Stability Analysis of an Inverting OPAMP Amplifier (Shunt-Shunt Feedback):

General model of the feedback system:

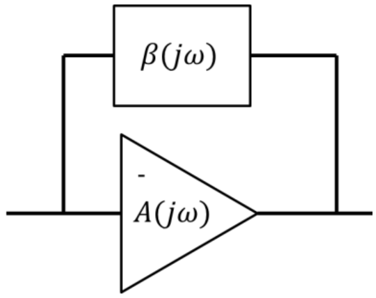


Fig. 1 General model of the feedback amplifier

With the overall transfer function:

$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)} \tag{1}$$

We would like to apply this to the inverting amplifier:

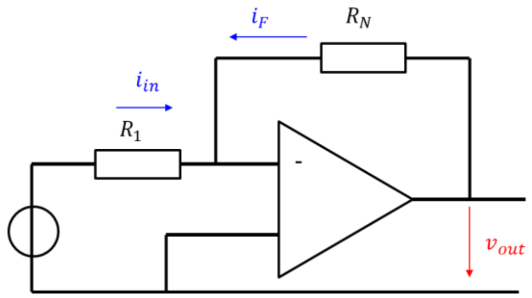


Fig. 2 Inverting OPAMP Amplifier

An appropriate way to model this amplifier would be to consider that the feedback circuit (R_N) samples the output voltage, and feeds back a current that is subtracted from the input current. If we redraw the circuit accordingly:

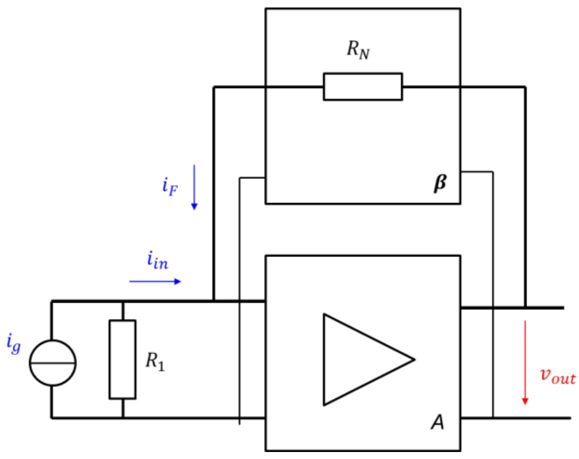


Fig. 3 Inverting OPAMP Amplifier modified drawing

In Fig. 3, the inverting amplifier from Fig. 2 is redrawn, clearly indicating the separation between the A and β circuits. Notice that the input source is also changed from a voltage source to a current source to match this drawing. In this case the closed loop gain will be:

$$A_f = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)} = \frac{v_{out}}{i_g} \tag{2}$$

Also notice that the β-circuit is connected in parallel to the A circuit both at the input and output; therefore, this kind of feedback is called a parallel-to-parallel feedback or a **shunt-shunt** feedback topology.

For the open-loop analysis, we will replace the β-circuit with its two-port representation using y-parameters. Y-parameters are especially suited for analyzing circuits with shunt connections (a shunt connection in admittance domain simply becomes addition).

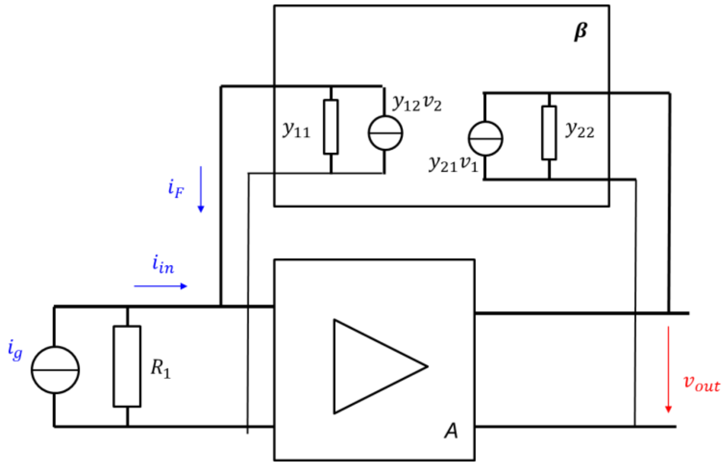


Fig. 4 Inverting OPAMP Amplifier modified drawing using y-parameters

In Fig. 4, we notice that the feedback network will be acting in both directions, from the output to the input as expected, but also naturally from the input to the output. As the amplification of the circuit is very high, we can easily assume that the feedback from the output to the input is much more significant than the coupling from the input to the output, therefore we will neglect the forward path through the feedback circuit, which results in the following:

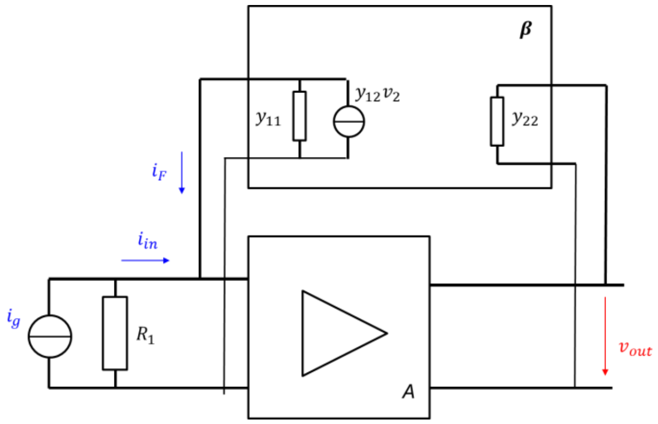


Fig. 5 Inverting OPAMP Amplifier modified drawing using y-parameters, forward path neglected

Now all that remains is to determine the y-parameters of the β-circuit, which simply consists of a single resistor, R_N. Therefore, it takes only a few simple steps to determine these parameters:

$$\begin{aligned} i_1 &= y_{11}v_1 + y_{12}v_2 \\ i_2 &= y_{21}v_1 + y_{22}v_2 \end{aligned} \tag{3}$$

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = 1/R_N \tag{4}$$

$$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = 1/R_N \tag{5}$$

$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -1/R_N \tag{6}$$

Following these steps we can redraw the circuit:

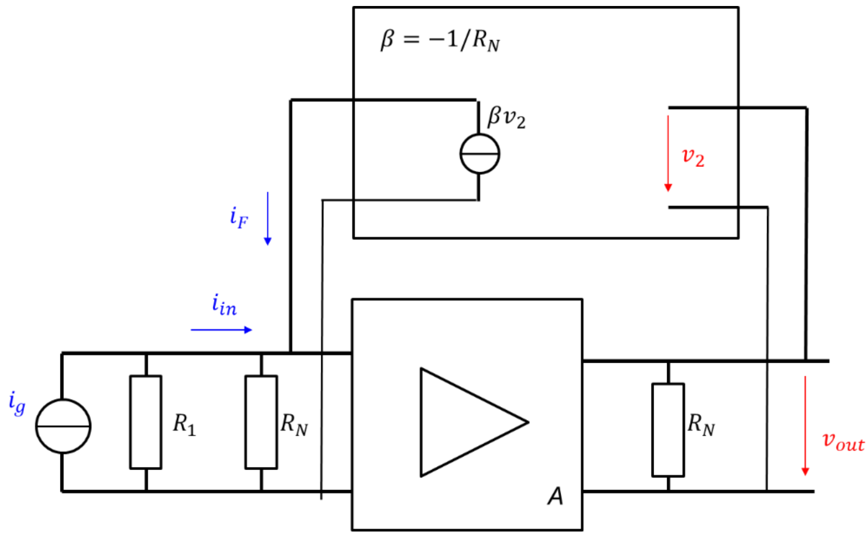
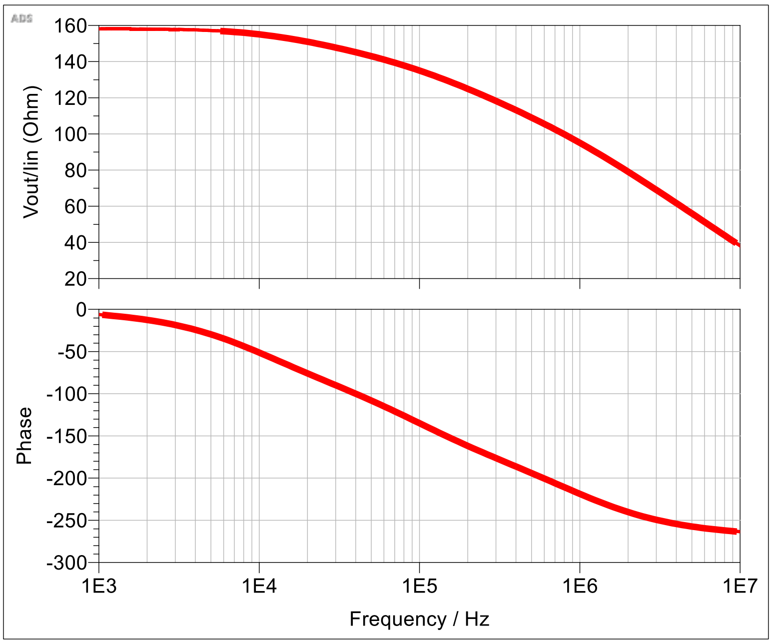


Fig. 6 Inverting OPAMP Amplifier modified drawing using y-parameters, forward path neglected

As can be seen in Fig. 6, the input and output admittances of the β-circuit are absorbed into the input and output terminals of the A-circuit, resulting in a very simple feedback loop, making open-loop analysis now straight-forward. For the open-loop analysis, all we need to do right now is to eliminate β and acquire the Bode plots of the open-loop circuit. This is shown in Fig. 7.



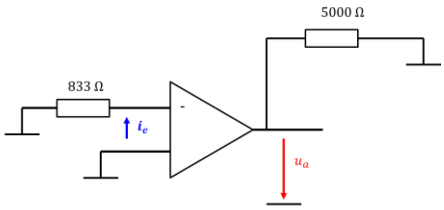


Fig. 7 Bode plots of the open-loop gain of the inverting amplifier with an OPAMP (pole locations: 10 kHz, 100 kHz, 1 MHz) and $R_1 = 1000\ \Omega$, $R_N = 5000\ \Omega$).

Notice that the unit is in Ω. The feedback circuit has a unit of $1/\Omega$, making the loop gain ($A\beta$) unitless. For stability analysis, we are interested in the magnitude condition $|A\beta| = 1$ or $|A| = |1/\beta|$. Therefore, if we plot $|1/\beta|$ on top of this Bode diagram, the intersection will be the point that fulfils the condition for instability. For $R_N = 5000\Omega$, $|1/\beta|$ is also $5000\ \Omega$, which is in logarithmic scale $74\ dB\Omega = 20\log(5000)$.

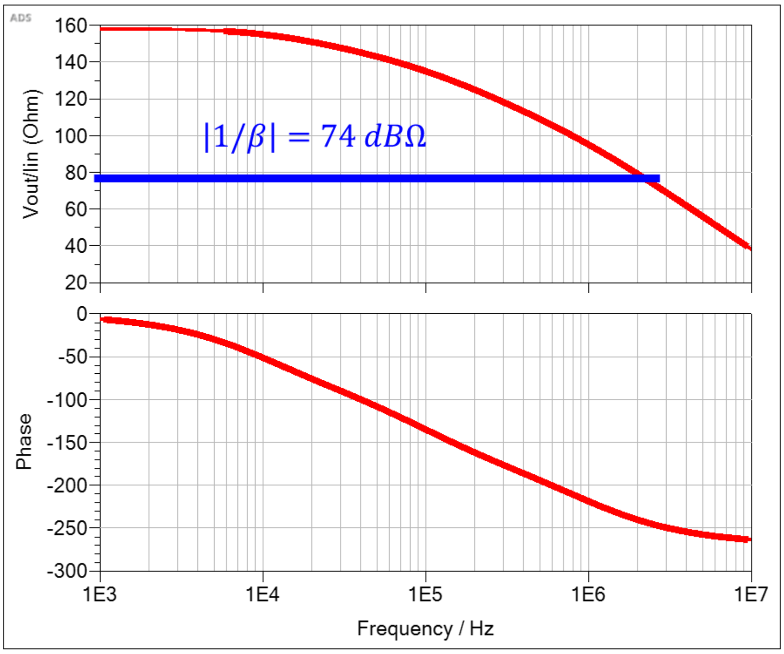


Fig. 8 Bode plots of the open-loop gain of the inverting amplifier with an OPAMP (pole locations: 10 kHz, 100 kHz, 1 MHz) and $R_1 = 1000\ \Omega$, $R_N = 5000\ \Omega$).

By observing the phase at the intersection point, we can determine the phase margin of the loop gain. In this case the phase exceeds 180°, therefore, this circuit in this configuration will be unstable.

Stability Analysis of a Non-Inverting OPAMP Amplifier (Series-Shunt Feedback):

We apply the same procedure for a non-inverting amplifier.

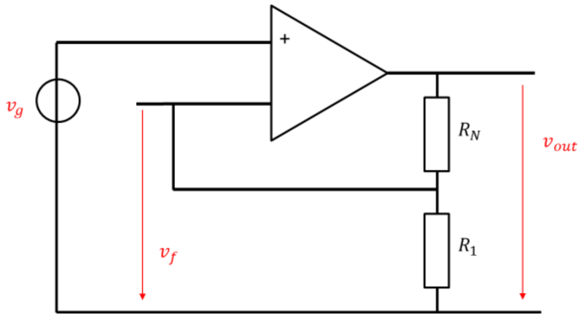


Fig. 9 Non-Inverting OPAMP Amplifier

For the non-inverting amplifier, the convenient way to model the β-circuit is that it samples the output voltage and feeds a voltage back to the negative terminal of the OPAMP. This kind of feedback topology is a **series-shunt** configuration. Now, the voltage transfer characteristics of the β-circuit becomes of interest, for which we employ the h-parameters, while neglecting the forward coupling through the feedback circuit as done previously:.

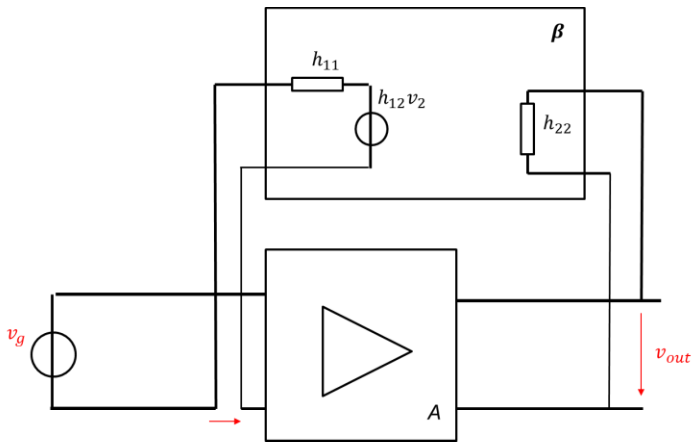


Fig. 10 Non-Inverting OPAMP Amplifier using h-parameters, forward path neglected

In a similar fashion, the h-parameters can be calculated in a simple way, as the β-circuit consists of only two resistors.

$$\begin{aligned} v_1 &= h_{11}i_1 + h_{12}v_2 \\ i_2 &= h_{21}i_1 + h_{22}v_2 \end{aligned} \tag{4}$$

$$h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2=0} = R_1 \parallel R_N \tag{5}$$

$$h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1=0} = R_1 + R_N \tag{6}$$

$$h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0} = \frac{R_1}{R_1 + R_N} \tag{7}$$

After this step, we absorb the input and output resistors in the A circuit, simplifying the β-circuit for the open-loop analysis:

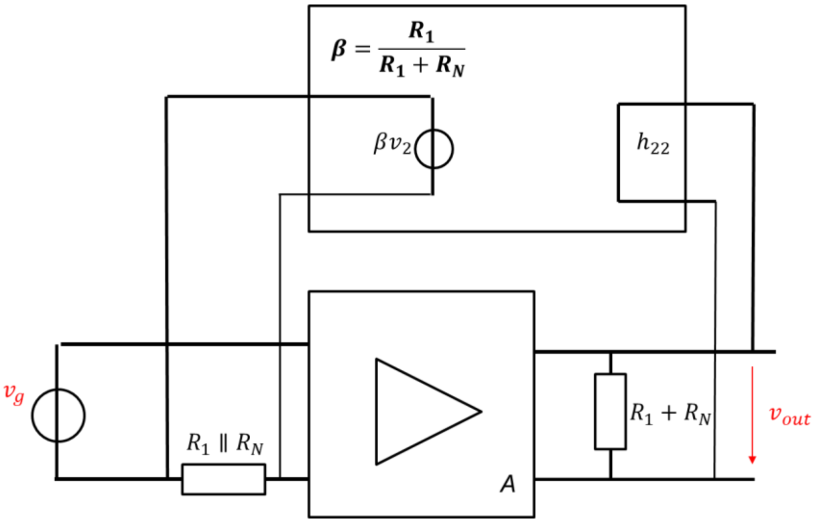


Fig. 11 Non-Inverting OPAMP Amplifier using h-parameters, forward path neglected

For the open loop analysis, all we have to do is to disconnect the β-circuit, and acquire the Bode plots.

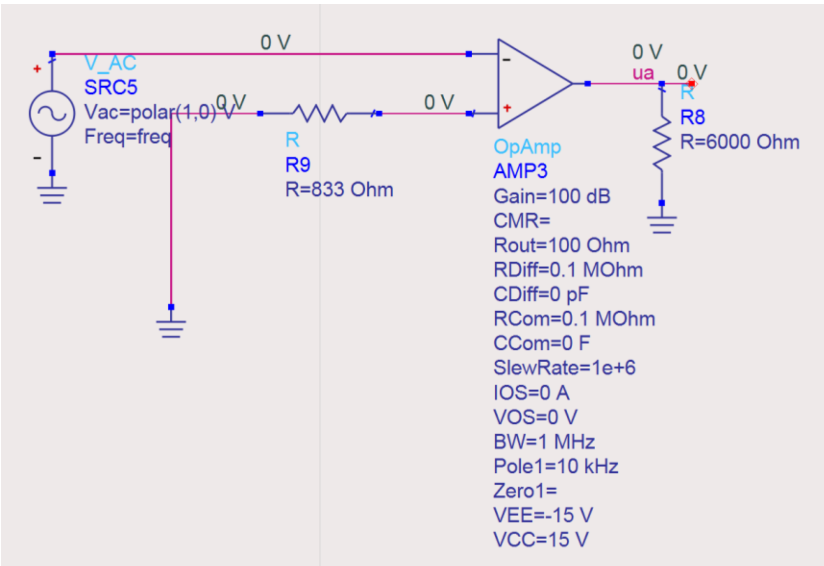


Fig. 12 Open-Loop simulation model of the non-inverting OPAMP with $R_1 = 1000\ \Omega$ and $R_N = 5000\ \Omega$ (A=+6)

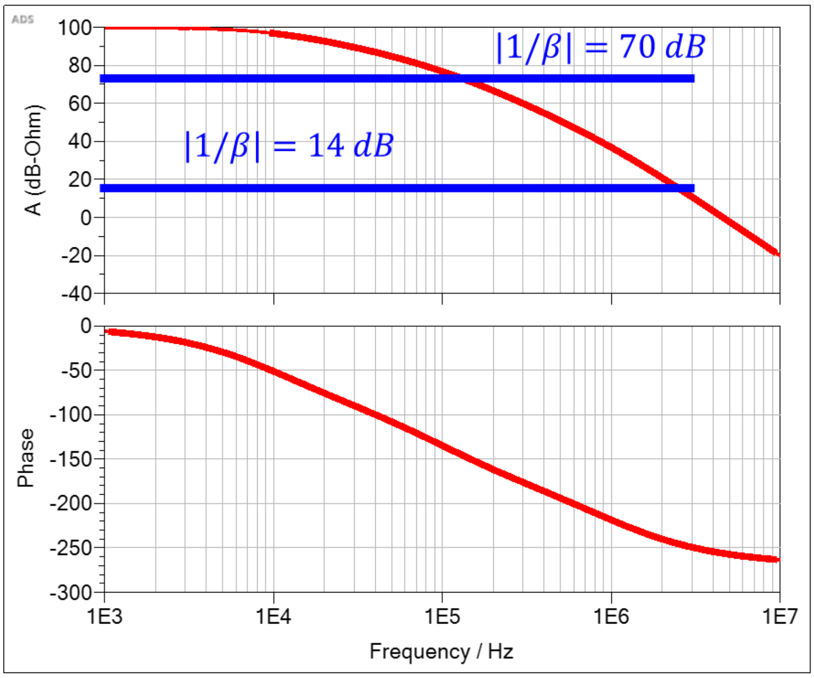


Fig. 13 Bode plots of the open loop circuit

At the intersection we see that the phase of the open-loop circuit exceed 180°, meaning the circuit in this configuration ($R_N = 5k\Omega, R_1 = 1k\Omega, \frac{1}{\beta} = 14dB$) will be unstable. In the same figure a stable condition is plotted as well for ($R_N = 3\ M\Omega, R_1 = 1k\Omega, \frac{1}{\beta} = 70dB$)

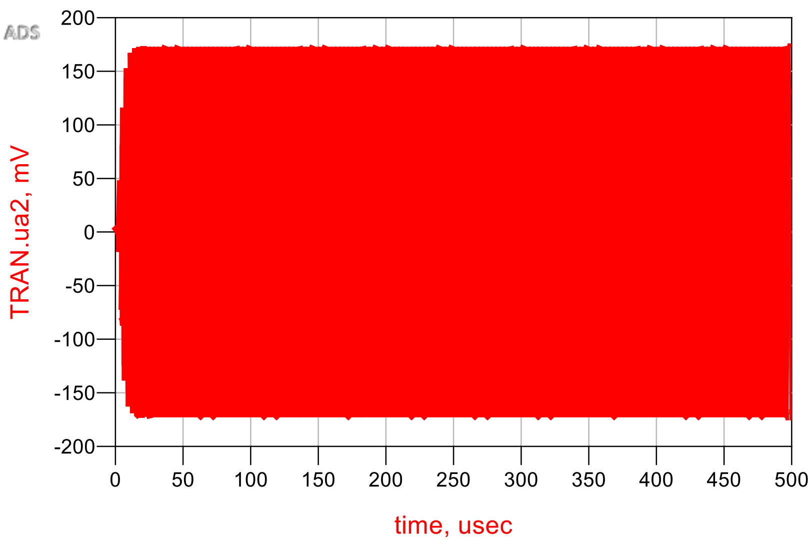


Fig. 14 Step response (1 µV step) of the non-inverting amplifier for ($R_N = 5k\Omega, R_1 = 1k\Omega, \frac{1}{\beta} = 14dB, A_f = +6$)

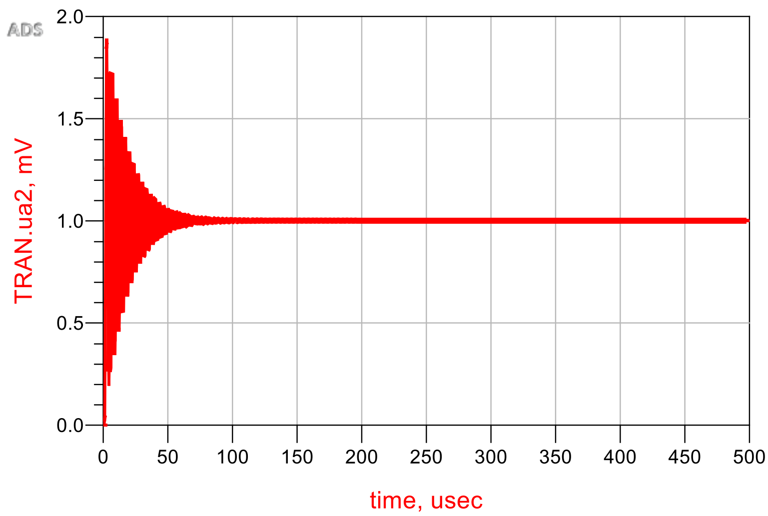


Fig. 15 Step response (1 µV step) of the non-inverting amplifier for ($R_N = 3\ M\Omega, R_1 = 1k\Omega,\ \frac{1}{\beta} = 70dB,\ A_f = +1000$)