FAKULTÄT FÜR PHYSIK

PHYSIKALISCHES PRAKTIKUM FÜR FORTGESCHRITTENE

PRAKTIKUM MODERNE PHYSIK

Gruppe Nr. 108 Kurs: ~~Mo~~ **Mi** **WS 2012 / 2013**

Versuch: Parity Violation

Namen: Tobias Renz

Raphael Schmager

Assistent: Alessio Porcelli

durchgeführt am: 07. January 2013

Protokollabgabe am: ______

______

Note gesamt + - 0

Datum: ______

anerkannt: ______

Bemerkung:

Karlsruher Institut für Technologie (KIT) WS 2012/13

Physikalisches Praktikum für Fortgeschrittene 1

Parity Violation

Protokoll von
Tobias Renz und **Raphael Schmager**

Gruppe: **108**

Durchgeführt am 07. January 2013

Inhaltsverzeichnis

1 Aim of the experiment

In this experiment we want to demonstrate the violation of parity with the beta minus decay. Also we will estimate the degree of polarization of bremsstrahlung generated by beta-particles. Finally we want to estimate the longitudinal polarization of electrons emitted during the beta decay.

2 Theoretical Principles

2.1 The parity

The parity operation is the inversion of the coordinates. For a given state with the wave function $\Psi(\vec{r})$ we get:

$$P\Psi(\vec{r}) = \Psi(-\vec{r}) \tag{1}$$

P is the parity operator and Ψ a wave function in position space. For any given state $|a\rangle$ the eigenvalues are given by:

$$P|a\rangle = \pi_a |a\rangle \tag{2}$$

A second inversion gives us the original state:

$$P^2|a\rangle = |a\rangle \tag{3}$$

This brings us the eigenvalues π_a:

$$P^2 = 1 \quad \Leftrightarrow \quad P = P^{-1} \quad \Leftrightarrow \quad \pi_a = \pm 1 \tag{4}$$

We can see that the eigenvalues of the parity operator are ± 1. The eigenvalue $+1$ indicates a symmetric state and -1 describes an asymmetric state. An operator O transforms under parity operations like:

$$POP^{-1} = \pi_O O \tag{5}$$

Let's have a look at the transformation behaviour of an operator in position state:

$$\langle P\vec{R}P\rangle = \langle\vec{r}|\,P\vec{R}P\,|\vec{r}\rangle = \langle\vec{r}|\,P\vec{R}\,|-\vec{r}\rangle = -\vec{r}\,\langle\vec{r}|\,P\,|-\vec{r}\rangle = -\vec{r}\,\langle\vec{r}|\,|\vec{r}\rangle = \langle -\vec{R}\rangle \tag{6}$$

In case of the operators of momentum $\vec{P}$, spin $\vec{S}$ and angular momentum $\vec{L}$ this leads to:

$$P\vec{P}P = -\vec{P} \qquad P\vec{S}P = +\vec{S} \qquad P\vec{L}P = +\vec{L} \tag{7}$$

We can see, that there are operators that change their sign. This shows, that space and momentum are vectors (so called polar vectors) and spin and angular momentum are pseudo-vectors (so called axial vectors).

2.2 Parity violation

The consequence of Noether's theorem is that any symmetry (invariance of an operator by a transformation in symmetry) leads to a conservation law. For example conservation of (angular) momentum or energy.
To proof a theory on parity conservation we had to measure the expected value of a pseudo-scalar. A pseudo-scalar is a quantity that behaves like a scalar, expect that it changes sign under a parity inversion. The expected value of a pseudo-scalar must be 0, otherwise the parity gets violated.
In this very experiment we want to measure the pseudo-scalar of spin $\vec{\sigma}$ and momentum $\vec{p}$ of beta particles. The projection of the electron spin onto the direction of momentum $\vec{p}$ is called helicity:

$$H = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{\sigma}||\vec{p}|} \tag{8}$$

In a parity invariant theory the expected value should be zero:

$$\langle H \rangle = \langle PHP \rangle = \langle -H \rangle \quad \rightarrow \quad \langle H \rangle = 0 \tag{9}$$

By measuring a helicity being different from zero we can proof the parity violation of β^--decay.

2.3 Polarization

The polarization towards a certain axis is given by the ratio of the expectancy-value in this direction and the absolute value of the spin.

$$P = \frac{\langle S_z \rangle}{S} \tag{10}$$

For a particle with spin 1/2 (such as an electron) the spin operator $\vec{S}$ is given by:

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma} = \frac{\hbar}{2}(\sigma_x, \sigma_y, \sigma_z) \tag{11}$$

with the Pauli matrices σ_i. In this notation the basic states are:

$$|\frac{1}{2}, \frac{1}{2}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \textit{and} \quad |\frac{1}{2}, -\frac{1}{2}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{12}$$

They are oriented parallel and anti-parallel to the z-axis.
The general state is a superposition of both:

$$\Psi = \begin{pmatrix} a_+ \\ a_- \end{pmatrix} = a_+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_- \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{13}$$

with: $a_+^2 + a_-^2 = 1$.

This brings us the polarization of on single electron:

$$P = \frac{\langle S_z \rangle}{S} = \langle \sigma_z \rangle = \langle \Psi | \, \sigma_z \, | \Psi \rangle = a_+^2 - a_-^2 \tag{14}$$

Therefore the polarization of the single electron can be between -1 and +1.

- $a_- = 0 \rightarrow$ P=+1 $\rightarrow$ spin is parallel
- $a_+ = 0 \rightarrow$ P=-1 $\rightarrow$ spin is anti-parallel

For the polarization for the entity of all particles we get:

$$P = \frac{N_+ - N_-}{N_+ + N_-} \tag{15}$$

$N_\pm$ represents the number of electrons in the state $|\pm\rangle$. Note that this equation applies only in non relativistic quantum mechanics, where is no coupling between the spin and angular momentum.

If we want to determine the polarization of photons (γ-quanta) we need a formula that is also valid for relativistic quantum mechanics. That's because the photon travels of course with the speed of light. The spin of photons is S=1. The possible spin orientations are parallel or anti-parallel to the propagation direction. Hence the polarization of all the photons together can be written equivalent to (15).

$$P_C = \frac{N_+ - N_-}{N_+ + N_-} \tag{16}$$

N_+ and N_- represents the number right- or left-handed circular polarized electrons. This means: $P_C = 1$ says, that all photons are right-handed circular polarized. Note that a single γ-quantum cannot be linear polarized!

2.4 β^--decay

For this experiment we use a β^--decay, which can be described as:

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{17}$$

In β^-−decay, the weak interaction converts an atomic nucleus into a nucleus with one higher atomic number while emitting an electron (e^-) and an electron antineutrino ($\bar{\nu}_e$).

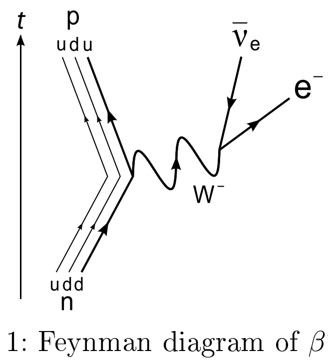

Figure 1: Feynman diagram of β^--decay

3 Measuring principle

3.1 Helicity transfer

Sadly we cannot measure the helicity directly, but we can measure them indircetly with the β^--decay of the electrons.
The electrons of the β^--decay are decelerated in a plumb layer. This generates bremsstrahlung. The brems-quanta have a circular polarization P_C. There will be a partial transfer of the helicity of electrons to the brems-quanta:

$$H = \frac{P_C}{L} \tag{18}$$

A negative helicity transfers to a left-handed polarisation for the photons. The more energy the photons have, the more helicity will be transferred. However, the carry is nearly independent of the electron energy.

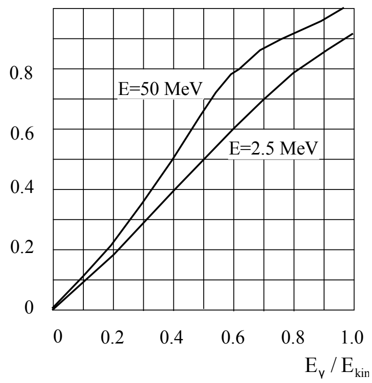

Figure 2: Helicity transfer over photon energy

From the diagram 2 we are able to estimate the helicity transfer L.

3.2 Measuring of circular polarized photons

We see, that the helicity gets transferred to the bremsstrahlung. Now we had to find a way to measure the polarization of the photons. For that we can use the Compton scattering at polarized electrons. As scattering body we use iron, which electrons gets polarized by a magnetic field. The cross section of the Compton scattering is:

$$\frac{d\sigma}{d\theta} = \frac{r_0^2}{2}\left(\frac{k}{k_0}\right)^2 (\Phi_0 + f \cdot P_c \cdot \Phi_c) \tag{19}$$

r_0 is the classical electron radius, k_0 / k is the impulse of the incoming / scattered photon. The degree of polarization is $f = \frac{2}{26}$. That's because only the two valence electrons are polarized.

The variables Φ_0 and Φ_c are given by:

$$\begin{aligned} \Phi_0 &= 1 + \cos^2\theta + (k_0 - k) \cdot (1 - \cos\theta) \\ \Phi_c &= -(1 - \cos\theta) \cdot [(k_0 + k) \cdot \cos\theta \cos\psi + k \sin\theta \sin\psi \cos\phi] \end{aligned} \tag{20}$$

θ is the scatter angle, psi is the angle between $\vec{k}_0$ and the electron spin $\vec{S}$, ϕ is the angle between the $(\vec{k}_0 \cdot \vec{S})$- and $(\vec{k}_0 \cdot \vec{k})$-layer.

Its obvious, that only Φ_c depends of polarization. The reason for that, is the dependency of ψ. If the polarity of the magnetic field gets reversed, the electron spin of the valence electrons turns around. So ψ becomes: $\psi + \pi$. This means:

$$\Phi_c(\psi + \pi) = -\Phi_c(\psi) \tag{21}$$

For the relative count rate E for spin switching we get:

$$E = \frac{N_- - N_+}{N_- + N_+} = f \cdot P_c \cdot \frac{\Phi_c^-}{\Phi_0} \tag{22}$$

with N_+ the number of scattered photons for electron spin parallel to the incoming photon $(0 \leq \psi \leq \frac{\pi}{2})$ and N_- the amount of photons, where the spin is anti-parallel $(\pi \leq \psi\frac{3\pi}{2})$.

With the asymmetry E of the count rates of the Compton scattered photons we can suggest to the circular polarization P_c. With P_c and the helicity transfer L we can compute the helicity H.

The parity violation can be shown by measuring a value unequal zero.

The factor $\frac{\Phi_c^-}{\Phi_0}$ dependence on the experimental set-up. In our case is it:

$$\frac{\Phi_c^-}{\Phi_0} = 0,52 \pm 0,05 \tag{23}$$

4 Experiment

4.1 Set-Up

An emitted electron from the source impact onto the lead-layer. There it generates bremsstrahlung. The bremsstrahlung reaches the iron core of the magnetic coil. There it gets scattered (Compton scattering). Under an angle of 60° the photon reaches a fiber-optic. The fiber optic leads to an scintillator, where the photons can be detected.

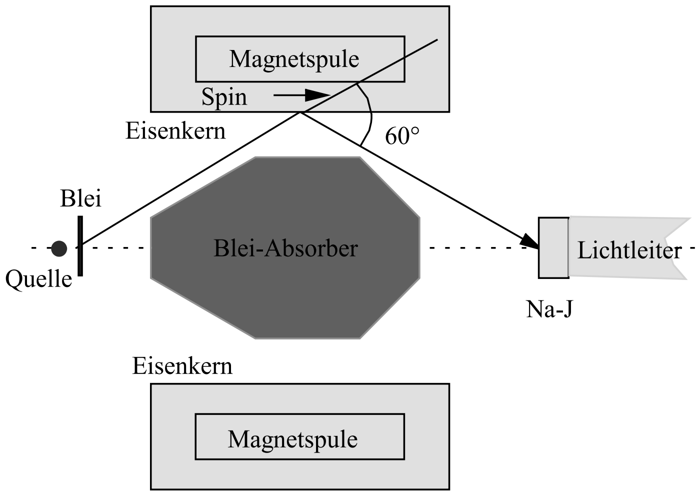

Figure 3: Sketch of set-up

By the difference of count rates for several magnetic field orientations, we can asses the asymmetry E.

4.2 Calibration

As we only want to count the γ-quanta with a higher energy that 1MeV, we got to calibrate our single-channel-discriminator. The γ-quanta lose a lot of energy during the Compton scattering. After the Compton scattering the γ-quanta with a starting energy of E=1MeV has:

$$E' = \frac{E}{1 + \frac{E}{E_0}(1 - \cos\theta)} = \frac{2E_0 E}{2E_0 + E} \approx 505keV \tag{24}$$

for $\theta = 60°$.

So we should calibrate our discriminator to only detect photons with an energy hight that 505keV. It is appropriate to use ^{22}Na. Sodium has its characteristic annihilation peak at 511keV.

So we lay the sodium (^{22}Na) near by our detector. The photo multiplier was connected to the oscilloscope. The intensity shows us something of the spectrum of the sodium. Now we calibrated our discriminator in such way, that it ignores all incoming photons below an energy of the biggest peak in the spectrum of sodium. This wasn't easy, so we had to guess a bit. We took our test series with a value of 1,67.

5 Analysis

to find out that the parity is violated, we have to measure N_+ and N_- to calculate E. With E we can calculate P_C and then H. To measure $N_\pm$ we need the magnetic field to align the spins. Which polarity of the magnetic field stands for N_+ was chosen by us at random. We defined one polarity for N_+ and after changing the polarity we measure N_-.

Measure	Number of detections
1	306
2	254
3	292
4	286
5	301

table 1: background

For Analysis at first we measure the background and then we measure with the source. For this we take 30 couples of measurement (N_+ and N_-). It is to say that we changed the polarity after each measurement to get these couples. Why this is important will be explained a little bit later.
To calculate E we choose two different ways. The first way is to calculate E from each couple with the statistic error and then calculate the mean (method 1). The second way is to sum up all the values for N_+ and N_- and then calculate E (method 2).
It is better to have two different ways of analysis which can be compared.

5.1 Background measurement

After the calibration we started the measurement of the background (N_b). Therefore we count the number of detections for 30 seconds without any source. For the background we get the following men with a statistical error:

$$N_b = 287,8 \pm 17,0 \tag{25}$$

The statistical error in this case and in the following as well is given by

$$\sigma = \sqrt{N} \tag{26}$$

because our values are following the Poisson distribution.
A reason for the background is natural radioactivity or maybe cosmic rays.
In the following we assume the background as a systematical error which is small and so negligible in the calculation for the helicity.

5.2 Measured data

Before we analyse our data we want illustrate the measurements.
The graphic shows our measurements. We can see a decrease of N_+ and N_- during the measure. The decrease of $N_\pm$ maybe can be explained with a heating of the magnet or a heating of the photomultiplier.

We can see that the decrease for N_+ and N_- is almost the same, therefore this effect does not influence the value of E so much. But it is important to measure couples of N_+ and N_- to make sure that the influence of this systematic effect is almost the same for N_+ and N_-. If we would measure at first all values for N_+ and then for N_- we would get a big systematic error.

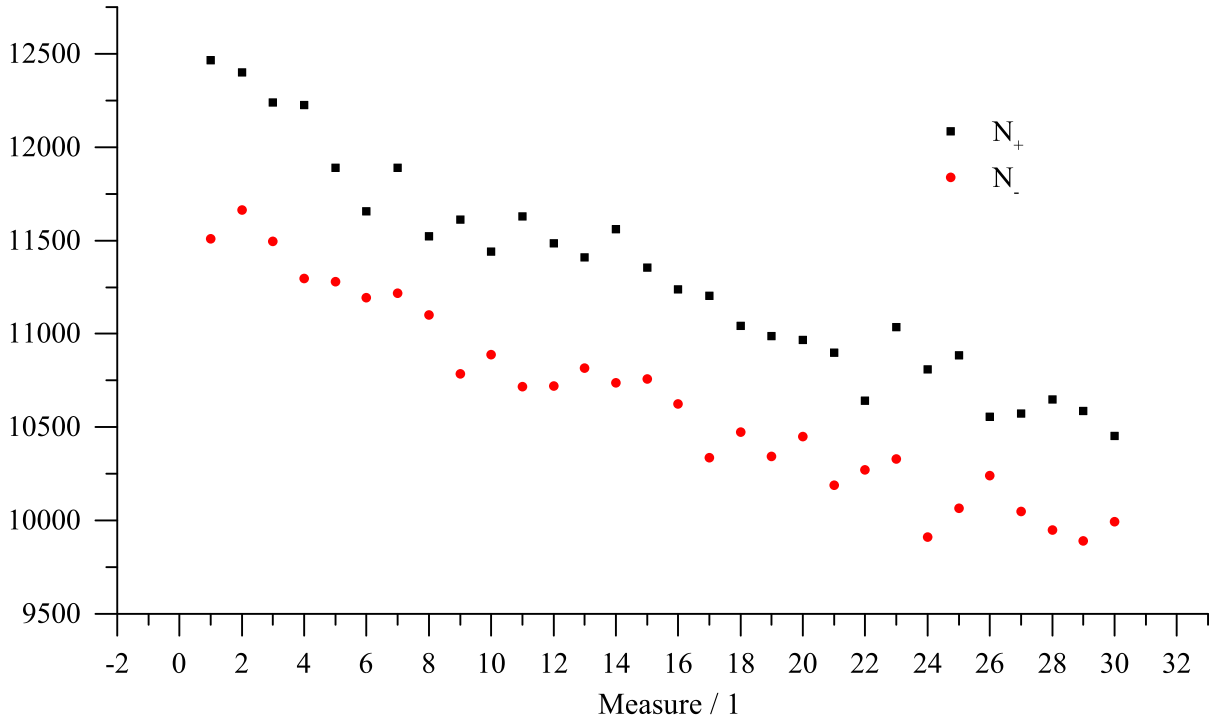

Figure 4: Measure data

5.3 Propagation of uncertainty - method 1

Here we will calculate the asymmetry E of every pair of variables. For each value of E we compute its statistical uncertainty.

So we start with the values of E for each pair:

$$E_i = \frac{N_{i,+} - N_{i,-}}{N_{i,+} + N_{i,-}} \tag{27}$$

For the statistical uncertainty we took the Gaussian error formula:

$$\sigma_{E_i} = \sqrt{\left(\frac{\partial E_i}{\partial N_{i,+}}\sigma_{N_{i,+}}\right)^2 + \left(\frac{\partial E_i}{\partial N_{i,-}}\sigma_{N_{i,-}}\right)^2} = 2\sqrt{\frac{N_{i,-}\cdot N_{i,+}}{(N_{i,+}+N_{i,-})^3}} \tag{28}$$

with the error on the counting rates: $\sigma_{N_\pm} = \sqrt{N_\pm}$

N_+	N_-	E	σ_E		N_+	N_-	E	σ_E
12465	11509	0,0399	0,0065		11239	10626	0,0280	0,0068
12402	11664	0,0307	0,0064		11203	10338	0,0402	0,0068
12240	11494	0,0314	0,0065		11042	10474	0,0264	0,0068
12224	11296	0,0395	0,0065		10989	10345	0,0302	0,0068
11890	11279	0,0264	0,0066		10968	10450	0,0242	0,0068
11655	11195	0,0201	0,0066		10900	10189	0,0337	0,0069
11888	11219	0,0290	0,0066		10643	10272	0,0177	0,0069
11522	11102	0,0186	0,0066		11036	10331	0,0330	0,0068
11613	10785	0,0370	0,0067		10810	9913	0,0433	0,0069
11441	10887	0,0248	0,0067		10886	10064	0,0392	0,0069
11628	10716	0,0408	0,0067		10557	10239	0,0153	0,0069
11486	10722	0,0344	0,0067		10574	10048	0,0255	0,0070
11410	10818	0,0266	0,0067		10648	9949	0,0339	0,0070
11562	10736	0,0370	0,0067		10586	9890	0,0340	0,0070
11353	10758	0,0269	0,0067		10452	9994	0,0224	0,0070

table 2: Measured data

We collected n=30 data pairs. The mean of the asymmetry $\bar{E}$ is:

$$\bar{E} = n\frac{1}{n}\sum_{i=1}^{n} E_i = \frac{1}{30}\cdot 0,9101 = 0,03034 \tag{29}$$

For the calculation of the error $\sigma_{\bar{E}}$ of $\bar{E}$ we need the standard deviation of the mean:

$$\sigma_{\bar{E}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(E_i - \bar{E})^2} = 0,00137 \tag{30}$$

Also we get an systematic error on the counting rates $\Delta N_\pm := \Delta_N = 100$. This is because, we are not sure, if the counter works right.

$$\begin{aligned} \Delta E_i &= \frac{\sqrt{2(1+E_i^2)}}{N_{i+}+N_{i-}} \cdot \Delta N \\ \Delta \bar{E} &\approx \frac{\sqrt{2(1+\bar{E}^2)}}{2\cdot \bar{N}_\pm} \cdot \Delta N = 0,00707 \end{aligned} \tag{31}$$

So we get with method 1:

$$\bar{E} = 0,03034 \pm 0,00137 \pm 0,00707 \tag{32}$$

5.4 Method 2

In method 2 we build the sum $N_\pm^{(s)}$ of all $N_\pm$ before we calculate the asymmetry E.

Out of our measured data we can calculate $N_\pm^{(s)}$:

$$\begin{aligned} N_+^{(s)} &= 339312 \\ N_-^{(s)} &= 319302 \end{aligned} \tag{33}$$

The statistical error for $N_\pm^{(s)}$ is $\sigma_{N_\pm^{(s)}} = \sqrt{N_\pm^{(s)}}$.

Now we can calculate the asymmetry $E^{(s)}$ (the “s“ because in method 2 we built the sum at first):

$$E^{(s)} = \frac{N_+^{(s)} - N_-^{(s)}}{N_+^{(s)} + N_-^{(s)}} = 0,03038 \tag{34}$$

and the statistical error:

$$\sigma_{E^{(s)}} = 2\sqrt{\frac{N_-^{(s)} \cdot N_+^{(s)}}{(N_-^{(s)} + N_+^{(s)})^3}} = 0,00126 \tag{35}$$

And the systematical error with: $\Delta N_\pm^s := 15 \cdot \Delta_N = 1500$ is:

$$\Delta \bar{E} = \frac{\sqrt{2(1+E^{s2})}}{N_+^{(s)} + N_-^{(s)}} \cdot \Delta N = 0,00322 \tag{36}$$

With method 2 we get the following value for E:

$$E^{(s)} = 0,03038 \pm 0,00126 \pm 0,00322 \tag{37}$$

6 Determination of polarization an helicity

The polarization is:

$$P_c = \frac{E}{f \cdot \frac{\Phi_c^-}{\Phi_0}} = \frac{E}{\frac{2}{26} \cdot 0,52} = 25 \cdot E \tag{38}$$

The factor $\frac{\Phi_c^-}{\Phi_0}$ has a systematic error of $\Delta \frac{\Phi_c^-}{\Phi_0} = 0,05$. So now we need both errors. The statistic error is:

$$\sigma_{P_c} = \frac{\sigma_E}{f \cdot \frac{\Phi_c^-}{\Phi_0}} \tag{39}$$

The systematic error is:

$$\Delta P_c = \sqrt{\left(\frac{\Delta E}{E}\right)^2 + \left(\frac{\left(\Delta \frac{\Phi_c^-}{\Phi_0}\right)}{\frac{\Phi_c^-}{\Phi_0}}\right)^2} \tag{40}$$

So we get the following values for the polarization:

$$\begin{aligned} &\text{method 1:} \quad P_c = 0,7585 \pm 0,0343 \pm 0,252 \\ &\text{method 2:} \quad P_c = 0,7595 \pm 0,0315 \pm 0,138 \end{aligned} \tag{41}$$

finally we can calculate the helicity H:

$$H = \frac{P_c}{L} \tag{42}$$

Where L = 0,8 ± 0,15 is the helicity transfer which can be determined from figure 2.

To calculate the statistical and systematical errors, we have to say, that only P_c has a statistical error.

$$\begin{aligned} \sigma_H &= \frac{\sigma_{P_c}}{L} \\ \Delta_H &= \sqrt{\left(H\frac{\Delta P_c}{P_c}\right)^2 + \left(H\frac{\Delta L}{L}\right)^2} \\ &= H\sqrt{\left(\frac{\Delta P_c}{P_c}\right)^2 + \left(\frac{\Delta L}{L}\right)^2} \end{aligned} \tag{43}$$

So we get the following values for the helicity:

$$\begin{aligned} &\text{method 1:} \quad H = 0,94813 \pm 0,0429 \pm 0,3814 \\ &\text{method 2:} \quad H = 0,94938 \pm 0,0415 \pm 0,2611 \end{aligned} \tag{44}$$

If we compare method 1 and method 2 we see, that both methods bring almost the same value.

We calculated the helicity on two different ways and got a value unequal zero. This means that the parity in the beta decay is violated.