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Namen: Fleig, Georg

Krause, Marcel

Assistent: Porcelli, Alessio

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Experiment:

Mean lifetime of positronium

from the subarea

Nuclear Physics

Lab report

of

Georg Fleig (georg@leech.it)
Marcel Krause (mrrrc@leech.it)

Group: **110**

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I. Preliminary

Abstract

Aim of the present work is the measurement of the mean lifetime of positronium by recording energy spectra of the positronium source 22-Na. In addition, we will measure the speed of light.

Theoretical background

Annihilation

The process of annihilation is observed whenever an elementary particle collides with its antiparticle. The conservation of momentum and energy indicates that the annihilation of any pair of particles and their antiparticles must create at least one new particle.

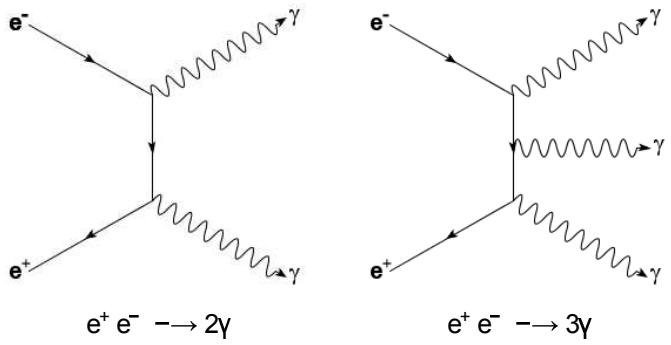

Figure 1: Feynman diagrams of electron-positron annihilation (source: [1]).

Let the particles be a pair of an electron and a positron. The collision annihilates both of them, thus creating a new particle, namely a photon. It is also possible that not only one, but two, three or even more photons are created, as it is shown in the Feynman diagrams in figure 1. Let the momentum of both the electron and the positron be zero at the time of the annihilation. Due to the conservation of momentum and energy and the fact that photons can never rest, it is obvious that the creation of one single photon can only take place in solid-state bodies which are able to take the inverse momentum of the created photon.

The second possible form of decay is that the electron-positron pair decays into two photons whose momenta are opposed. The conservation of energy predicts that every of the two photons has an energy of approximately $511\,\mathrm{keV}$ which matches the rest mass of an electron.

The third form of decay is the annihilation into three photons. Due to the laws of conservation, these photons can have arbitrary energies and momenta. In the following, we want to neglect the annihilation into one single photon because of its low cross section and therefore low probability of taking place.

Let us now additionally have a look at the spins of the two particles. It should be mentioned that we can neglect the orbital angular momenta completely because of the fact that they are very low compared to the spins in solid-state bodies. By adding two particles with spin $1/2$ one can analytically show that there exists one eigenstate for the overall spin $S = 0$ and three different eigenstates for $S = 1$, therefore calling them singlet and triplet state, respectively.

Photons are gauge bosons with a spin of 1. Two photons moving in opposed directions can therefore add

to an overall spin of either 0 or 2. However, three photons moving in arbitrary directions have a chance of adding their spins to an overall spin of 1. Thus, the singlet state can only decay into two photons and the triplet state into at least three, which means that the decay of the triplet state is a process of higher order, making it less probable than the decay of the singlet state. Let us have a look at the cross sections of both forms of annihilation

$$\sigma_{2\gamma} = \alpha \frac{\pi\hbar}{v} \quad , \quad \sigma_{3\gamma} = \alpha^2 \frac{3\pi\hbar}{8v} \tag{1}$$

where $\alpha \approx 1/137$ is the fine-structure constant and $v \ll c$ the relative velocity of the particles. The ratio of both cross sections is as follows:

$$\frac{\sigma_{3\gamma}}{\sigma_{2\gamma}} \approx \frac{1}{372} \tag{2}$$

Consequently, in free space we want to neglect the decay of the triplet state and only consider the singlet state in the following.

Generation of positronium

Instead of one electron e^- and one positron e^+ annihilating each other, it is possible that both can form a new hydrogen-like atom called positronium. One can think of positronium as an ordinary hydrogen atom, with the positron replacing the proton. Due to the fact that both the electron and the proton have the same mass and the same absolute charge, they orbit each other around their common center of mass as it is shown in figure 2.

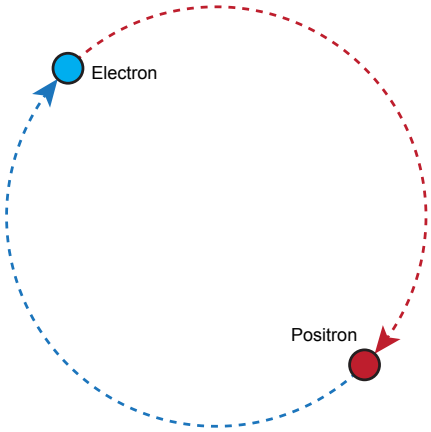

Figure 2: Structure of positronium (source: [2]).

The calculation of the eigenstates and the binding energies of positronium is analogous to that of the hydrogen atom. However, due to the fact that the reduced mass is now exactly $\mu = m_e/2$ where m_e is the rest mass of an electron, the binding energy of the ground state of positronium is exactly the half of that of hydrogen, namely $E_0 = -6.8\,\mathrm{eV}$.

Strictly speaking, because of the existance of two possible spins $S = 0$ and $S = 1$, positronium also has two different ground states: the singlet state is called para-positronium whereas the triplet state bears the name ortho-positronium. Just as before, only para-positronium is able to decay into two photons. Because of the different spins, the mean lifetimes of both forms of ground state differ over a factor of thousand, with para-positronium being much more ephemeral than ortho-positronium.

Positronium can be created in solid-state bodies in a rather simple process. In the beginning, a free

positron travels through the body. As a result of its high velocity, its kinetic energy is too high to either create positronium or to annihilate with a effectively free electron of the solid body. Instead, the velocity of the positron is reduced by numerous inelastic collisions with the atoms of the solid-state body until its kinetic energy is within the range of a few eV.
As a result of the inelastic collisions, the atoms have a chance of being ionised. The positron can now either combine or annihilate with the free electron of the ionised atom. There exists a certain range ΔE of energy for the positron where only the building of positronium but no other type of inelastic collision is possible. The limits of this range is the ionisation energy V minus the binding energy of positronium (due to its creation) as a minimal energy on one side and the first stimulation energy E_a of the atoms on the other side. This certain energy range is often called the Ore gap:

$$\Delta E = E_{\mathrm{max}-\mathrm{min}} = E_\mathrm{a} - (V - 6.8\,\mathrm{eV}) \tag{3}$$

Positronium in solid-state bodies

The question arises how the existence of positronium can be proven. One could try to detect the annhiliation of positronium and therefore the creation of two or three gamma quanta. However, it is also possible that these quanta originate from the annihilation of a free electron-positron pair. Thus, it is necessary to consider another way of positronium detection.
When building the ratio of the two possible forms of decay, one basically builds the ratio between the two cross sections. For a free electron-positron pair the result is given in equation (2). However, during the creation of positronium the triplet state has a three times higher chance of being built than the singlet state, which stems from the fact that the triplet state consists of three different eigenstates. Therefore, the ratio for the creation becomes:

$$\frac{\sigma_{3\gamma}}{\sigma_{2\gamma}} = \frac{3/4}{1/4} = 3 \tag{4}$$

In reality, the ratio is lower than given in (4) because of interaction between the rather long-living ortho-positronium and the solid-state body. When measuring positronium in a solid-state body like a polymer, three different components are found.
The first is a rather long-living component with a mean lifetime of $2\,\mathrm{ns}$ - $4\,\mathrm{ns}$ making up around 30% of all annihilations. It belongs to ortho-positronium. The second component is the most intense with an annihilation ratio of approximately 60% and a mean lifetime of $0.5\,\mathrm{ns}$. It stems from the decay of free electron-positron pairs. The last component with a mean lifetime of approximately $0.12\,\mathrm{ns}$ is the shortest-living and with 10% of all annihilations the least intense one. It belongs to para-positronium.
The difference of the mean lifetimes of ortho-positronium in a solid-state body compared to a positronium atom in free space is remarkable. Basically, this difference is explained with two processes in the solid body, the first being so-called pick-off processes.
Ortho-positronium interacts with electrons from the molecules of the solid body or their inner magnetic fields. With the ortho-positronium now having an impact partner, the solid body can take up energy and angular momentum of the triplet state. Therefore, it is possible that even the triplet state can decay only into two photons instead of at least three. This increases the chance of the triplet decay considerably. The energy of the created photons are continuously distributed up to maximum of $511\,\mathrm{keV}$.
The second possible interaction is the conversion between ortho-positronium and para-positronium made

possible by interchanging electrons from the positronium and from molecules of the solid body. The conversion can happen in both directions with the same probability. However, the probability of decay of the singlet state is higher and that of the triplet state is lower than the conversion probability, thus the conversion causes a higher rate of annihilation than one of creation of ortho-positronium. Due to the conservation of energy and angular momentum, the conversion process leads to the creation of photons with a fixed energy of $511\,\mathrm{keV}$.
Overall, the reduction of the mean lifetime τ_{ortho} of ortho-positronium due to pick-off processes τ_{p} and conversion τ_{c} is given by

$$\frac{1}{\tau_{\text{ortho}}} = \frac{1}{\tau_0} + \frac{1}{\tau_{\text{p}}} + \frac{1}{\tau_{\text{c}}} \tag{5}$$

where τ_0 is the mean lifetime of undisturbed ortho-positronium without any interaction.

Source of ^{22}Na

In order to induce the creation of positronium in e.g. polymers, free positrons have to be created. One way of achieving this is the β^+ decay

$$p \rightarrow n + e^+ + \nu_{\text{e}} \tag{6}$$

where a proton p in a nucleus decays into a neutron n, a positron e^+ and an electron neutrino ν_{e}. This reaction is always possible if the resulting nucleus has a greater binding energy than the original one. However, it is not easily possible to detect the exact time of creation of the positron during the β^+ decay in general.
In order to solve this problem, ^{22}Na is used as a source of positrons. Its β^+ decay is as follows:

$$^{22}\text{Na} \rightarrow\ ^{22}\text{Na}^* + e^+ \rightarrow\ ^{22}\text{Na} + \gamma + e^+ \tag{7}$$

The whole reaction takes place within a very small time span, therefore the creation of the positron and a gamma quantum γ with an energy of $E_\gamma = 1.275\,\mathrm{MeV}$ is effectively synchronous. Due to the fact that the time needed to decelerate this gamma quantum in solid bodies is almost the same as the time needed for the positronium to slow down to reach the Ore gap, the gamma quantum may serve as some kind of stopwatch.

Exercise 0: Experimental set-up

The β^+ source, namely ^{22}Na, is enclosed by acrylic glass and lies between two movable detectors positioned in an angle of 180°. One detector registers the gamma quantum of the ^{22}Na source, giving a start signal. The other detector shall be moved in order to allow measurements at different distances. It registers the whole spectrum of the source as well as the spectrum of all processes happening in the acrylic glass.
In order to process the data, the two detectors are connected with a computer.

Exercise 1: Time calibration and time resolution

The time pulse converter (TPC) is only working correctly if there is a certain time span between start and stop of at least $\Delta t = 2\,\mathrm{ns}$. Because of the fact that the TPC is only able to display events per channel number but not per time as it is needed during the experiment, we have to calibrate it. We will detect a whole spectrum of ^{22}Na and afterwards only the positronium decay with different delay times Δt. The peak of the positronium decay can be approximated as a Gaussian bell curve with its maximum at a certain channel number.

By increasing the delay time, we also move the maxima to higher channel numbers. When plotting the delay time over the channel number, we expect to see a linear relationship between those two values. With the help of a linear fit, we are then able to calibrate the TPC. The time resolution is then given as the product of the full width at half minimum (FWHM) of the bell curve with the delay time per channel.

Exercise 2: Mean lifetime of positronium

In order to measure the mean lifetimes of the different positronium states we will record another spectrum. Due to the fact that we calibrated the TPC before, we can now change the x-axis from channel numbers to time. Because of the limited time resolution it will not be possible to differ between the two short-living states of para-positronium on one hand and the annihilation between free electron-positron pairs in the acrylic glass on the other. Therefore, the spectrum will be of the form

$$N(t) = A \exp\left(-\frac{t}{\tau_1}\right) + B \exp\left(-\frac{t}{\tau_2}\right) + C \tag{8}$$

with the constants A, B and C and the τ_i being the mean lifetimes of long-living and generally short-living positronium respectively. With the help of appropriate fits it is possible to determine the lifetimes. In order to get rid of the random coincidences C we will consider large times where the exponential contributations of the positronium have vanished. Afterwards, we are able to subtract C from the spectrum.

Exercise 3: Speed of light

Finally, we also want to measure the speed of light with the help of the β^+ source. In order to achieve this, we will record the positronium decay at two different distances between the detector and the source. The maxima of the two spectra will then also be found at different times. We can immediately calculate the speed of light by building the ratio of these values.

References

[1] Blaues Buch zur Kernphysik

[2] `http://en.wikipedia.org/wiki/File:Positronium.svg`

II. Results and Discussion

Exercise 1: Time calibration and time resolution

First of all, we fixed the vial out of acrylic glass, which contained the ^{22}Na, between the two detectors. The movable detector was fixed at a position of zero from the ^{22}Na, meaning that it reached its minimum distance from it. We set the manual delay of the TPC to a time of $\Delta t = 2\,\mathrm{ns}$ and started the first measurement. On the computer, we were able to see a plot of the number of events over the channel number of the TPC as it is shown in figure 3.

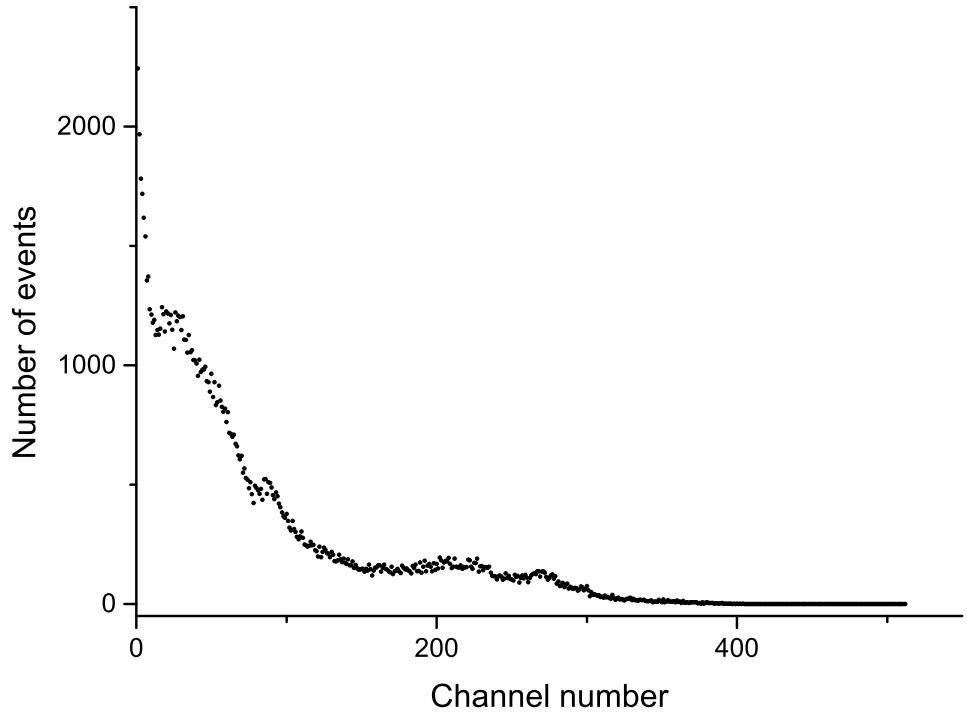

Figure 3: Full spectrum of ^{22}Na.

On the spectrum, we are able to see a few characteristic peaks, with the most interesting one around channel number 100. The high number of events in the channels below mostly stem from the free positron-electron annihilation, which is not interesting for measuring the mean lifetime of positronium. Therefore, we increased the lower level of the trigger in order to neglect all events from low channels. The resulting spectrum is shown in figure 4.

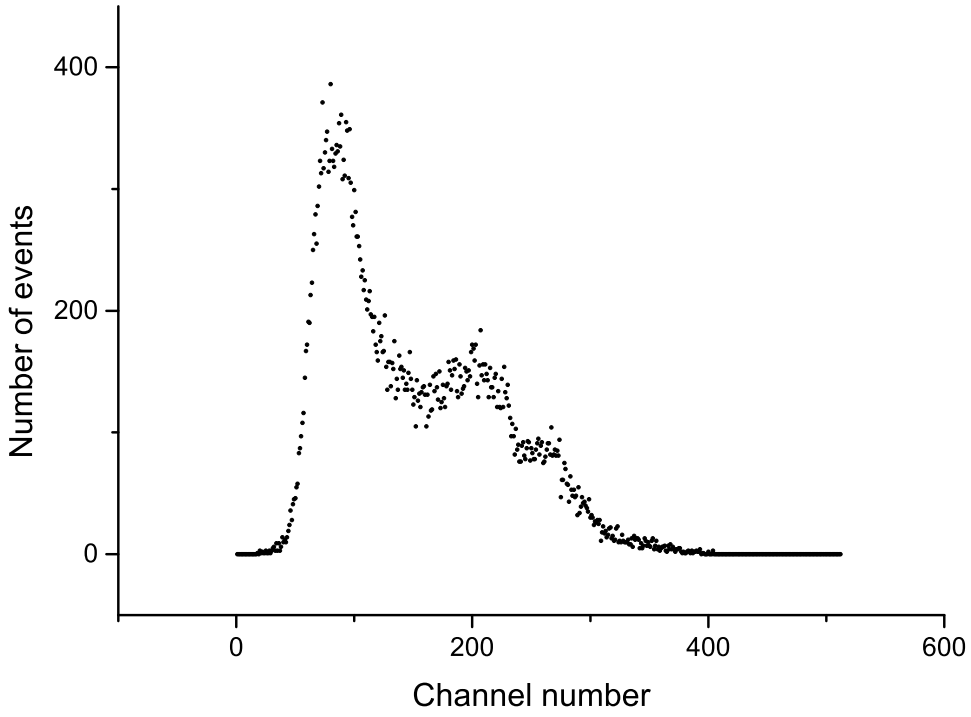

Figure 4: Reduced spectrum of ^{22}Na.

In the chart, the expected peaks can be seen. The huge peak from the decay of the positronium belonging to an energy of approximately $511\,\mathrm{keV}$ that can be seen in figure 3 is now cut off in figure 4. The other peaks that are actually visible in figure 4 are the gamma quanta γ originating from equation (7).
In order to calibrate the TPC, we used the signals we obtained from both the decay and the gamma quantum as triggers. In the following, we measured the number of events for different manual delay times, where the delay time $\Delta t = 2\,\mathrm{ns}$ should be our zero value. The resulting figures 8 to 13 are shown in the appendix. With the help of Origin, we fitted Gaussian bell curves of the form

$$N = N_0 + \frac{A}{\sigma\sqrt{\pi/2}} \exp\left(-2\frac{(x - x_\mathrm{c})^2}{\sigma^2}\right) \tag{9}$$

into each chart in order to find the maximum channel number x_c of the distribution. The results are shown in table 1, with σ_{x_c} being the standard deviation given by Origin.

Δt in ns	0	4	8	12	16	20
x_c	145.314	193.657	244.483	294.390	346.103	392.634
σ_{x_c}	0.050	0.059	0.064	0.059	0.060	0.058

Table 1: Maximum channel numbers against delay times

Plotting the delay time against the maximum channel numbers, we are able to linearly fit the data with a

function of the form

$$t = mx + t_0 \tag{10}$$

as shown in figure 5. The required parameters m and t_0 with their standard deviations for the axis transformation from channel numbers x to times t are printed in table 2. With respect to them, we get:

$$t = (0.080 \pm 0.005)\ \text{ns} \cdot x - (11.627 \pm 1.355)\ \text{ns}\ . \tag{11}$$

Note that the errors of the channel numbers from the Gaussian fit are very small compared to the respective channel. Therefore, the error bars in figure 5 may not be misinterpreted as error bars with respect to the time, but rather as such with respect to the channel numbers.

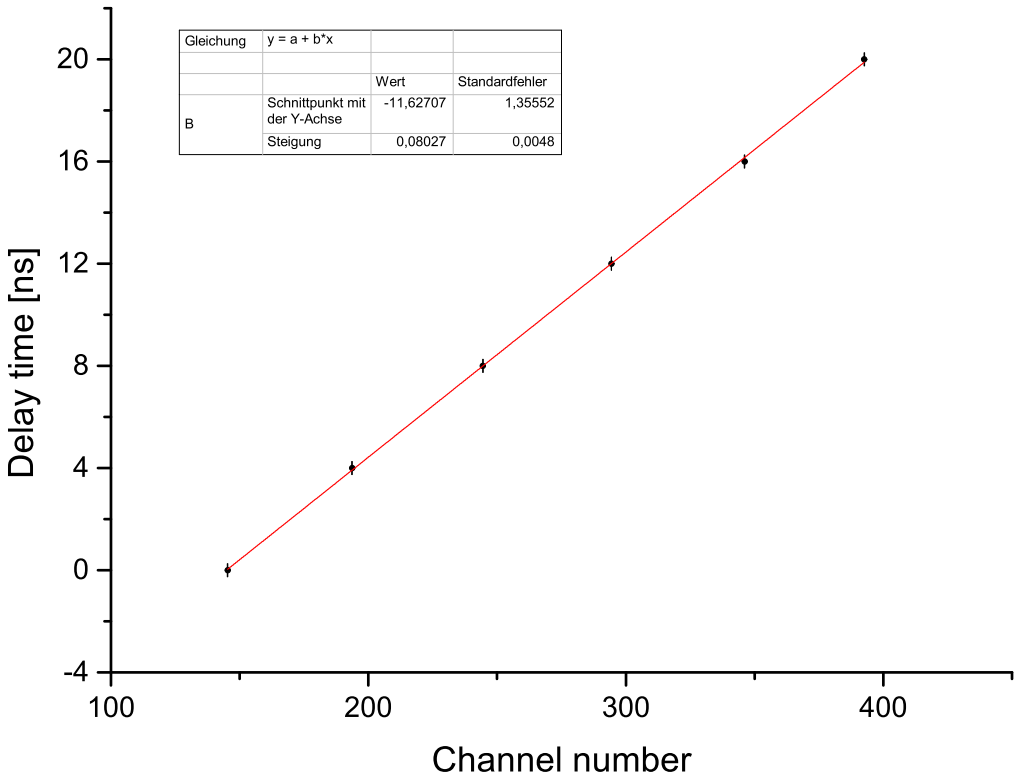

Figure 5: Time calibration.

m in ns	σ_m in ns	t_0 in ns	σ_{t_0} in ns
0.080	0.005	-11.627	1.355

Table 2: Calibration parameters.

Now that the time calibration is done, we are able to determine the time resolution of the TPC. The needed full widths at half maximum (FWHM) and their respective errors given in the charts in the appendix and abstracted in table 3. With regard to the fact that the time resolution δt is given as the product of FWHM

and the time per channel number m, we can calculate it as follows:

$$\delta t = \text{FWHM} \cdot m \tag{12}$$

The result of these calculations are also shown in table 3.

Δt	0	4	8	12	16	20
FWHM	14.944	14.658	14.347	14.007	13.711	13.983
σ_{FWHM}	0.119	0.141	0.152	0.141	0.143	0.138
δt	1.196	1.173	1.148	1.121	1.097	1.119
$\sigma_{\delta t}$	0.063	0.063	0.063	0.063	0.063	0.063

Table 3: Determination of the time resolution.

The errors $\sigma_{\delta t}$ in δt were calculated with the Gaussian error propagation:

$$\begin{aligned} \sigma_{\delta t} &= \sqrt{\left(\frac{\partial \delta t}{\partial m}\sigma_{\text{m}}\right)^2 + \left(\frac{\partial \delta t}{\partial \text{FWHM}}\sigma_{\text{FWHM}}\right)^2} \\ &= |\delta t| \sqrt{\left(\frac{\sigma_{\text{m}}}{m}\right)^2 + \left(\frac{\sigma_{\text{FWHM}}}{\text{FWHM}}\right)^2}. \end{aligned} \tag{13}$$

Building the average of all the time resolutions, we finally get:

$$\delta t = (1.142 \pm 0.063)\ \text{ns} \tag{14}$$

It should be noted that the time resolution is remarkably higher than the error we got in the calibration parameter m. Consequently, we want to neglect the error σ_{m} and only regard a common error σ_{t_0} for every time t.
With the time calibration being done, we can now transform the channel numbers into times for every following exercise. By doing so, we want to neglect times below zero.

Exercise 2: Mean lifetime of positronium

After calibrating the TPC, we are now able to determine the mean lifetime of positronium. The measurement shown in figure 8 was very precise, therefore, we decided to use the data again to obtain the lifetimes. First, we transformed the x-axis from channel numbers to times according to equation (11). In addition, we neglected any times below zero.

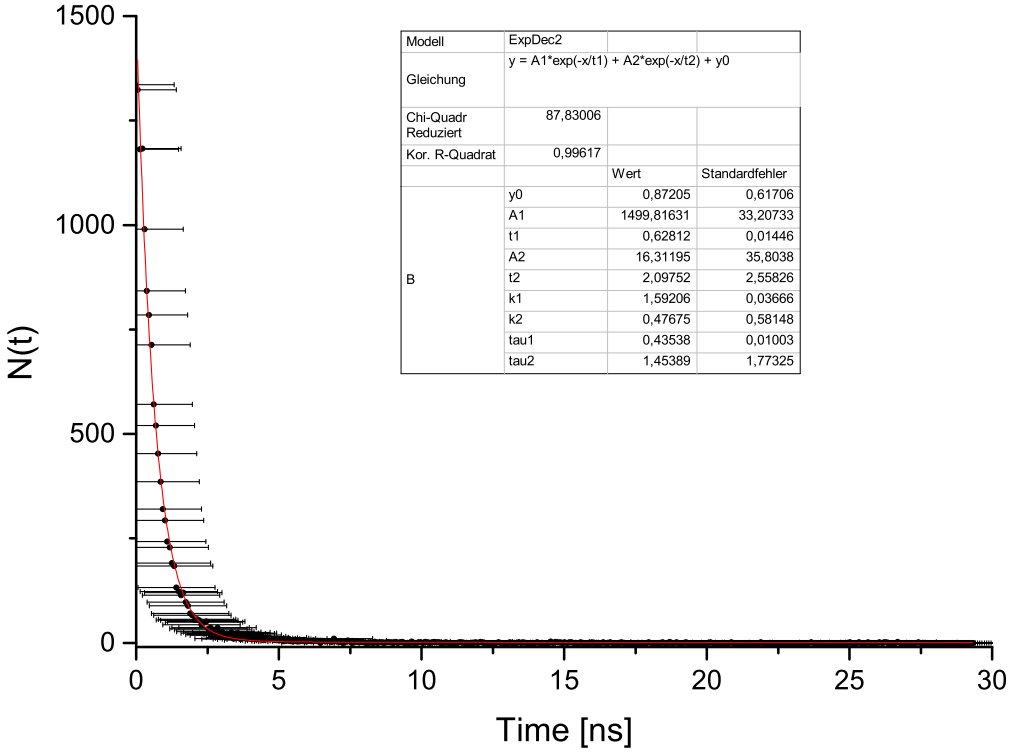

Figure 6: Double exponential fit.

Afterwards, we fitted the function (8) to our data. All the parameters as well as the data and the fit are printed in figure 6 and the important parameters concerning the lifetime are sumarized in table 4.

τ_1 in ns	σ_{τ_1} in ns	τ_2 in ns	σ_{τ_2} in ns
0.628	0.015	2.098	2.558

Table 4: Mean lifetimes of positronium

The errors shown in the table above stem directly from the conversion of the x-axis from channel numbers to times. The errors in equation (11) serve as weights for the double exponential fit, therefore we do not have to consider the error propagation here. Thus, the mean lifetimes of positronium are:

$$\begin{aligned} \tau_{\text{para, free}} = \tau_1 &= (0.628 \pm 0.015)\ \text{ns} \\ \tau_{\text{ortho}} = \tau_2 &= (2.098 \pm 2.558)\ \text{ns} \end{aligned} \tag{15}$$

Both para-positronium and the free positron-electron annihilation share the same measured value because the detectors are not precise enough to separate them from each other. Comparing these to the literature values $\tau_{\text{para,lit}} = 0.12\,\text{ns}$, $\tau_{\text{free,lit}} = 0.5\,\text{ns}$ and $\tau_{\text{ortho,lit}} = 2\,\text{ns} - 4\,\text{ns}$ we see that our results are within the expected range. The biggest source of error is most likely the precision of the used detectors. As we have seen before, the time resolution is relatively high, therefore we can not expect very precise measurements with respect to the mean lifetimes.

Exercise 3: Speed of light

In the end, we performed further measurements in order to calculate the speed of light. As described in the preliminary, we recorded the spectra at four different distances d between the start and the stop photomultiplier. Increasing the distance resulted in lower event rates and a shift of the signal peak towards higher channel numbers. Again, with the help of the time calibration in equation (11), we were able to transform the channel numbers on the x-axis to times. The recorded plots can be found in graph 14 to 17 in the appendix. To determine the time of the peak we fitted Gaussian bell curves of the form (9) into the charts. The error of time calibration has already been included in the charts and was regarded by the applied fit. The results and the corresponding errors σ_t are listed in table 5.

d in cm	t in ns	σ_t in ns
0	-0.00193	0.00399
7.5	0.00911	0.00481
15	0.31985	0.00442
20	0.55314	0.00422

Table 5: Position of the peaks at different distances.

Since we know now the distance and the time needed to travel this distance, we can directly calculate the speed of light by applying a linear fit of the form

$$c = mx + c_0 \tag{16}$$

to the data. Besides the error for the time, we also assumed a systematical error $\Delta d = 0.5\,\mathrm{mm}$ for the distance d between the two photomultipliers. Both errors are added to chart 7 and respected by the linear fit performed by Origin.

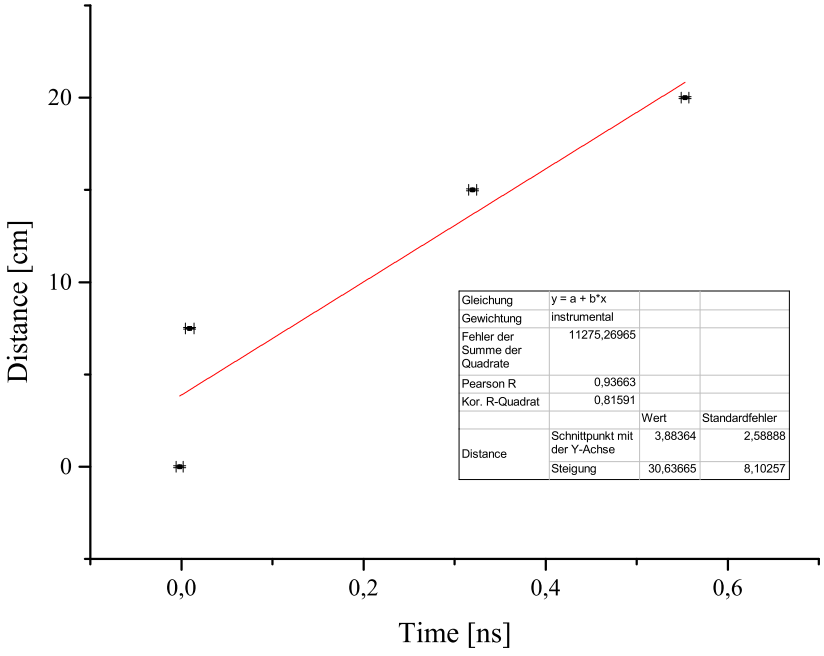

Figure 7: Linear fit for determining the speed of light.

As the slope of the straight line is linked to the speed of light, we finally get

$$c = (3.06 \pm 0.81) \cdot 10^8 \, \frac{\mathrm{m}}{\mathrm{s}} \tag{17}$$

as a value for the speed of light. Compared to the literature value $c_{\mathrm{lit}} = 3.00 \cdot 10^8 \, m/\mathrm{s}$ we only have a small relative error of 2.0%. Despite this presentable value, our result is not that satisfying since the error range is huge and the data points do not seem to follow a linear distribution. Again, the main error seems to stem from the high time resolution of the detector with respect to the small time differences measured in this procedure. Another factor could be the increased measurement time for longer distances since other disturbing signals have more time to reach the detector.

III. Appendix

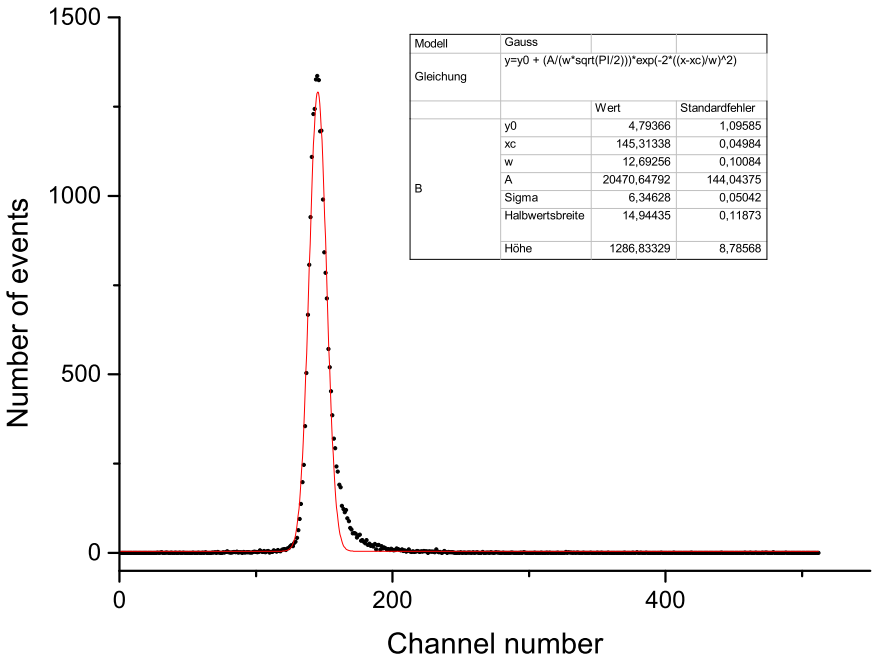

Figure 8: Calibration curve for $\Delta t = 0\,\mathrm{ns}$.

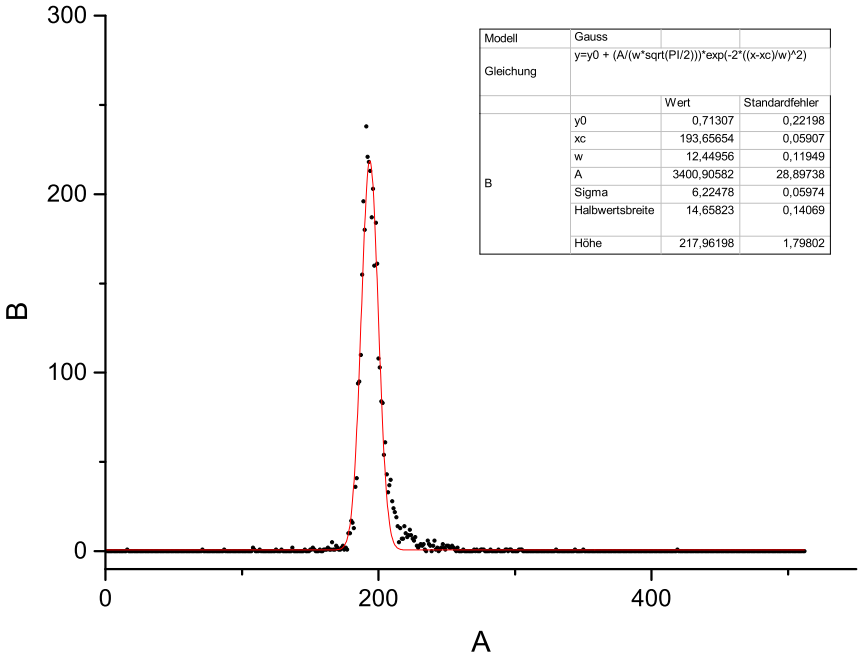

Figure 9: Calibration curve for $\Delta t = 4\,\mathrm{ns}$.

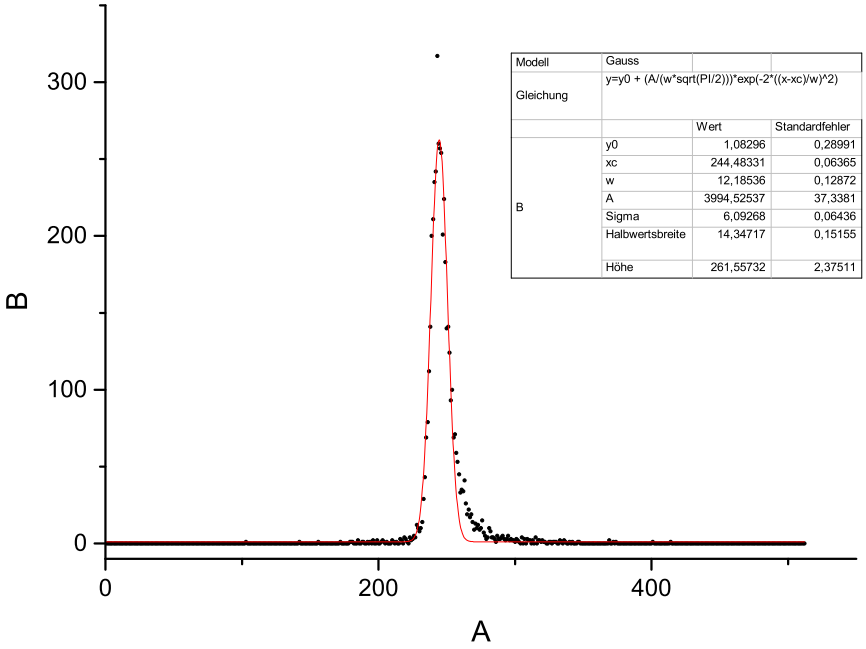

Figure 10: Calibration curve for $\Delta t = 8\,\mathrm{ns}$.

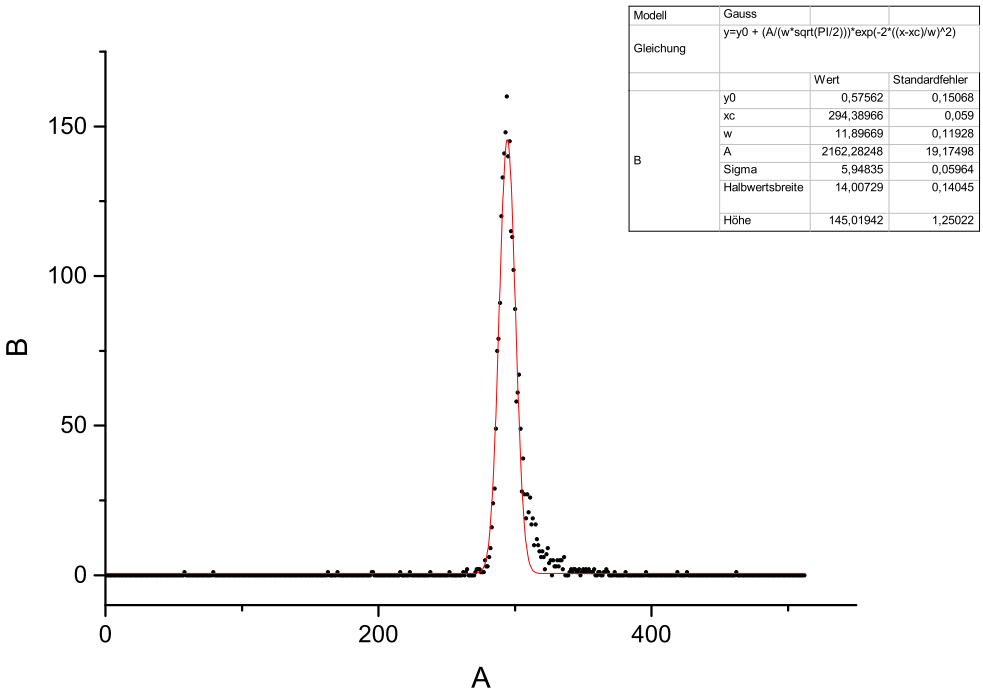

Figure 11: Calibration curve for $\Delta t = 12\,\mathrm{ns}$.

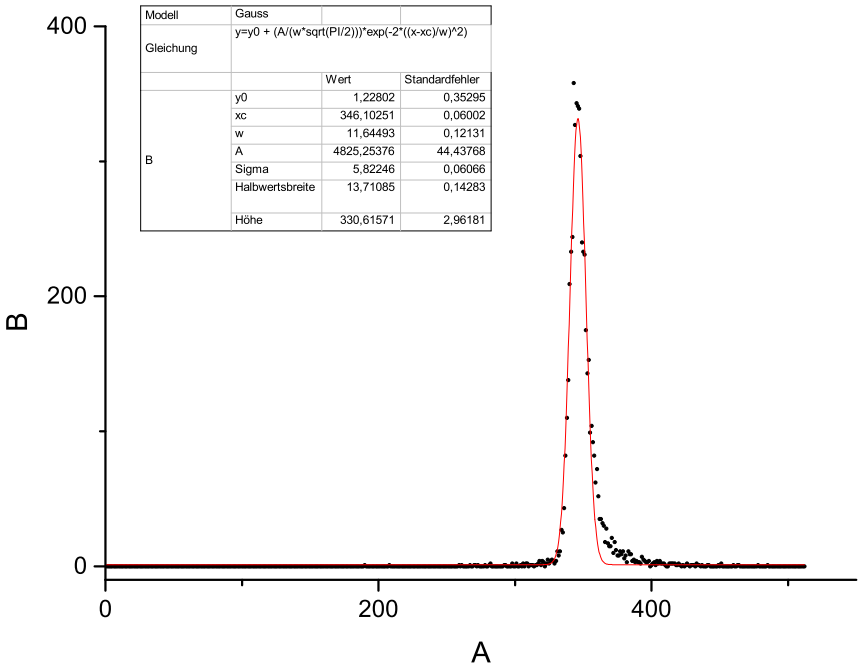

Figure 12: Calibration curve for $\Delta t = 16\,\mathrm{ns}$.

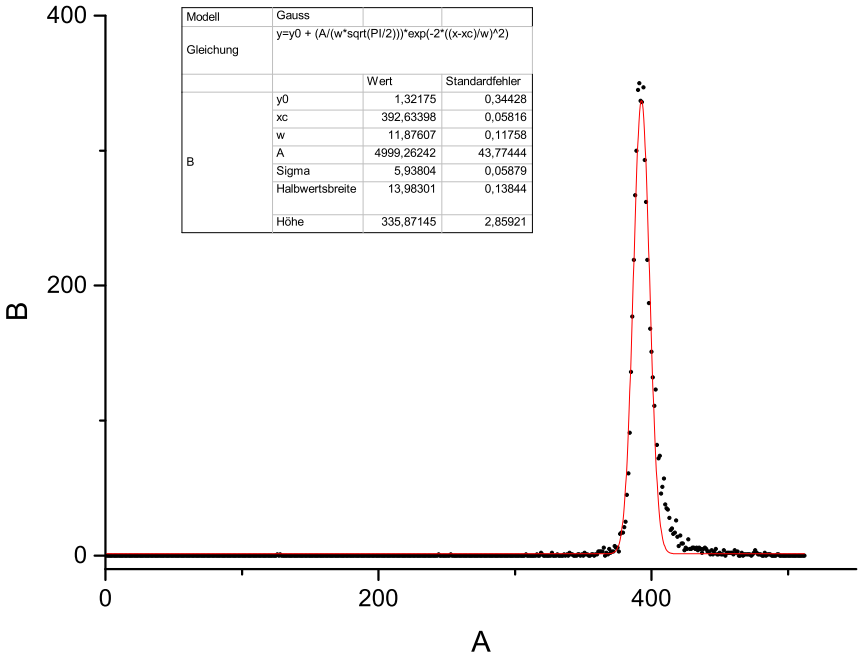

Figure 13: Calibration curve for $\Delta t = 20\,\mathrm{ns}$.

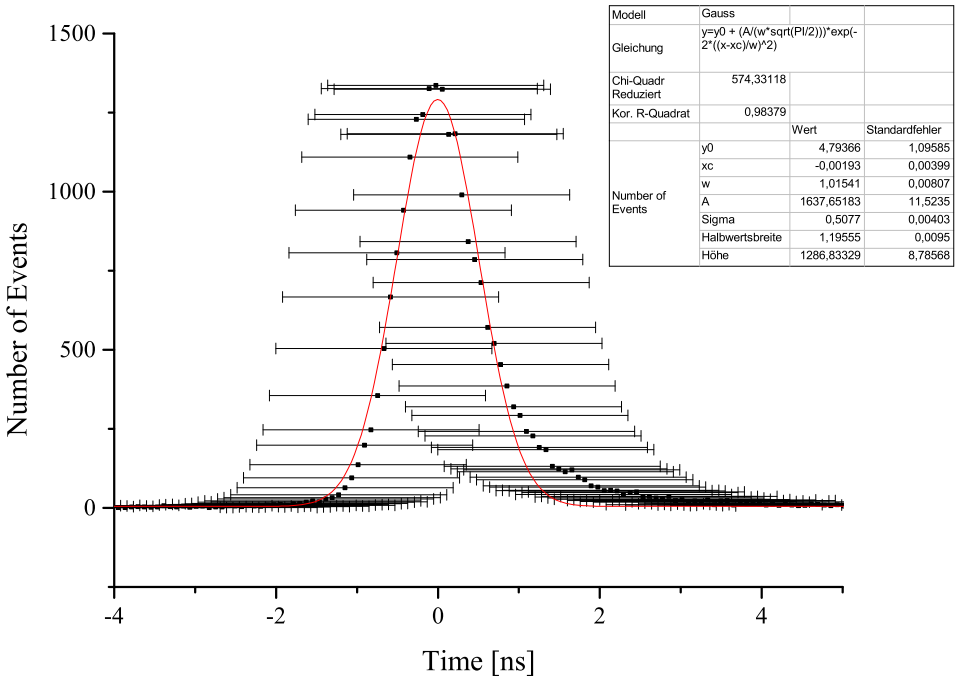

Figure 14: $d = 0\,\mathrm{cm}$.

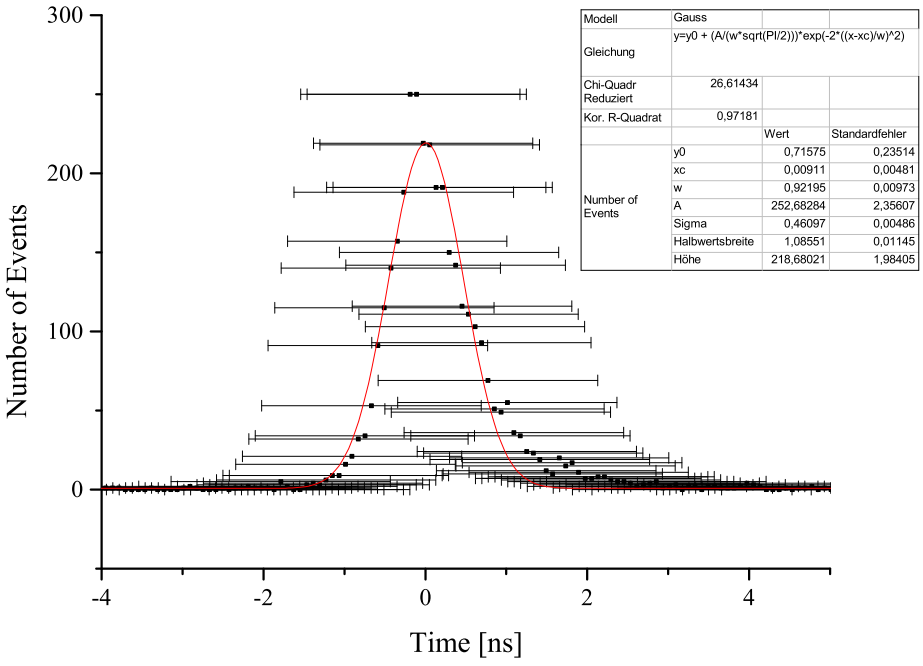

Figure 15: $d = 7.5\,\mathrm{cm}$.

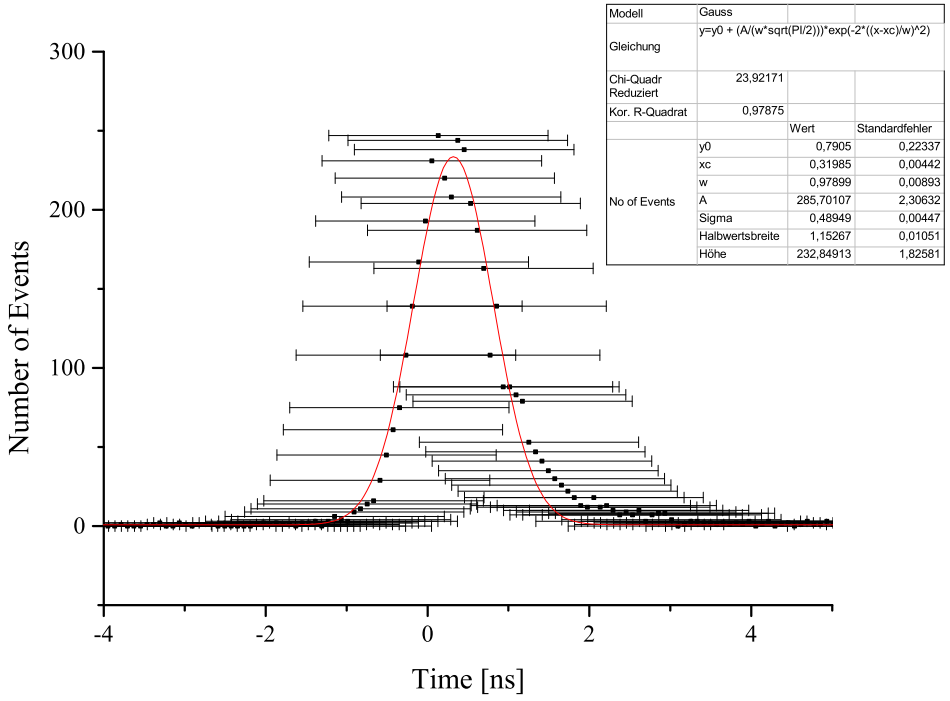

Figure 16: $d = 15\,\mathrm{cm}$.

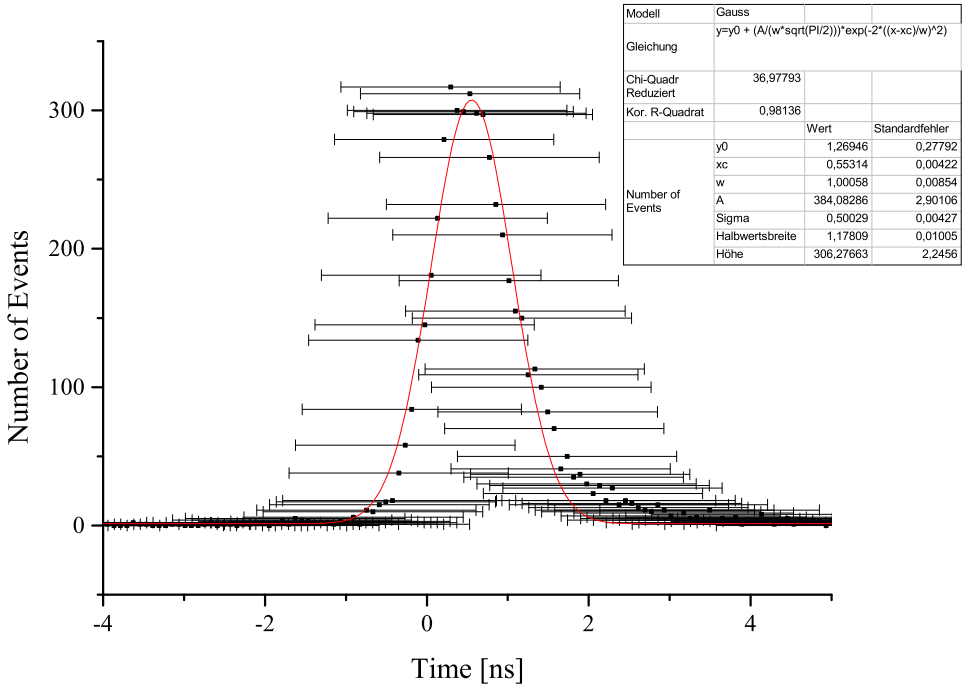

Figure 17: $d = 20\,\mathrm{cm}$.