P4 SS 2012

Semiconductor spectroscopy

Marco A. Harrendorf und Thomas Keck marco.harrendorf@googlemail.com, t.keck@online.de Group: 1 Karlsruhe Institut of Technology, Bachelor Physik

Date: 18.06.2012

Contents

1	Basics		3
	1.1	Explanation of semiconductors based on the energy-band model	3
	1.2	Effective mass of electrons and defect electrons	4
	1.3	Transmission and absorption in matter	5
	1.4	Fabry-Perot-Interferences in thin layer	5
	1.5	State densities	6
	1.6	Excitons	7
	1.7	Creation of semiconductur structures with reduced dimension	8
		1.7.1 Quantum well	9
		1.7.2 Quantum dot	9
2	Experiments 10		
	2.1	Scope of work	10
	2.2	Experimental setup	10
	2.3	Exercise 1	11
	2.4	Exercise 2	11
	2.5	Exercise 3	14
	2.6	Exercise 4	15
	2.7	Uncertainties of the experiment	17
Re	References		

1 Basics

1.1 Explanation of semiconductors based on the energy-band model

For the understanding of semiconductors fundamental knowledge of the energy-band model is essential.

The energy-band model describes the quantum-mechanical energy state of electrons in an ideal single-crystal with a big amount of periodically arranged atoms. While a single atom posses an energy spectrum with discrete energy levels the big amount of atoms in the crystal results in an interaction between the energy levels of the single atoms and thus a split-up of the energy levels in a energy band.

The highest completely occupied energy band at temperature T = 0 K is called valence band and the next higher band is called conduction band.

The conduction band of conductors is characterized by its not completely emptiness. For example in a monovalent metal the conduction band is half-filled and in multivalent metals the outer energy bands overlap each other. Thus the energy gap between the conduction and valence band is very small and electrons can easily change in a higher energy state and for this reason electrons can nearly freely move inside the conduction band of the metal. This is the reason why metals are good conductors. It should also be noted that by increasing the temperature of the metal the scattering of the electrons in the crystal is increased resulting in a smaller average velocity of the electrons and a smaller conductivity.

The energy gap between the conduction and valence band of insulators is greater than approximately 3 eV, so that only a negliglible amount of electrons are thermally excited and reach the conduction band. For this reason the specific resistance of an insulator is high.

The energy gap of semiconductors lays with approximately 1 eV to 3 eV between the energy gap of the conductors and insulators. In contrast to the insulators the energy gap is small enough that electrons can reach the conduction band by thermally excitation while creating a hole in the valence band. The hole or so-called defect electron can be filled by an other nearby electron resulting in a movement of the hole in the valence band, the so-called p-type conduction. Please note that the defect electron is a quasi particle and is used for the reason, that the movement of a hole can be easilier described than the movement of the many nearby electrons. The properties like charge, spin or impuls of the defect electron are totally opposite to its corresponding electron. Due to the thermally excited electrons in the conduction band and the defect electrons in the valence band semiconductors possess a small intrinsic conduction. Beside the thermally excitation doping of semiconductors is technically used to provide semiconductors with a given amount of charge carriers. For example doping of a quadrivalent crystal with a small amount of trivalent atoms or rather pentavalent atoms results in an acceptor niveau above the valence band and accordingly a donator niveau below the conduction band. By excitation of electrons from the valence band into the acceptor niveau defect electrons inside the valence band are created, while by excitation of electrons from the donator niveau into the conduction band these electrons can directly contribute to the occuring current conduction. Furthermore the excitation of electrons from the valence band to the conduction band can be caused by photons which is called photon induced conduction.

The figure 1 shows schematically the structure of the energy band for the three different material

types.





1.2 Effective mass of electrons and defect electrons

To characterize the electrons and defect electrons in crystals the effective mass is used. One can show that particles like electrons or defect electron react to electrical or magnetic fields in nearly the same manner as free particles in vacuum when one takes a modified particle mass into account.

In analogy to the second newton law

$$F = m \cdot a$$

a quantum mechanic description of a crystal electron in an outer electrical field E yields the following equation of motion:

$$a = \frac{1}{\hbar^2} \frac{\partial^2 E(k)}{\partial k^2} \cdot q \cdot E$$

In contrast to this the equation of motion of a free electron in vacuum is

$$a = \frac{q}{m} \cdot E$$

The effective mass m^* of the crystal electron (or other particle) follows from the comparison of both above-mentioned equations and is defined by the curvature of the corresponding energy band:

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{\partial^2 E(k)}{\partial k^2}$$

One often finds in cubic semiconductors that the valence band at k = 0 is divided into two partial energy bands due to the spin orbital interaction. Because of the different curvature of the two energy bands the effective mass of defect electrons in these bands are different and for this reason they are called heavy (hh) and light (lh) holes.

1.3 Transmission and absorption in matter

When an electromagnetic wave encounters a material so either transmission, reflection or absorption occur depending on the material properties.

The transmission through a specific material is given by the transmission coefficient in which the transmission coefficient T is defined by the quotient of the input light intensity I_0 and the transmitted light intensity I_T

$$T = \frac{I_T}{I_0}$$

The numerical values of the transmission coefficient T lay in the range from 0 to 1. In the same way the reflection coefficient R is defined by the quotient of the reflected light intensity I_R and the input light intensity I_0 :

$$R = \frac{I_R}{I_0}$$

The numerical values of the reflection coefficient R lay also in the range from 0 to 1. When electromagnetic radiation enters a material an amount of the radiation is normally absorbed. The exponential decrease of light intensity depends of the penetration thickness x and the absorption coefficient of the material α . For this reason the light intensity I(x) as a function of the penetration thickness x is proportional to:

$$I(x) \propto e^{-\alpha \cdot x}$$

Due to the energy band structure of semiconductors absorption in a semiconductor should only occur when the energy of the penetrating photons is at least sufficient to excite electrons from the valence band to the conduction band. By measuring the minimum photon energy at which abruptly the absorption of light occurs (this is the so-called absorption edge due to the abruptly step-up in the absorption spectrum) it is possible to determine the energy gap between the valence band and conduction band. But also absorption of photons with less energy can occur when the photon energy is used to excite a exciton, a quasi particle consisting of an electron and a defect electron. This excitation can be seen as a peak in the absorption spectrum.

1.4 Fabry-Perot-Interferences in thin layer

Fabry-Perot-Interferences occur by the pass of light through an optical resonator like two coplanar mirrors with a high reflection coefficient or also a thin layer of a reflective material. While the passage of the material the light is multiple reflected in the material which works as a resonator and constructive or destructive interference occurs depending on the wave length of the light λ and thickness d of the resonator. When the thickness of the resonator d is exact a multiple of the half wave length of the light which enters the resonator the constructive interference is maximal and thus for thin layers the extrema with order l in the transmission spectrum are given by:

$$2nd = \left(m + \frac{l}{2}\right) \cdot \lambda$$
, $l = 0, 1, \dots$, $m = const.$

Within these equation the wave length depending refraction index n is used. By transforming the equation in respect of l follows

$$l = 4d \cdot \frac{n}{\lambda} - 2m \qquad (1)$$

The layer thickness d of the material can now be determined by plotting l over the quotient of refraction index n and wave length λ .

1.5 State densities

The state densities N(k) or rather D(E) specify how many states in a given volume V per k interval or rather energy interval are available.

From the for a state in every spatial direction required space in the k space

 $\frac{\pi}{l}$

follows that the number of states N(k) in a spherical shell with radius k and thickness dk is

$$N(k)dk = \frac{l^3 \cdot 4k^2}{\pi^2}dk$$

We can generalize these equation to the state density of a *d* dimensional system:

$$N(k)dk \propto l^d \cdot k^{d-1} \cdot dk$$

This equation is valid for all free particles and quasi particle which can described by plane waves.

Often it is more useful to describe the state density according to the energy E. By using the following relationship

$$D(E)dE = \frac{N(k(E))}{|v_g|}dE$$

and taken only particles with a quadratic energy impuls relationship given by

$$E(k) = \frac{\hbar^2 k^2}{2m}$$
$$= \frac{p^2}{2m}$$

into account we obtain the state density

$$D(E)dE \propto E^{\frac{d}{2}-1}dE$$

For this reason the state density

- in one dimension is proportional to $\frac{1}{\sqrt{E}}$,
- in two dimensions consists of a series of heaviside functions,
- in three dimensions is proportional to \sqrt{E} .

1.6 Excitons

In figure 2 the excitation of an electron from the valence band VB to the conduction band CB and the energy gap E_g is shown.



Figure 2: Excitation of an electron from the valence band VB to the conduction band CB: Taken from [Vorbereitungshilfe].

As aforementioned in section 1.3 beside the excitation of an electron from the valence band to the conduction band when the energy of the photon is at the least the size of the energy gap E_g the creation of an exciton can occur for photons with less energy.

Excitons are quasi particles consisting of an electron and defect electron with properties similar to a hydrogen atom. In almost the same manner as the "'normal" excitation the electron of an exciton is excited from the valence band to the conduction band creating a defect electron in the valence band but the wave functions of the electron and defect electron are so entangled that both particle form a bounded state. This means that the electron and defect electron cannot move as single particles in the crystal however they can move freely together in the crystal.

Through the bound between both particles the required excitation energy is reduced due to the additional coulomb energy resulting in an energy smaller than the energy gap E_g . The decrease of energy can be described similar to the hydrogen atom by using a quantum number $n_B = 1, \ldots, \infty$. Thus for the energy of the exciton E_X in a semiconductor with a direct energy gap follows:

$$E_X = E_g - \underbrace{E_X^b}_{\text{Rvdberg energy}} \cdot \frac{1}{n_B^2} + \frac{\hbar^2 k^2}{2M}$$

In this connection $k = k_e + k_d$ is the k vector of the electron and defect electron and $M = m_e^* + m_d^*$ is the effective translation mass of both particles.

The figure 3 shows schematically the different energy states of the excitons and their corresponding energy level. Note that above the energy of the energy gap E_g lays the ionisation continuum of the excitons.



Figure 3: The different energy states of the excitons and their corresponding energy level: Taken from [Vorbereitungshilfe].

1.7 Creation of semiconductur structures with reduced dimension

Quantum wells and quantum dots are semiconductor potentials which are limited in one or rather three dimensions. They are created by advanced growing methods by the growing of a thin layer ($\approx 10 \text{ nm}$) of a material with small energy gap between thicker layers of a material with bigger energy gap. The material with the bigger energy gap acts as a potential wall while the material with the small energy gap forms a quantum well.

The quantum wells produce quantized state in the particular spatial direction whereas in other spatial directions the in the quantum well capturated particle can move freely or better its wave function can prolong due to the missing potential walls.

In part a) of figure 4 a single quantum well with potential walls of infinite height is shown. When the height of the quantum walls is not infinite tunneling and of overlapping of wave functions can occur as shown in part b) of figure 4. For this reason it would be ideal to have potential walls with infinite height but walls with sufficient height reduce these effects fair enough so that we can consider the real quantum wells and quantum dots in this laboratory experiment as ideal quantum structures.



Figure 4: A single quantum well with a) infinite height and b) finite height: Taken from [Vorbereitungshilfe].

1.7.1 Quantum well

When we choose the z axis as the quantized axis of the quantum well the energy of the in the two dimensional (ideal) quantum well captured particle is

$$E = \frac{\hbar^2}{2m^*} \left(\frac{\pi^2}{l_z^2} n_z^2 + k_x^2 + k_y^2 \right)$$

in which m^* is the effective particle mass, l_z the thickness of the quantum well or better the thickness of the thin layer and $n_z = 1, 2, \ldots$ the quantum number of the corresponding energy level.

1.7.2 Quantum dot

If the free moving space is further reduced one obtains first a quantum wire and if the free movement is limited in all three spatial directions one obtains a quantum dot. These quantum dot corresponds to a three dimensional potential well and the energy ground state of a particle in a spherical quantum dot with radius R is given by:

$$E=\frac{\hbar^2\pi^2}{2m^*R}$$

2 Experiments

2.1 Scope of work

In this laboratory experiment the properties of semiconductors shall be examined using spectroscopic methods.

The radius of the smallest colloids of a CdS/CdSe probe, the layer thickness of this probe, the energy of the energy gap corresponding to different polarisations in this probe are of special interest in this connection.

Furthermore multiple quantum well systems shall be investigated and the thickness of the thin layer in a two dimensional quantum well as well as the radius of a spherical quantum dot shall be calculated.

2.2 Experimental setup

The experimental setup consists mainly of an optical system in which the light of a halogen lamp is focused on a probe and afterwards a mesh spectrometer is used to gain the transmission spectrum corresponding to the probe. For the cool-down process needed in the later experiments the probe is mounted in a cryostat.

The transmission spectrum is processed by a computer program which directly supplies the transmission spectrum adjusted to background light and to the frequency depending light intensity of the halogen lamp. For that reason we undertake a measurement without the halogen lamp and a measurement without a probe in the light path previous to each measurement of the probe. The figure 5 shows schematically the experimental setup.



Figure 5: The experimental setup

2.3 Exercise 1

The radius of the smallest colloids in the CdS/CdSe probe can be calculated by the ground state energy of a quantum dot. The effective mass m^* was taken from the [Vorbereitungshilfe].

$$E = \frac{\hbar^2 \pi^2}{2m^* R} \tag{2}$$

$$m^* = 0.2m_e \tag{3}$$

This energy E corresponds to the energy difference E_g between the blueshifted and redshifted absorption edge, which is equal to the energy gap of the material according to the [Vorbereitungshilfe]. The error of the measured absorption edge was estimated with ± 5 nm.

$$\lambda_{blue} = (472 \pm 5) \text{nm} \tag{4}$$

$$\lambda_{red} = (614 \pm 5) \text{nm} \tag{5}$$

$$E_g = hc \left(\frac{1}{\lambda_{blue}} - \frac{1}{\lambda_{red}}\right) = (0.61 \pm 0.03) \text{eV}$$
(6)

$$R = \sqrt{\frac{h}{8m^*E_g}} = (1.76 \pm 0.05) \text{nm}$$
(7)

The measurement is shown in figure 6. The radius of the smallest colloids in the domain of nano meters seems reasonable.



Figure 6: Redshifted and blueshifted absorption edge measurement

2.4 Exercise 2

As mentioned in the basics the thickness of the CdS-crystal can be obtained by using the fabry perot interference equation 1. The refractive index $n(\lambda)$ is according to the [Vorbereitungshilfe]

$$n(\lambda) = 1.94 \sqrt{1 + \frac{\frac{4696}{2\pi c}}{\left(\frac{1}{390}\right)^2 - \left(\frac{1}{\lambda}\right)^2}} \qquad (8)$$

We measured the position of the maximum and minimum light transmission of the probe, see figure 7. We executed this measurement two times, first time for parallel polarisation $\vec{E} \parallel \vec{c}$, where \vec{c} is the optical axis of the material, and the second time for perpendicular polarisation $\vec{E} \perp \vec{c}$. With a linear regression as shown in figure 8 we obtain the thickness *d*:

$$d_{\parallel} = (1970 \pm 15) \text{nm} \tag{9}$$

$$d_{\perp} = (1951 \pm 12) \text{nm} \tag{10}$$

Both results are equal within the error limits. Therefore we can combine both results to get a final result for the thickness of the CdS crystal:

$$d = (1960 \pm 10) \,\mathrm{nm} \tag{11}$$



Figure 7: Minima and maxima of the CdS crystal acting like a fabry perot interferometer.



Figure 8: Linear regression for d according to equation 1.

The energy gap of the CdS crystal can be estimated independently by the absorbation edge of the perpendicular and the parallel orientation:

$$\lambda_{\perp} = (507.2 \pm 5) \text{nm} \tag{12}$$

$$\lambda_{\parallel} = (510.6 \pm 5) \text{nm} \tag{13}$$

This is equal to an energy E_g of:

$$E_{q,\perp} = (2.445 \pm 0.024) \text{eV}$$
 (14)

$$E_{g,\parallel} = (2.428 \pm 0.024) \text{eV}$$
(15)

We can assume that both energies are equal when we take an uncertainty intervall of one sigma into account. As above we combine these values to a final result:

$$E_g = (2.436 \pm 0.017) \text{eV} \tag{16}$$

From the theory of optics the connection between absorption and refractive index is known. The refractive index n is in general a complex number. The imaginary part describes the absorption. It holds:

$$n = n' + in'' \tag{17}$$

$$\alpha = 2n''\frac{\omega}{c} \tag{18}$$

Where α is the absorption index. All variables in the above equations, except c, depend on the wavelength λ .

2.5 Exercise 3



Figure 9: Transmission over wavelength

In figure 9 we see a heavyside function-like curve as expected for a two dimensional state density. Therefore the material forms a two dimensional quantum well, as shown in the basics.

$$\lambda_{lh} = (768.9 \pm 5) \text{nm}$$
 (19)

$$\lambda_{hh} = (781.0 \pm 5) \text{nm}$$
 (20)

From the absorption peaks for heavy and light holes one can calculate the photon energy that was needed for the transition E. This energy corresponds to the energy gap E_g and the energy of a particle in a 2 dimensional quantum well. The following equation is valid for both, light and heavy holes, where m is the effective mass of the hole, and E the photon energy which was necessary to stimulate the transition of this hole.

$$E = E_g + \frac{h^2}{8ml^2} - 13.6 \text{eV} \frac{m}{m_0 \epsilon^2}$$
(21)

The three terms in this equation refer to: The energy gap, the binding energy of the quantum well, the ionisation energy of the exciton. The energy gap and the effective mass for light and heavy holes is known form the [Vorbereitungshilfe].

$$E_g = 1.511 \text{eV} \tag{22}$$

$$m_{hh} = 0.48m_e \tag{23}$$

$$m_{lh} = 0.087 m_e$$
 (24)

So we have two unkown values l and ϵ and two independent values for the light and the heavy holes. This leads to the following formula for l, if we insert equation 21 for light holes into the same equation for heavy holes.

$$l = \frac{h}{2\sqrt{2}} \sqrt{\frac{\frac{m_{lh}}{m_{hh}} - \frac{m_{hh}}{m_{lh}}}{(E_{lh} - E_a)m_{hh} - (E_{hh} - E_a)m_{lh}}}$$
(25)

$$= (6.91 \pm 0.42)$$
nm (26)

The result seems reasonable, as the typical thickness of a quantum well is in the region of nano meters.

2.6 Exercise 4

In our measurement we could not identify the exciton series. Only one peak per measurement was visible. The measurement was taken two times for every sample. The results are shown in the figures 10 and 11.



Figure 10: Cu₂O Absorbance measurement



Figure 11: Second Cu_2O Absorbance measurement

With the formula, where the left side refers to the photon energy ,the first term on the right side to the gap energy, the second on the right side to the binding energy of the exciton level

$$E = E_g - \frac{E_b}{n^2} \tag{27}$$

and two independent results for two different exciton levels one can calculate the energy gap E_g and the rydberg energy E_b of the excitons. However we did not measured two independent values. Only one peak was visible. This peak corresponds to the 2p excitonlevel, because the transition to 1s is forbidden. In higher orders the 1s transition also takes place, this transition can be identified in the thick sample in the figure 10. There are two edges visible, which are caused by the 1s transition. We use the average of the two edges to estimate the photon wavelength E which was needed to induce the transition to the first exciton level. As mentioned this was only possible for the thick sample.

$$\lambda_{1s} = \left(\frac{620 + 612}{2} \pm 5\right) \,\mathrm{nm} \tag{28}$$

$$= (616 \pm 5)$$
nm (29)

The peak of the 2p transition is for this thick sample located at:

$$\lambda_{2p} = (582.1 \pm 5) \text{nm} \tag{30}$$

Now we have two independent transitions and we can calculate E_g and E_b :

$$E_{1s} = (2.01 \pm 0.02) \text{eV} \tag{31}$$

$$E_{2p} = (2.13 \pm 0.02) \text{eV} \tag{32}$$

$$E_g = \frac{4}{3}E_{2p} - \frac{1}{3}E_{1s} \tag{33}$$

$$= (2.17 \pm 0.02) \text{eV}$$
(34)

$$E_b = \frac{4}{3} \left(E_{2p} - E_{1s} \right) \tag{35}$$

$$= (0.16 \pm 0.03) \text{eV} \tag{36}$$

Both the energy gap E_g and the binding energy E_b match the exspected order of magnitude. For example we exspect that the energy gap of a semiconductor is between 1eV and 3eV. Therefore our result seems reasonable.

2.7 Uncertainties of the experiment

Due to the given experimental setup, the transmission and absorption spectra are directly calculated by a computer software using a closed mesh spectrometer, it is quite difficult to declare concrete uncertainties for the measurement results: The results for the transmission and absorption spectra taken from the computer software do not contain any statistical uncertainty even though the computer software seems to use averaging to correct for background light and frequency dependent light of the lamp. Systematical uncertainties should normally only occur in the mesh spectrometer and computer software due to the recalibration before every single measurement but both uncertainties are unknown.

For this reason we cannot calculate advanced uncertainty budgets for the single measurements but we can mention possible sources of error:

Lighting The experiment was setted in a room which contained also other experiments and was not in a darkroom. The room lights were off during the whole measurement time and the mesh spectrometer was calibrated to the background light before every measurement was started. But even with this precaution the ambient light which flooded into the room through the open windows of a neighbouring room seemed to change some times the amount of light in the room and maybe changed the background light.

Another difficulty is the use of a halogen lamp which changes slightly its light intensity due to heating up. For this reason the halogen lamp should be switched on at least one hour before the start of the first measurement according to our advisor. We tried to reduce the effect of the changing lamp light intensity by undertaking a light calibration before the start of every measurement.

Temperature dependance of the measurements A part of the measurements are undertaken at room temperature while the other part is undertaken when the sample temperature is nearly 77 K.

Despite the cooling the transmission and absorption curves were especially in exercise 4 eroded

due to thermically excited phonons and it was difficult to determine single absorption peaks. A measurement of the absorption spectra at room temperature was even impossible. For a better determination of the exciton series in exercise 4 further cooling would be required.

But also from the cooling down problems arise: The thin and thick samples in exercise 4 should be measured at the same temperature. In this exercise we waited until we saw fog around the cryostat before we started the measurement of both samples. In my opinion a not existing thermometer should be used to surely determine if the cool down process has stopped. Otherwise it is not certainty that both sample which are mounted in different heights at the sample holder have the same temperature at the time of measurement. One have to note that the fog around the cryostat must be removed by a small fan which can also be a cause for errors.

Lightpath through the single samples Discrepancies between different measurements of the same sample can occur due to the spacious samples and different possible light paths through the sample. The light path through the sample can be adjusted by an adjusting bolt by lowering or heightening of the cryostat. It would be an improvement if the cryostat would have discrete height positions.

Another problem can arise when the light is not correctly focussed on the sample by using the optical lenses. Then the sample holder can scatter some amount of the light directly to the fiber glass which is connected to the mesh spectrometer.

References

- [Kuiper] Pierre Kuiper: Comparison of the electronic band structures of metals, semiconductors and insulators. http://en.wikipedia.org/wiki/File:Isolator-metal.svg, Reference date: 16.06.2012
- [Vorbereitungshilfe] Fakultät für Physik: Vorbereitungshilfe zum Praktikumsversuch Halbleiterspektroskopie

P4 SS 2012

Semiconductor spectroscopy - Revision

Marco A. Harrendorf und Thomas Keck marco.harrendorf@googlemail.com, t.keck@online.de Group: 1 Karlsruhe Institut of Technology, Bachelor Physik

Date: 18.06.2012

1.6 Exercise 4

In our measurement we could not exactly identify the exciton series at room temperature, because of the thermal fluctuations. At low temperature T = 77K we could identify some peaks of the exciton series, due to the reduce of the thermal noise by cooling with liquid nitrogen. The measurement was taken two times at low temperature for every sample. The results are shown in the figures 1 and 2.



Figure 1: Cu₂O Absorbance measurement



Figure 2: Second Cu_2O Absorbance measurement

With the formula, where the left side refers to the photon energy ,the first term on the right side to the gap energy, the second on the right side to the binding energy of the exciton level

$$E = E_g - \frac{E_b}{n^2} \tag{1}$$

and two independent results for two different exciton levels one can calculate the energy gap E_q and the rydberg energy E_b of the excitons. However we did not measured two independent

values. Only one peak was visible. This peak corresponds to the 2p excitonlevel, because the transition to 1s is forbidden. In higher orders the 1s transition also takes place, this transition can be identified in the thick sample in the figure 1. There are two edges visible, which are caused by the 1s transition. We use the average of the two edges to estimate the photon wavelength E which was needed to induce the transition to the first exciton level. As mentioned this was only possible for the thick sample.

$$\lambda_{1s} = \left(\frac{620 + 612}{2} \pm 5\right) \,\mathrm{nm} \tag{2}$$

$$= (616 \pm 5)$$
nm (3)

The peak of the 2p transition is for this thick sample located at:

$$\lambda_{2p} = (582.1 \pm 5) \text{nm} \tag{4}$$

Now we have two independent transitions and we can calculate E_g and E_b :

$$E_{1s} = (2.01 \pm 0.02) \text{eV} \tag{5}$$

$$E_{2p} = (2.13 \pm 0.02) \text{eV} \tag{6}$$

$$E_g = \frac{4}{3}E_{2p} - \frac{1}{3}E_{1s} \tag{7}$$

$$= (2.17 \pm 0.02) \text{eV}$$
(8)

$$E_b = \frac{4}{3} \left(E_{2p} - E_{1s} \right) \tag{9}$$

$$= (0.16 \pm 0.03) \text{eV} \tag{10}$$

Both the energy gap E_g and the binding energy E_b match the exspected order of magnitude. For example we exspect that the energy gap of a semiconductor is between 1eV and 3eV. Therefore our result seems reasonable.