(x) = x^2$

$(Ap_1)(x) = -1+x+1 = x$

$(Ap_2)(x) = 1-(1+x)+(1+x)^2 = -x+1+2x+x^2 = 1+x+x^2$

$(Ap_3)(x) = (1+x)-(1+x)^2 = 1+x-1-2x-x^2 = -x-x^2$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathcal{E}_3} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathcal{P}} \qquad {}_{\mathcal{E}_3}A^{\mathcal{P}} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix}$$

$A\,p(x) = q(x) = a_1 l_1(x) + a_2 l_2(x) + a_3 l_3(x)$

$p(x) = b_1 p_1(x) + b_2 p_2(x) + b_3 p_3(x)$

$${}_{\mathcal{E}_3}U^{\mathcal{P}} = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right) \Rightarrow {}_{\mathcal{P}}U^{\mathcal{E}_3} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$${}_{\mathcal{P}}A^{\mathcal{P}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 3 & -2 \\ 1 & 2 & -1 \end{pmatrix}$$

Auf 24)

$${}_{\mathcal{E}_1}\phi^{\mathcal{E}_3} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \qquad {}_{Y}\mathcal{U}^{\mathcal{E}_1} = -\frac{1}{3} \leftarrow y = -3e_1 \Leftrightarrow -\frac{1}{3}y = e_1$$

$${}_{Y}\mathcal{U}^{\mathcal{E}_1}\,{}_{\mathcal{E}_1}\phi^{\mathcal{E}_3}\,{}_{\mathcal{E}_3}\mathcal{U}^{X} = \begin{pmatrix} -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 4 & -2 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{4}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$