Karlsruher Institut für Technologie

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Namen: Carl Jarschke ()

Jona Umlauf ()

Assistent: Dr. Charanpreet Singh

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Contents

3. Performing the experiments / tasks

3.1. Generate sound and record it

• Make sure that the speaker and the microphone are working as expected, i.e., connect the function generator to the *QUANTUM ANALOGS* device, which is used as an adapter between the BNC connector and the speaker jack. Program the function generator to generate a sine wave with a frequency of 1 kHz and a peak-to-peak amplitude of 1 V. You should be able to hear a tone. Change the frequency manually - what are the highest and lowest frequencies you can hear?

• Connect the microphone to the *QUANTUM ANALOGS* device, which supplies the microphone with a constant bias voltage and pre-amplifies the signal. Connect the output to the oscilloscope. When you touch the metal cylinder which contains the microphone, the oscilloscope should show the noise recorded by the microphone. Bring the microphone and speaker in close proximity. You should observe the sine wave generated by the speaker with the scope.

3.2. Measure the sound velocity

• Build up a tube of ~ 60 cm length and measure the resonances of the standing waves in the tube. Use the function generator to drive a sinusoidal sound wave and find the maxima in the amplitude by manually changing the frequency.

• Why is this method superior to measuring the traveling time of a pulse or the phase difference between speaker and microphone? Estimate the uncertainty of every method or try to measure it.

• Calculate the speed of sound from these resonances (homework). Remember to note the temperature, see e.g., http://www.sengpielaudio.com/Rechner-schallgeschw.htm for the temperature dependence of the sound velocity.

3.3. Band structure in periodic potential

• Connect speaker and microphone to the computer. Bring them in close proximity without any tube in between and measure the transmission function of the whole setup (speaker → microphone), see chap. III for setup.

• Build up a tube from multiple identical segments without apertures between the segments.

• Decrease the aperture size which corresponds to a decrease of the coupling strength between the tube segments. Measure the spectrum for each aperture size and compare them (homework). How does this setup correspond to the nearly free electron model and to the phonon model?

• **Homework:** Extract the reduced band structure for these setups. Get the individual points (states) for, at least, the black apertures. Estimate the band gap for all apertures. How does the dispersion relation compare of the periodic tube compare to the dispersion relation of nearly free electrons and to phonons? Hint: look at the bending at $k = 0$ and $\omega = 0$ and explain in which cases states with $k > \pi/a$ make sense.

• Now start with one segment consisting of one tube and one aperture and elongate it segment by segment. Measure for each number of segments. Do this for at least three different aperture sizes. How does this setup correspond to the tight-binding model?

• **Homework:** Extract the reduced band structure for these setups. Get the individual points (states) at least for one aperture. How does the number of states per Brillouin zone evolve?

• Combine two different tube lengths with identical apertures to simulate the states in a solid with a two-atomic unit cell. Measure the spectrum of the two atoms. Build a crystal of these atoms and measure the spectrum of the crystal.

• Assign the emerging bands to the corresponding tube length and the corresponding mode number (= number of nodes in the wave inside one tube).

• Combine two different apertures with identical tube lengths to simulate the states in a solid with a two-atomic unit cell. Measure the spectrum of the two atoms. Build a crystal of these atoms and measure the spectrum of the crystal.

• In this periodic system of acoustic elements another analogy can be found: Build up a microwave filter cavity with passband and stopband.

(a) Choose one tube length and one aperture diameter and build a periodic chain of resonators. Measure the transmission of these coupled resonators as usual (linear frequency sweep).

(b) Now let the speaker generate white noise and record that noise after it has passed your filter. What do you expect? Compare the spectrum of both measurement techniques.

3.4 Build a hydrogen atom

• Build a hydrogen atom with $\alpha = 180°$, i.e., the speaker and the microphone are located at opposite positions of the hollow sphere. Make connections to the sound card of the computer. Find all resonance peaks in the amplitude vs frequency spectrum (1-12000 Hz) at $\alpha = 180°$.

• Connect the sphere to the *QUANTUM ANALOGS* device, function generator and oscilloscope.

(a) Find the first 6 resonance peaks (between 1-9000 Hz). Please ignore the peak around 400 Hz which is an artefact of the setup. Use the spectrum measured with the computer to estimate the positions of the resonances and note the frequencies for further use. On the

oscilloscope you can observe a phase shift between speaker and microphone signal while crossing each resonance.

(b) Tune to the second resonance. Rotate the upper hemisphere with respect to the lower one to change the angle α from 180° to 0° and record the amplitude in steps of 10°. Make smaller steps close to nodes, where the amplitude drops to zero. Convert values of α to corresponding values of θ using equation (7). Create polar plots and use them to identify the angular momentum *l* and the spherical harmonics function of each resonance. Hint: Quantify the amplitude dependence by comparing it to Legendre-polynomials from table (1).

(c) Repeat at the 3rd and 4th resonance positions.

• For the following use the sound card of the computer for part (a) and (b); oscilloscope, function generator and quantum analog device for part (c):

(a) Measure the spectrum of the spherical resonator up to the first 3 resonance peaks. Insert spacer rings of thickness 3 mm, 6 mm, and 9 mm, one after the other and remeasure the spectrum. What does the splitting of the resonance peaks indicate? Plot the splitting vs the spacer ring thickness.

(b) Measure a highly resolved spectrum with 9 mm spacer rings of the $l = 2$ resonance. It will split into 3 peaks (at about 3435 Hz, 3455 Hz, and 3600 Hz). In order to distinguish the first two it is necessary to minimize the amplitude of the first resonance by rotating α into a node. (Hint: Observe the phase shift.)

(c) For each peak, measure the amplitude as a function of α and identify the magnetic quantum number. Rotate the upper hemisphere in steps of 10° to change the azimuthal angle φ = α and measure the amplitude. Use smaller steps close to nodes.

3.5. Build a hydrogen molecule

• Build a hydrogen molecule by using the other two hemispheres. Use the sound card. Measure the amplitude vs frequency spectrum in a frequency range 1-4000 Hz. Use various irises with diameters 5 mm, 10 mm, 15 mm, and 20 mm and remeasure the spectra. Compare with the result obtained for the atom. How do the resonance frequencies depend on the iris diameter?

1 Experimental goal , theoretical principles

1.1 Expermiental goal

The experimental goal of the experiment is, to get an better understanding of the nature of quantum physics especially the quantum physics in crystals. This goal is reached by comparing sound waves to waves in crystals and analysing the behaviour of sound waves in a pile tube.

1.1.1 Resonation in a pile tube

Initially, when the sound wave enters the tube, it travels down the length and reflects off the closed end of the tube. The reflected wave combines with the incident wave, creating regions of constructive and destructive interference.

In a closed tube, the closed end acts as a node, where the displacement of particles is minimal. This creates a standing wave pattern with alternating nodes and antinodes along the length of the tube.
If the length of the tube L is such that the distance traveled by the wave is an exact multiple of half its wavelength λ:

$$L = \frac{n}{2}\lambda, \qquad n\epsilon\mathbb{N}$$

the incident and reflected waves align with each other at the respective boundaries. This alignment causes a reinforcement of the wave, resulting in a standing wave pattern.

However the wave function of the pressure p in a pile tube is:

$$\frac{d^2}{dt^2}p(x,t) = \frac{1}{\rho\kappa}\frac{d^2}{dx^2}p(x,t)$$

where ρ is the density and κ is the comprehensibility of the material (in our experiment of air). If the neumann-boudary conditions are used, this leads to:

$$p(x,t) = 2p_0 cos(kx)cos(\omega t)$$

The dispersion relation is

$$\omega(k) = kc = 2\pi f$$

Where $k = \frac{2\pi}{\lambda}$ is the wave-vector.

1.1.2 resonation in a quantum mechanic square well potential

In general the wave equation of an free particle in one dimension is given by the Schrödinger-equation with potential $V(x,t) = 0$:

$i\hbar\frac{d}{dt}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x,t)$

where ψ is the wave function of the particle, x is the position coordinate, m is the mass of the particle, E is the total energy of the particle, V is the potential energy within the well, and $\hbar$ is

the reduced Planck's constant.
For a particle in a square well potential, the Schrödinger equation can be written as:

$$\frac{d^2\psi}{dx^2} = -\frac{2m(E-V)\psi}{\hbar^2} \tag{1.1}$$

To solve this equation, it is needed to consider the two regions within the square well potential: the region where the potential is zero and the region where the potential is infinite.

Region where $V = 0$: In this region, the potential is zero, and the Schrödinger equation simplifies to:

$$\frac{d^2\psi}{dx^2} = -\frac{2m(E)\psi}{\hbar^2} \tag{1.2}$$

Region where $V = \text{inf}$:
In this region, the potential is infinite, meaning the particle cannot exist in this region. Therefore, the wave function must be zero, i.e., $\psi(x) = 0$.

All possible solution can be written as:

$$\psi(x) = \sum_i A_i sin(ik_i x + \alpha_i + B_2 cos(-ik_i x + \alpha_i), \tag{1.3}$$

where is $k = \sqrt{\frac{2mE}{\hbar}}$ and $A_i, B_i \alpha_i$ are constants that depend on the boundary conditions.
To fullfil these, B_i and α_i have to be set to zero.
So it follows, that

$$k = \frac{n}{L}\pi, \qquad n \epsilon \mathbb{N}. \tag{1.4}$$

1.1.3 Analogies and differences between the classical wave of pressure and the quantum mecanic wave

Like it can be seen if the two equations 1.1.1 and 1.1.2 are compared to each other, the Schrödinger equation has only one time derivative AN complex part, while the Differential equation of the pressure has a second time derivative and is real.
What can also be seen in there solutions.

1.1.4 Basics of solid-state physics

To understand the experiment we have to understand at fist some basics of the solid-state physic it is made to be an analogy to.

Building of crystals

In solid state physics, a crystal is a periodic arrangement of atoms or molecules in three-dimensional space.

A crystal exhibits translational symmetry, meaning that the arrangement of atoms or molecules repeats periodically throughout space. This periodicity is characterized by a lattice vector G, which is an vector representing the positions of the repeating unit cells.

A crystal can be splitted up into primitive cells the simplest repeating unit within the crystal lattice that allows for the reconstruction of the entire crystal by translation.
The primitive cell is chosen such that all essential symmetries of the crystal are accounted for.

A really importaint primitive cell is the Wigner-Seitz Cell. The Wigner-Seitz cell is a construction that allows to identify the neighbors of a given lattice point in the crystal lattice. It is defined as the region of space that is closer to a particular lattice point than to any other lattice point. The Wigner-Seitz cell has the property that it contains only one lattice point at its center.

Symmetry of the Crystal Lattice: The symmetry of a crystal lattice refers to the set of symmetry operations that leave the lattice unchanged. These operations may include translations, rotations, and reflections.

Symmetry operations can be used to generate the complete lattice from a smaller set of basis vectors, known as the lattice basis. By applying symmetry operations to the basis vectors, the entire crystal lattice can be reconstructed.

Reciprocal lattice

In solid state physics, the reciprocal lattice is a mathematical construct that provides a systematic way to describe the periodicity of a crystal in reciprocal space. It is closely related to the direct lattice, which represents the periodic arrangement of atoms or molecules in real space.

The reciprocal lattice is defined by the set of lattice vectors that satisfy the condition:

$$\mathbf{b}_i \cdot \mathbf{a}j = 2\pi\delta ij$$

where $\mathbf{b}_i$ represents the reciprocal lattice vectors, $\mathbf{a}j$ represents the direct lattice vectors, and δij is the Kronecker delta.

The reciprocal lattice is useful in several ways:

Calculating wave vectors: In quantum mechanics, the wave vector ($\mathbf{k}$) is the momentum of a particle in reciprocal space. The reciprocal lattice vectors provide a basis for expressing the wave vector and allow for the calculation of electronic or vibrational properties of the crystal.

Band structure: The reciprocal lattice is also employed in calculating the electronic band structure of crystalline materials. The electronic energy as a function of the wave vector ($\mathbf{k}$) in the reciprocal lattice determines the allowed or forbidden energy levels (bands) for electrons in the crystal.

1.1.5 1.st Brillouin-Zone

The first Billouin Zone ist the Wigner-Seitz-zell of an reciprocal lattice of an crystal.

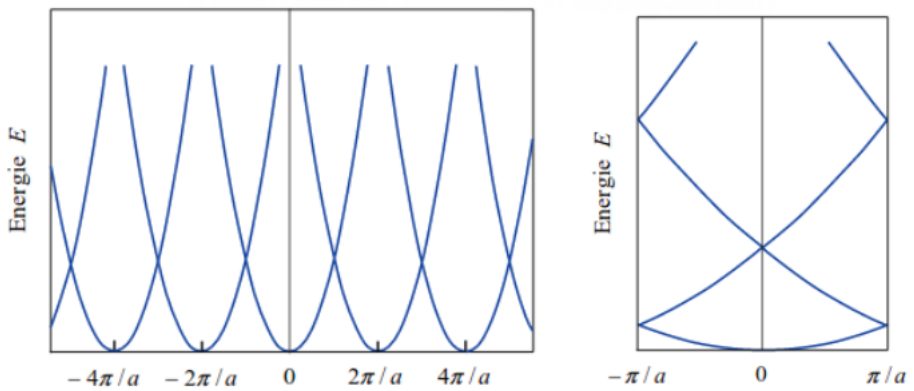

Figure 1.1: zone-spectrum, reduced zone-spectrum [Ger17]

Bloch waves

The solutions of formula 1.1 in an periodic potential can be suspected to follow an periodic shema. So in one dimension we use:

$$\psi_k(x) = u_k(x)e^{-ikx} \tag{1.5}$$

where $u(x)$ is the so called Bloch-function, and the periodicity of the lattice is saved by the relation.

$$u_k(x) = u_k(x + a) \tag{1.6}$$

Cause of this periodicity the wave-function and the Eigenvalues can be reduced into an reduced zone spectrum which only includes the first Brillouin-zone (picture 1.1).

1.2 Models

There are two different approaches to get an model to describe the behavior of solid state crystals, that are able to explain the way the measurements of band gaps in the zone spectrum.

1.2.1 Nearly free Electron model

In the nearly free Electron model you start with an couple of nuclei which are bound together. However in the beginning approximation, that is used for the model the valence electrons are free, which means that they do not interact with the nucleus's. Now step for step you increase the binding of these electrons to the nuclei, which changes the eigenvalues of the energy. The difference of the resulting energys gives you the band gaps.
Interesting to see is that the number of states in the first Brillouin zone are equal to the number of atoms in the crystal.

1.2.2 Tight-Binding model

The Tight-Binding-model starts with an single nucleus and one electron. An formula that is already solved. To get from there to an crystal an second atom is taken from infinite distance and is brought near to the first, so the two charge distributions overlap. These overlaps create

a new term in the energy level, which describe the band gap between the first and the second level.
Like in the free electron model to get the further band gaps (or further energy states) it is needed to bring more atoms close together.
Mathematical the energy distribution is given by:

$$E(k) = E_0 + 2W cos(ka) \tag{1.7}$$

Where W is the strength of the interaction between the two atoms.

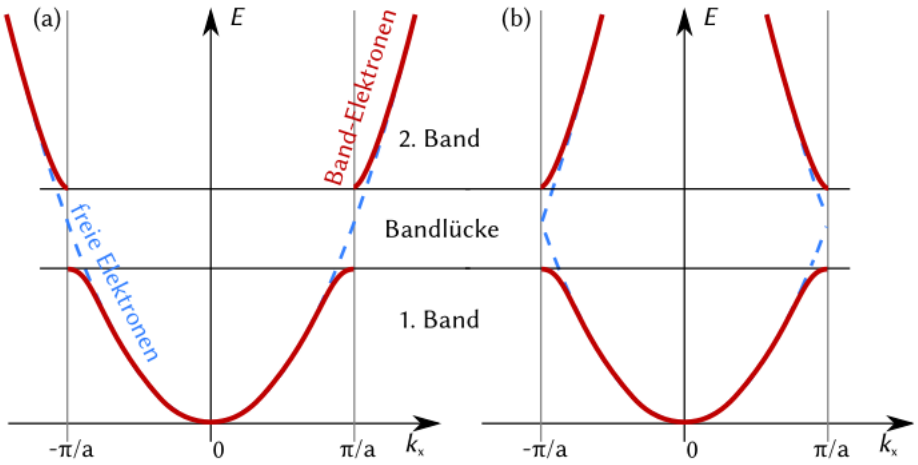

Figure 1.2: band gaps [Ger17]

1.3 Phononic and acoustic band structure

The experimental setup is made out of single metal pipes, that are coupled by apertures.
Or if you look at it in a mathematical way, coupled harmonic oscillators.
For these harmonic oszilators there are two different solutions:

$$\omega_1 = \sqrt{k^2/\rho\kappa}, \tag{1.8}$$

$$\omega_2 = \sqrt{k^2/\rho\kappa + 2b} \tag{1.9}$$

Where b is a constant that depends on the coupling of the pipes.
This can be seen as a splitting of the Eigenfrequencies just like in the quantum harmonic case.
Also just like in the quantum harmonic case the number of splits is equal to the number of oscillators (pipes)

1.4 Spherical resonator

In three dimensions the solution of the Schrödinger equation for an hydrogen atom and the solution of the Helmholtz equation for an sphere have different eigenfunctions in respect to the radial component but same in respect to the angular components.
This leads to the same eigenvalues for the quantum numbers l and m in both cases, while the radial equations and therefor its quantum number n' differ.
However it is possible to describe the resonances of the spherical resonator with the same radial

quantum numbers as the solution of the hydrogen atom.

$$n' \geq 0, l \geq 0, -l \leq m \leq l \tag{1.10}$$

$$n', l, m \in \mathbb{N} \tag{1.11}$$

The sources for the theory are:
[Ger17] [unk21]

2 Experimental setup

The experimental set up has to parts.

- the pipe pieces and apertures with different radius
- the spherical oscillator.

In the first part, there are different pipes (differ in length) and different apertures (differ in radius) which are composed to analogies for coupled atoms in one dimension.
In the second part the spherical oscillator is used to create an analogy to an hydrogen atom in three dimensions and even two coupled hydrogen atoms.
In both cases on the one side there is an microphone and on the other side an speaker, which makes it possible to measure the constructive and deconstructive interference inside the experimental set up and therefor to evaluate the eigenfunctions of the build crystals.

3 Execution

3.1 Modeling Bloch waves with an acoustic resonator

At first it was needed to test the measurement equipment was working fine, for this the microphone and the speaker where brought together without anything between them to change the soundwaves.

3.2 Sound velocity

To get the sound velocity a pipe of 60 cm made our of 8 pipe parts with 7.5 cm length was build. Then the speaker created sounds from 1-12000 Hz and the microphone recorded them. To find the sound velocity it is needed to get the resonance peaks.
With them and the formula:

$$c = \frac{2Lf}{n} \tag{3.1}$$

as well as the measured temperature 19.8-20°C, it is possible to get an velocity of the sound.

3.3 band structure in periodic potential

The band structure in a periodic potential can be studied in two different approaches one with the free electron model in mind and one with the thight binding model in mind.

3.3.1 Free electron model

For the free electron model it is used that the coupling strength of the electrons corresponds to the radius of the apertures.
To measure this, it is needed to connect 5 7.5 cm pipes with apertures between them and an aperture between the last pipe and the speaker, where all the apertures have the same radius.
Like before the resonance in an spectrum form 1-12 kHz is measured. The used aperture radius in cm are 2, 1.6, 1, 1.3, 0.7, 0.3.

3.3.2 Tight binding model

In analogy to the tight binding model we start with an single atom (an pipe of 7.5 cm and an aperture
In the next steps new atoms get connected to the first, all with the same aperture.
This is repeated with other apertures, while in each step the resonance in an spectrum form 1-12 kHz is measured.

3.4 Defects

There are different kinds of defects, that can occur in an crystal// To get an impression how defects substitutional and intersectional defects influence the band structure an atom with an

pip length of 5 cm and an aperture of 1.6 cm is used.
This atom is build into the cystal at different places and the effect on the resonance spectrum is looked at.
For the substitution 4 "normal" atoms where used, while for the intersectional defect 5 "normal" atoms had to be there.
An other defect is measured by the use of 4 atoms with pipe length 7.5 cm and an aperture x of 1.3 cm. An pipe of 5 cm is inserted in the middle of the crystal, this time with an aperture y that differs from the aperture of the others. The effect on the resonance is recorded.

3.5 Unit cells with more than one atom

Instead of using one atom (pipe) to construct an unit cell, two pipes 7.5 cm and 5 cm, that are separated thew an aperture with 1.6 cm are used.
5 of these unit cells are used like before to build an crystal.

3.6 Hydrogen atom

The hydrogen atom is made out of two hollow hemispheres, that can be rotated to each other.
First the resonance peaks get measured, than the second and third resonance frequency are taken, to evaluate the dependency of the resonance from the angle between the two hemispheres, to get l.
In the next step spacers of 3,6,9 mm are build into the set up.Then the third peak gets evaluated a second time.
Because of the lifting of the degeneracy of m there are three peaks. For all of them there dependency on α gets measured. To identify m.

3.7 Hydrogen molecule

Two hydrogen atoms get connected to an hydrogen molecule. (Two Spheres separated by an aperture.)
Again the resonance peaks get measured an the resonance peaks are taken. In difference to the hydrogen atom now the influence of the apertures is interesting, so there are different apertures used to separate the two spheres.

4 Evaluation, error analysis, and discussion of the measurement results

4.1 Modeling Bloch waves with an acoustic resonator

For further evaluation the microphone and spectrum of speaker and microphone have to be calibrated/ checked for additional resonance peaks that should be ignored in analysis of the resonance spectrum. For perfect measurement the spectrum of the system should be constant but varies because of the properties of microphone and speaker. The spectrum can be seen in figure 4.4.

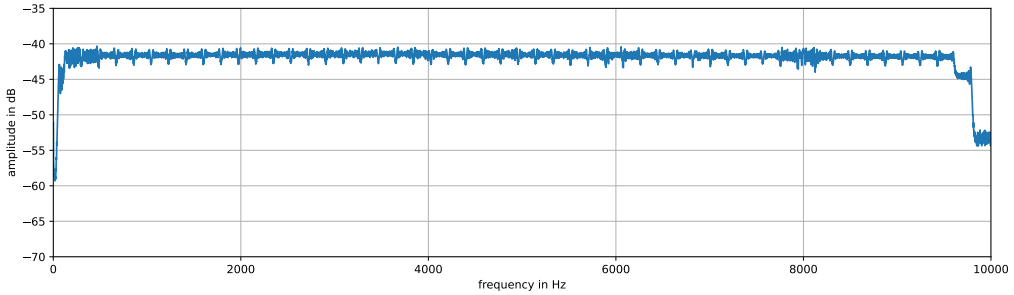

Figure 4.1: Spectrum of calibration

4.1.1 Sound velocity

To calculate the sound velocity there are three ways. First it is calculated with the resonance peaks measured with the oscilloscope and the Quantum Analogs device. Second with the resonance peaks measured with the sound card and Audacity. The last calculation is by measuring the temperature and using the given website to determine the theoretical sound velocity. For the first two options equation 3.1 is used to perform a linear fit with $2Lf$ on the y-axis and the order n on the x-axis.

Table 4.1: Calculated sound velocities

measurement	sound velocity
oscilloscope	$343.25\,\frac{\mathrm{m}}{\mathrm{s}} \pm 0.83\,\frac{\mathrm{m}}{\mathrm{s}}$
sound card	$341.27\,\frac{\mathrm{m}}{\mathrm{s}} \pm 2.48\,\frac{\mathrm{m}}{\mathrm{s}}$
temperature	$[343.304\,\frac{\mathrm{m}}{\mathrm{s}}, 343.412\,\frac{\mathrm{m}}{\mathrm{s}}]$

Overall all calculated sound velocities fulfill the expected sound velocity of $343\,\frac{\mathrm{m}}{\mathrm{s}}$. Depending on the method of measurement the error on the calculated value varies.

Oscilloscope data

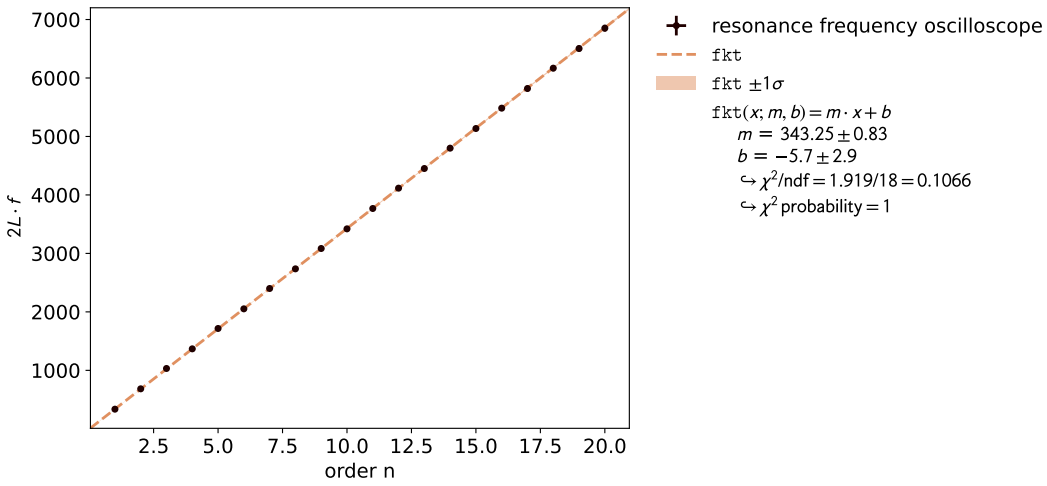

Figure 4.2: Fitted oscilloscope data

Table 4.2: Measured resonace peak from oscilloscope

peak	frequency (kHz)
1	5.710000
2	5.420000
3	5.140000
4	4.850000
5	4.570000
6	4.280000
7	4.000000
8	3.710000
9	3.430000
10	3.140000
11	2.850000
12	2.570000
13	2.280000
14	2.000000
15	1.710000
16	1.430000
17	1.140000
18	0.860000
19	0.570000
20	0.280000

Sound card data

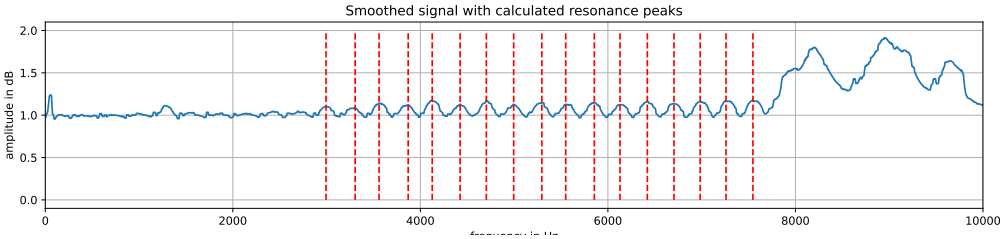

Figure 4.3: sound card spectrum

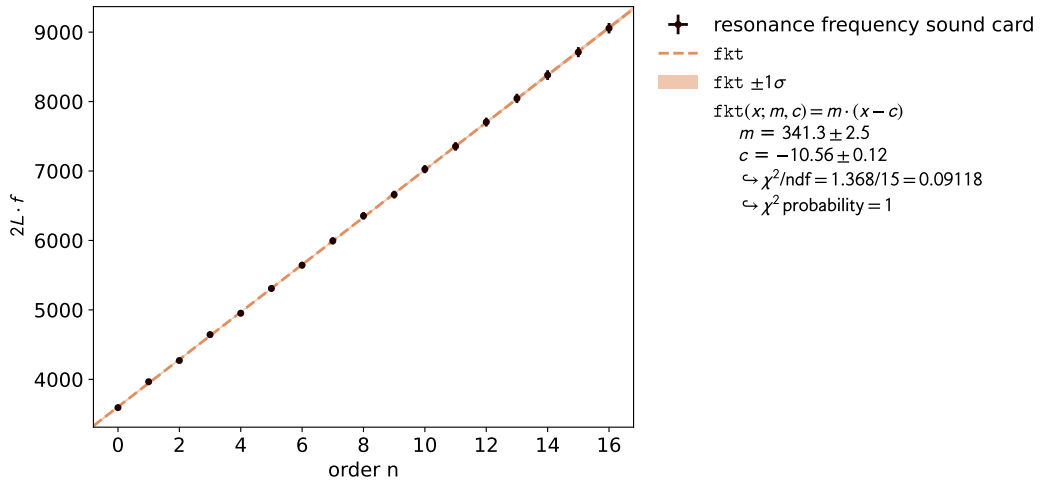

Figure 4.4: Fitted sound card data

4.2 band structure in periodic potential

4.2.1 free electron model

For the free electron model the resonance peaks of the adjusted spectrum are identified and based on the peaks the band structure model created.
Looking and the band structure, as the potential increases and therefore the diameter gets smaller, the energy gaps within the free electron model widen. This observation aligns with the principles of quantum mechanics, where the interplay between the external potential and the behavior of electrons in a crystal lattice creates distinct energy bands. The widening of energy gaps signifies a more pronounced separation between the electron energy levels, influencing the overall electronic properties of the material.

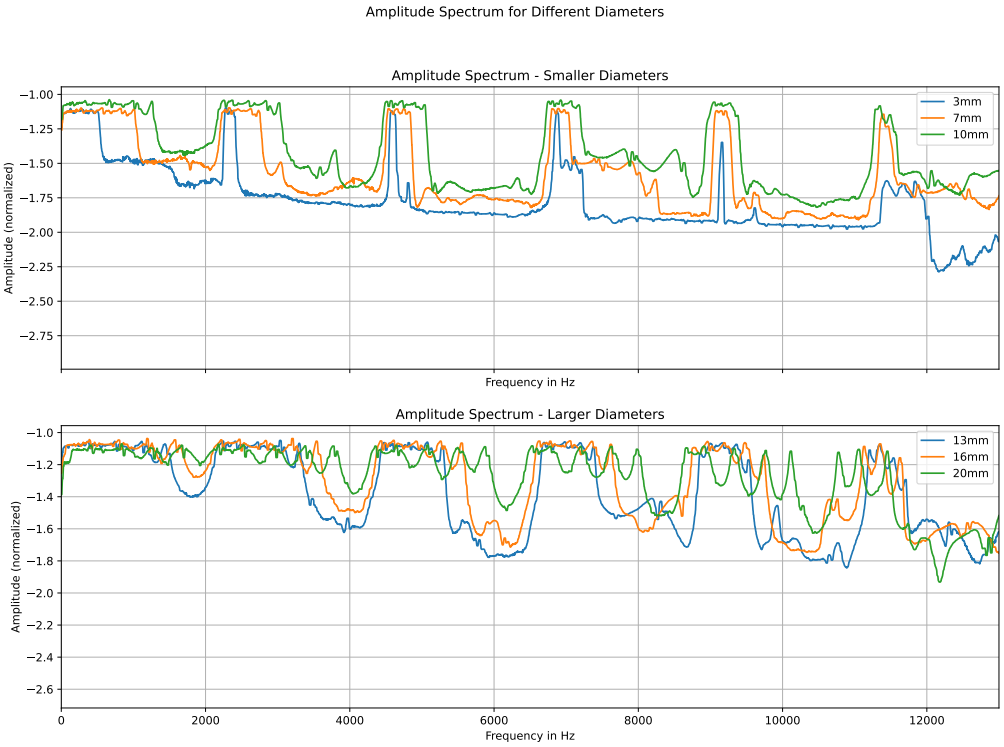

Figure 4.5: free electron model spectrum

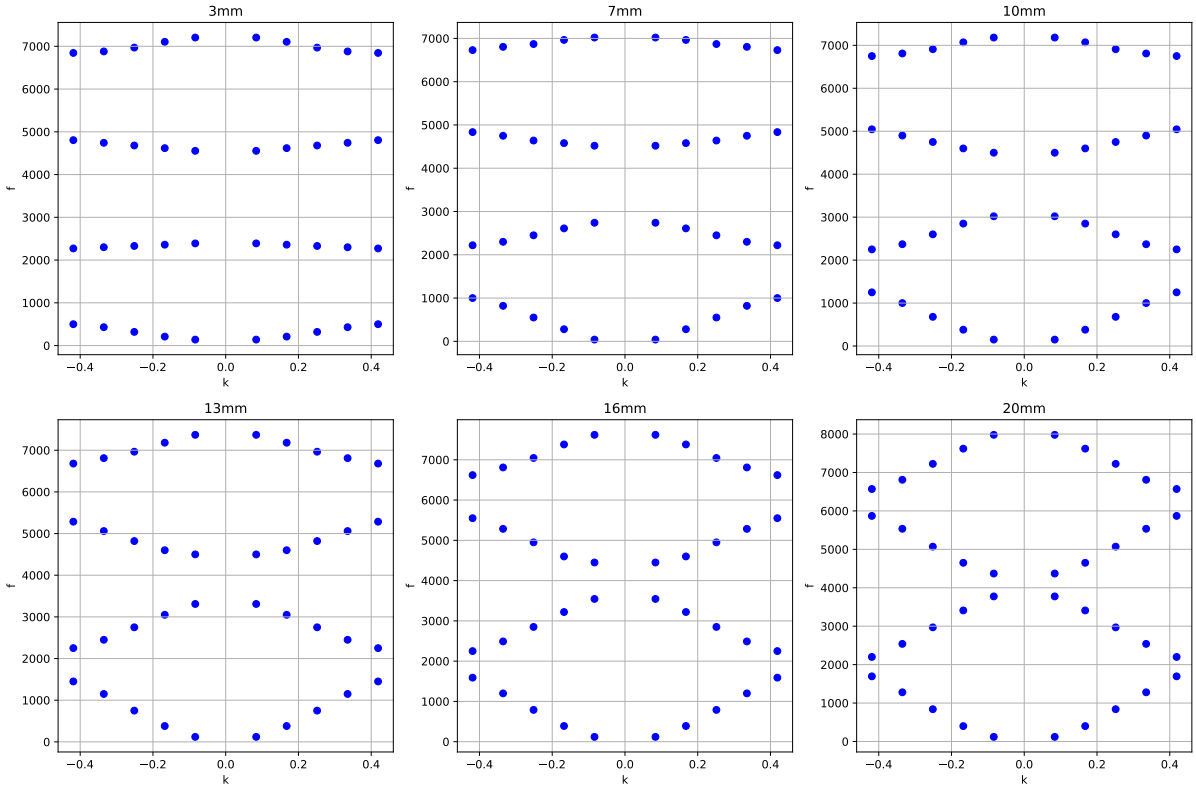

Figure 4.6: free electron model band structure

4.2.2 Tight binding model

In the tight binding model, for each new segment that is added the spectrum gets additional peaks because of the overlapping electron wave functions of the different segments. Again the spectrum gets visualised and the resonance peaks get identified to look at the change of peaks and band structures in the tight binding model.

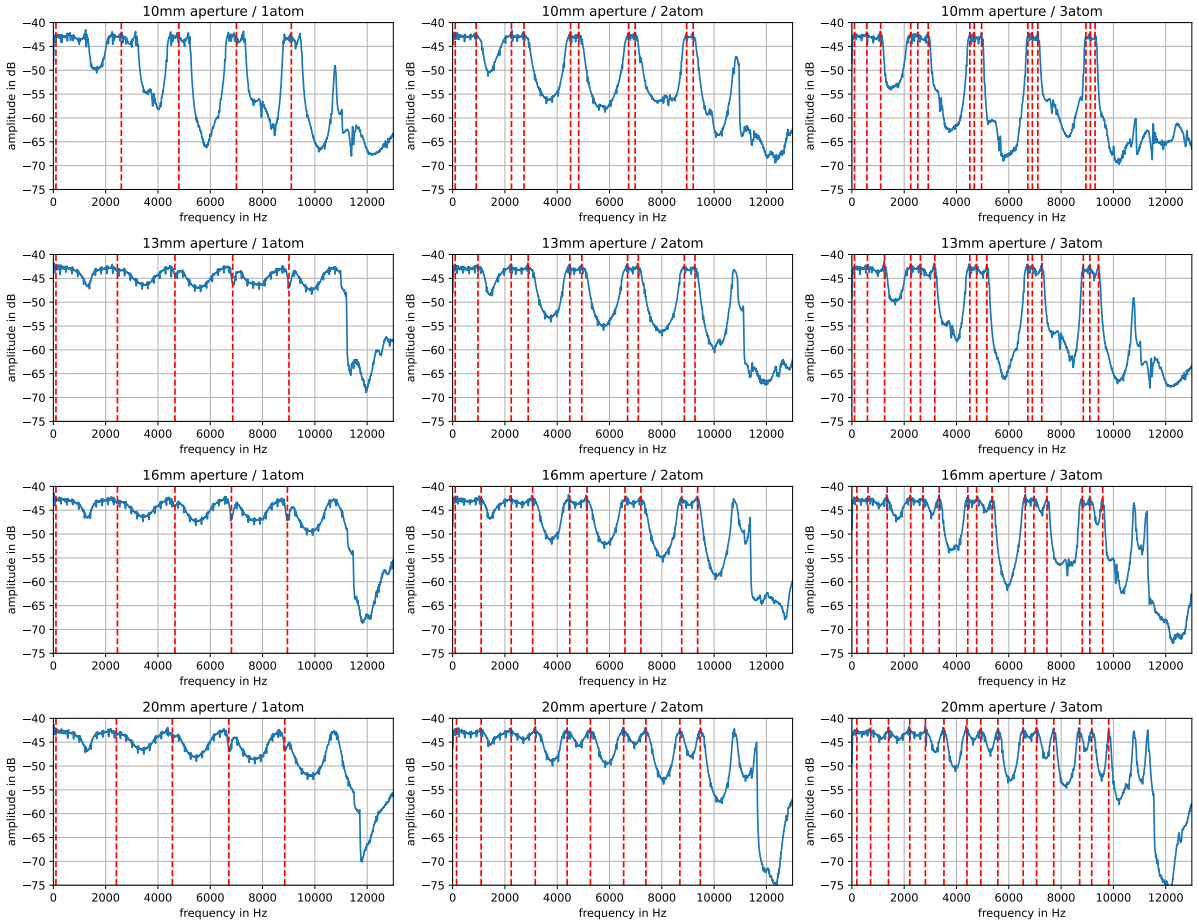

Figure 4.7: Tight binding model spectrum

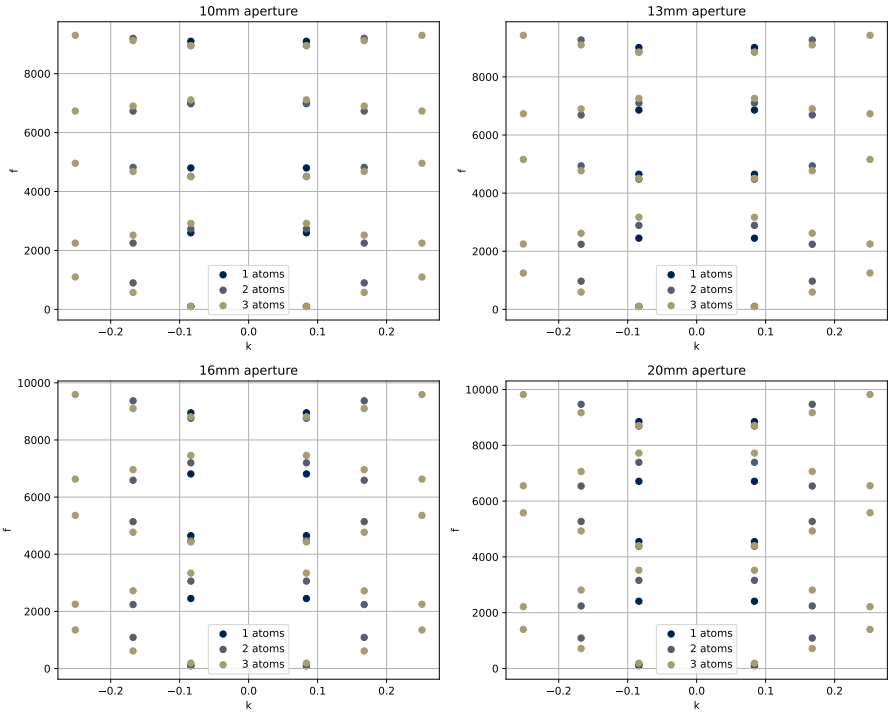

Figure 4.8: Tight binding model spectrum

4.2.3 Defects

This section visualizes two distinct crystallographic defects that can occur within a crystalline structure. The first defect under scrutiny is the substitutional defect, a phenomenon wherein one segment of the crystal lattice undergoes substitution by a segment with a smaller length. In the substitutional defect scenario, the regular lattice structure experiences a modification, introducing a different segment that deviates in size from the original lattice. This substitution can result from various factors such as impurities, thermal fluctuations, or external influences. The second defect explored in this section is the interstitial defect, where an additional smaller segment integrates into the crystal lattice. This introduction of an extra segment into the lattice structure creates what is known as an interstitial site, disrupting the regular arrangement of atoms or ions. Defects in crystals induce new states in the band structures, manifesting as additional peaks in resonance spectra compared to a crystal with no defects, as seen in figure 4.9.

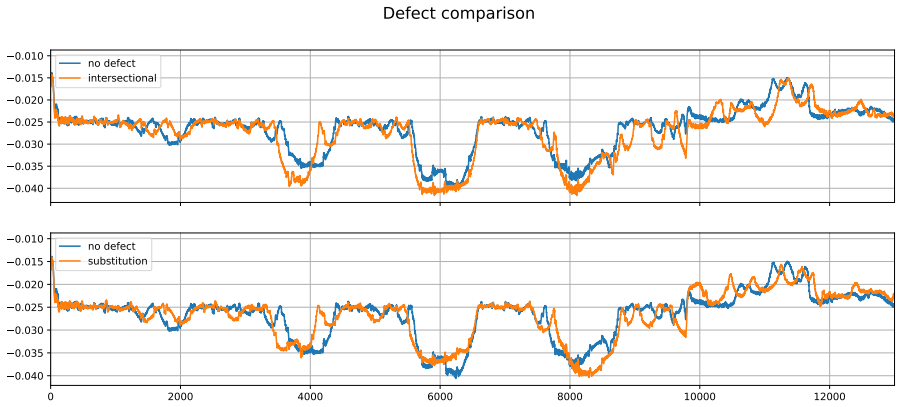

Figure 4.9: Defects

4.2.4 Two-atom cells

The current elementary cell has undergone a doubling in size compared to its monatomic counterpart, resulting in a significant shift in its properties. This enlargement causes a notable reduction in the size of the first Brillouin Zone, which is now only half of its previous dimensions, as the width is determined to be $\frac{\pi}{a}$.
Observing the resonance spectrum in Figure 4.10, each band within the spectrum undergoes a split into two smaller sub-bands. Remarkably, each of these sub-bands exhibits a resonance corresponding to each utilized cell.

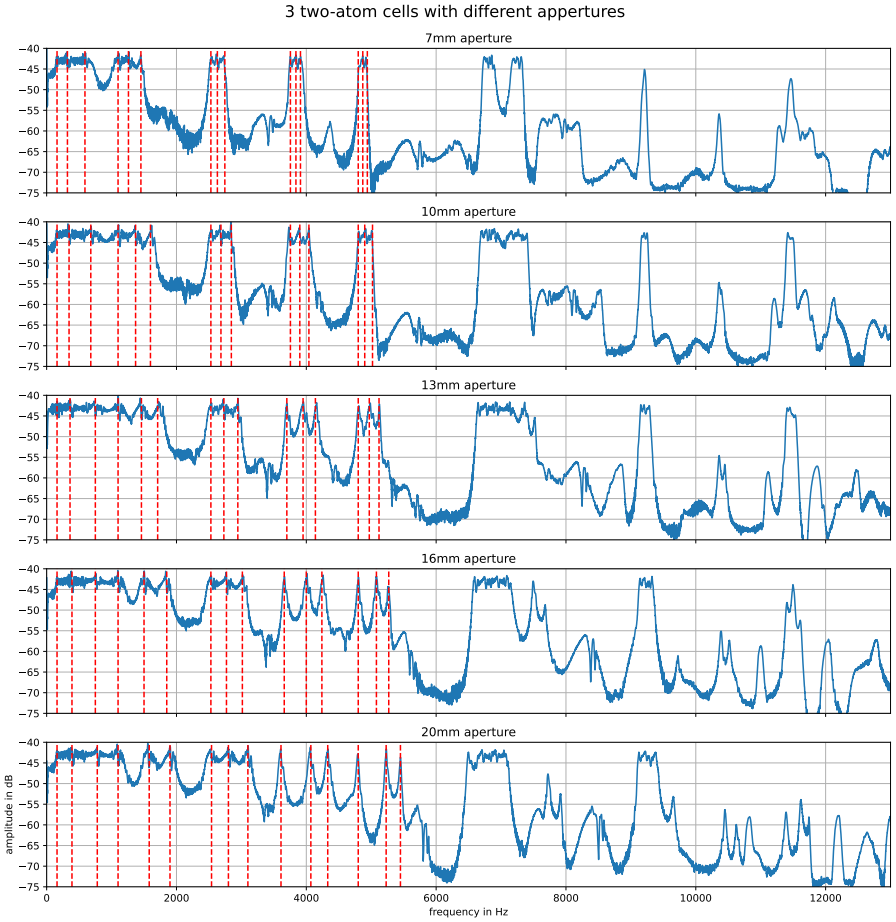

Figure 4.10: Two-atom cell spectrum with different coupling

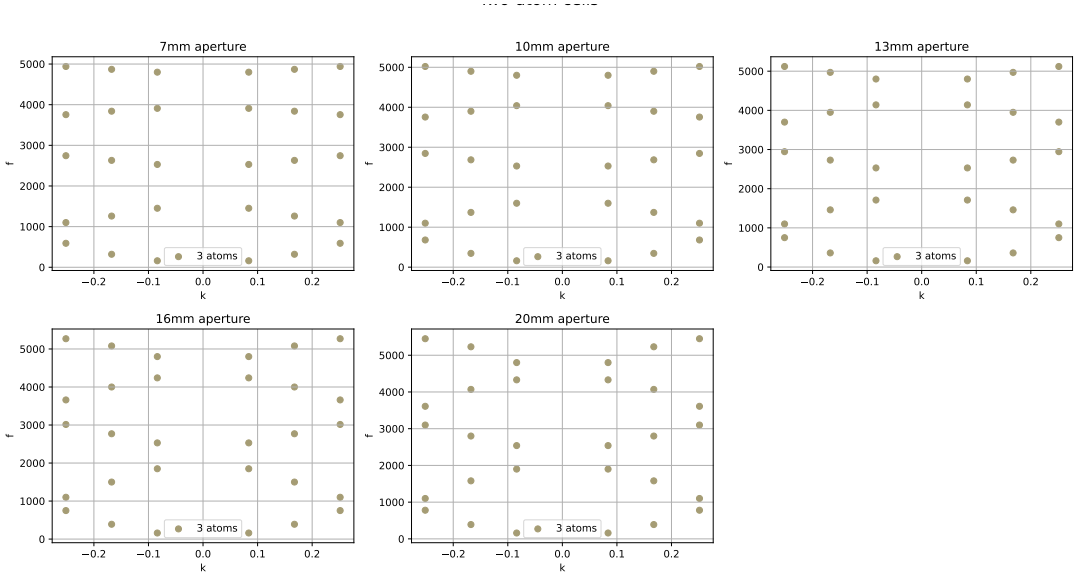

Figure 4.11: Band structure for two-atom cells with different coupling

4.3 Hydrogen atom

4.3.1 Maxima of the resonance in an spherical resonator

With the sound card the maxima in the amplitude spectrum of frequencies in an range of 1-12000 Hz at an angle $\alpha = 180°$ get evaluated and with the quantum analogs device this frequencies get specified to the values in table 4.3. In the process, the second peak got confused with the first one, so the rest of the experiment has been done for the peaks 3,4,5 instead of 2,3,4.
The peak at around 400 Hz (495 Hz) was ignored like the experimental map said.

peak	(qa) frequency kHz	(sc) frequency kHz
1	-	1.898
2	2.27	2.269
3	3.660	3.649
4	4.93	4.922
5	6.16	6.154
6	7.36	7.354
7	8.44	8.532

Table 4.3: peaks of hdrogen measured with the quantum analog device(qa) an the sounscard of the computer (sc)

4.3.2 Angular momentum l

With this knowledge over the peaks the dependency of the amplitude of the resonance peaks from the angle α can be measured.
To get θ out of α the formula

$$\theta = arccos(\frac{1}{2}cos(\alpha) - \frac{1}{2}) \tag{4.1}$$

is used, and the values get mirrored along the axis to get the angles of θ from 0°to 360 °.
The magnetic quantum number $m = 0$, because only such resonances as $m = 0$ are measured without an aperture.
To find out the angular momentum l the nodes in the figures 4.12 -4.14 are counted.
For the peak at 3660 Hz there are two nodes so $l = 2$.
For the peak at 4930 Hz it is not totally clear if the peak in at 90°is relevant,because it is so small in comparison to the others, but because we measured definitively that there was such a peak, there are for nodes so $l = 4$.
For the peak at 6160 Hz there are five nodes so $l = 5$.

4.3.3 Magnetic quantum number

To verify that the degeneracy of m the resonance up to the first three resonance peaks get measured for different spacers of 3,6,9 mm like it can be seen in figure 4.16 to 4.18.
As can bee seen in figure 4.19 the split of the peaks depends positive on the thickness of the spacers.

To get the magnetic quantum number m it is needed to measure the height of the resonance peak for the different spitted resonance peaks of the former third peak at 3432 Hz ,3451 Hz and 3596 Hz with the spacer of 9 mm, between the tow spheres
This means to measure the degeneracy of the $l = 2$ spherical harmonics. The plots can be seen

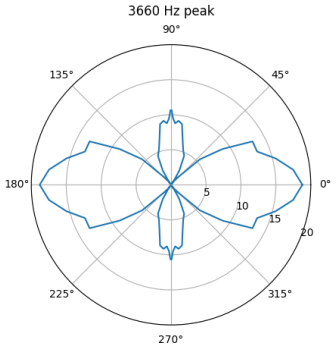

Figure 4.12: 3660 Hz peak

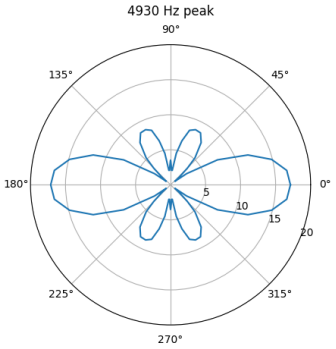

Figure 4.13: 4930 Hz peak

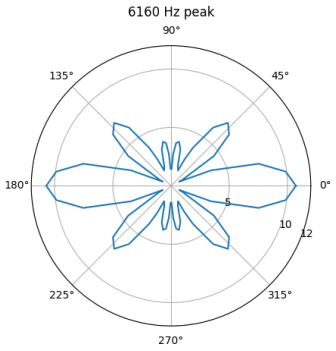

Figure 4.14: 6160 Hz peak

in figure 4.20 - 4.22

However the comparison of 4.20 - 4.22
with the spherical harmonics $Y_l^m(\theta, \varphi)$ as they can be seen in figure 4.23.
Brings us to the conclusion that for the different resonance peaks:

- 3432 Hz: $m = \pm 1$
- 3451 Hz: $m = \pm 2$
- 3596 Hz: $m = \pm 0$

4.4 Hydrogen molecule

At last different apertures of 5,10,20,25mm get used to build an hydrogen molecule. And the resonance from 1-4000 Hz is measured like it can bee seen in figure 4.24. And in the figures 4.25 to 4.29.
It can bee recognised, that the height of the peaks depends on positive on the aperture radius, and in a way that for apertures ≥ 15 mm there can be seen a peak around 1kHz, while there is none for lesser radii.
Also in comparison to the hydrogen atom 4.30 there are two peaks around 2.2Hz instead of one. Which brings us to the conclusion that the apertures correspond to the degeneracy of the resonance peaks.

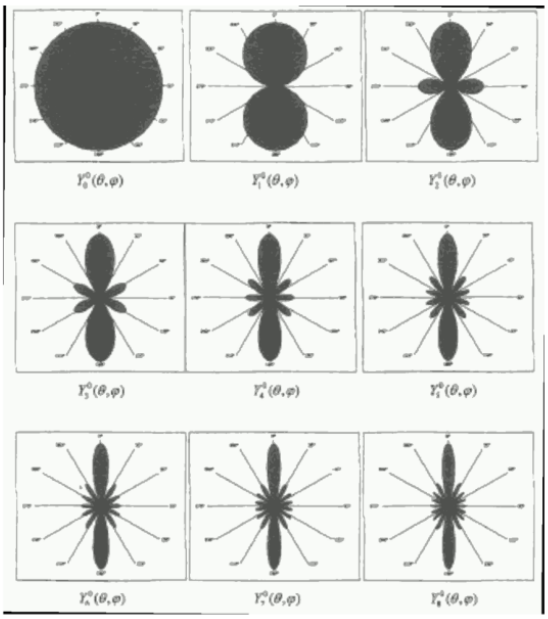

Figure 4.15: Cut threw the spherical harmonics with magnetic numbers m = 0

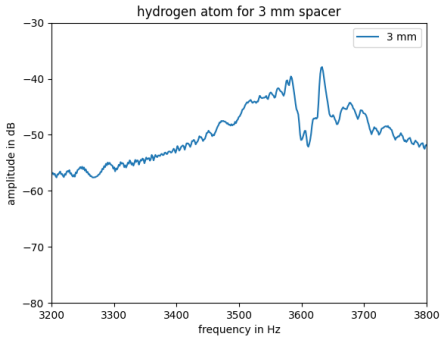

Figure 4.16: hydrogen atom for 3mm spacer

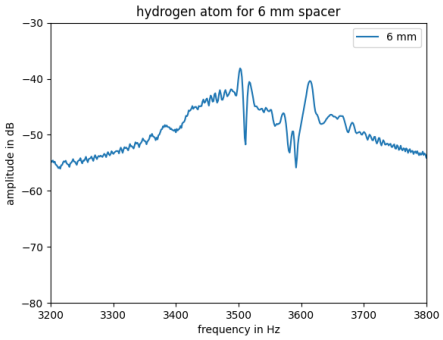

Figure 4.17: hydrogen atom for 6mm spacer

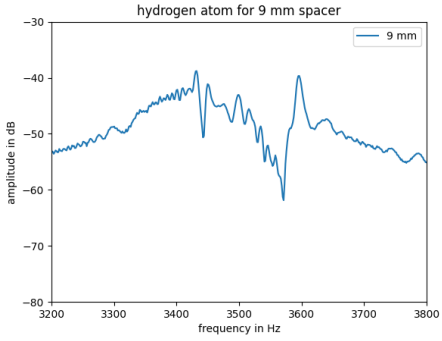

Figure 4.18: hydrogen atom for 9mm spacer

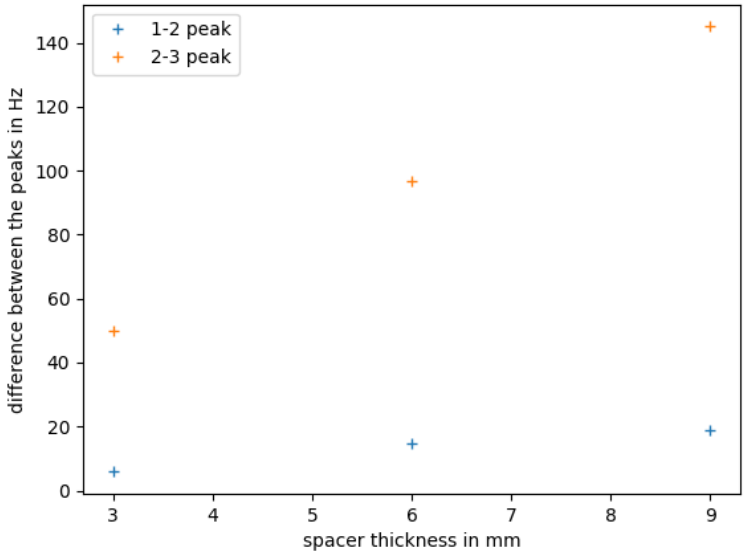

Figure 4.19: splitting of the different peaks for variable spacer thickness

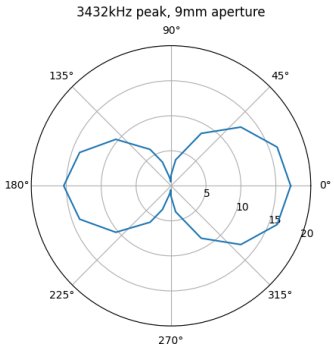

Figure 4.20: 3432 Hz peak

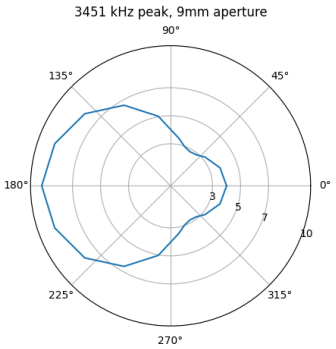

Figure 4.21: 3451 Hz peak

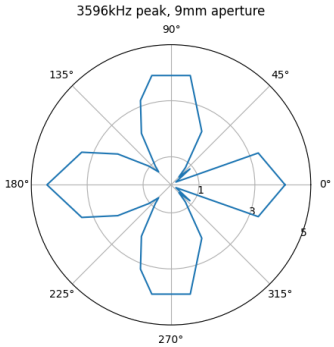

Figure 4.22: 3596 Hz peak

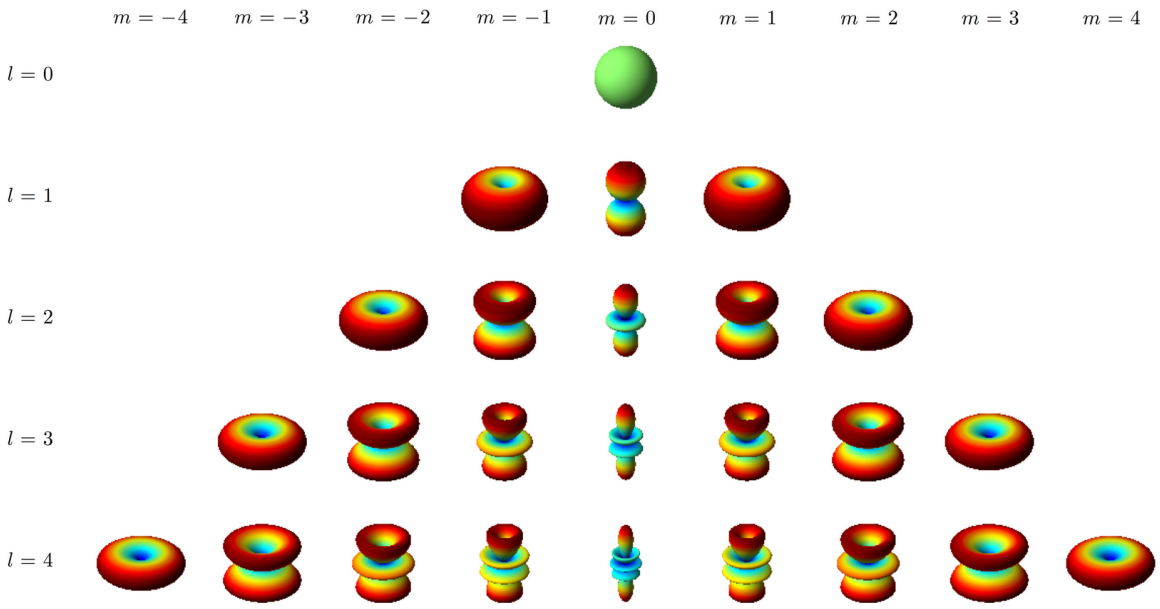

Figure 4.23: spherical harmonics with different m, l from [PHJ]

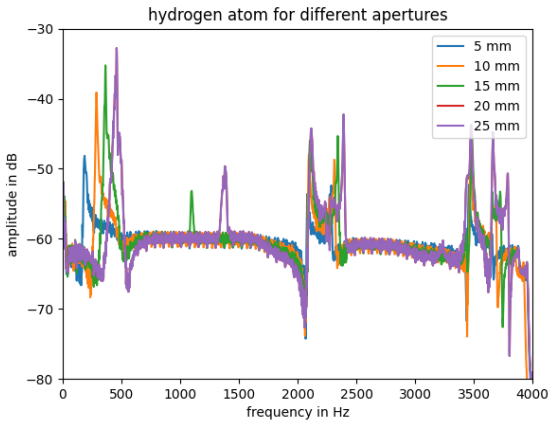

Figure 4.24: hydrogen atom for different apertures

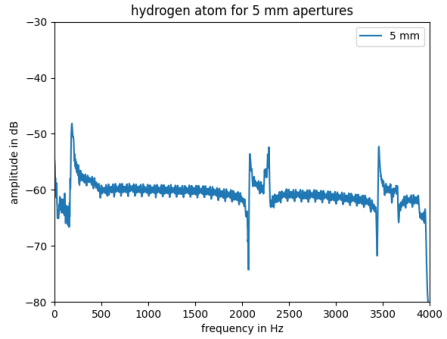

Figure 4.25: hydrogen atom for 5mm apertures

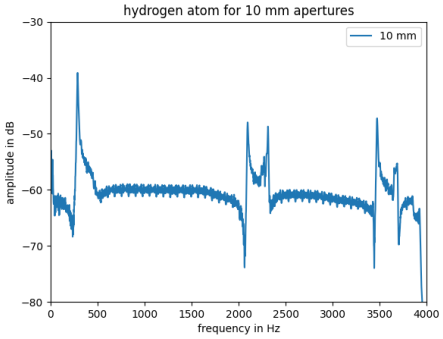

Figure 4.26: hydrogen atom for 10mm apertures

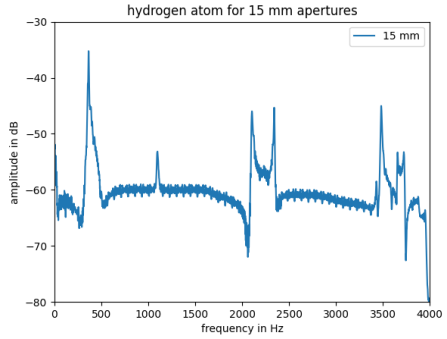

Figure 4.27: hydrogen atom for 15mm apertures

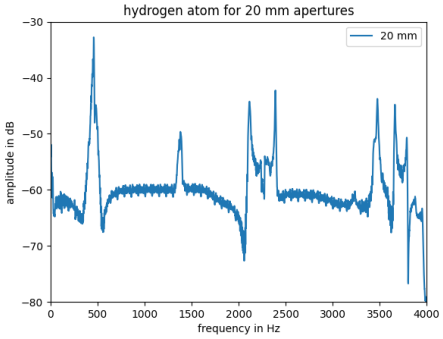

Figure 4.28: hydrogen atom for 20mm apertures

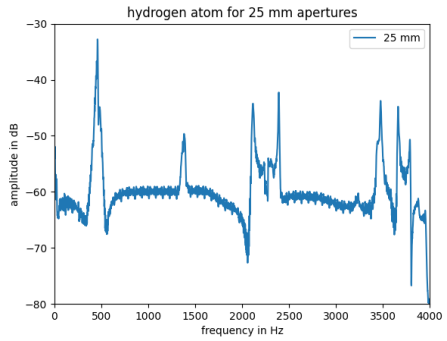

Figure 4.29: hydrogen atom for 25mm apertures

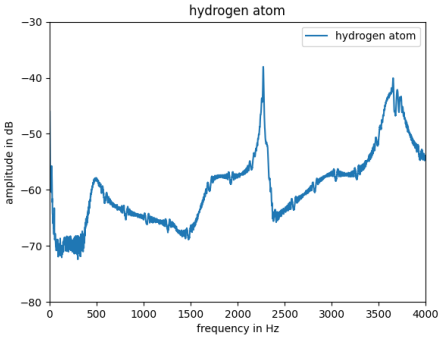

Figure 4.30: hydrogen atom

Sources

[Ger17] Gerst, Angelika: *Schallwellenexperimente zur Illustration fundamentaler Quantenphysik Versuchsvorbereitung.* 28. September 2017. Karlsruhe, 2017

[PHJ] `http://opticaltweezers.org/chapter-5-electromagnetic-theory/figure-5-2-spherical-harmonics/`, letzter Zugriff 06.01.2024

[unk21] unknown: *Quantum Analogs.* 2 November 2021. Karlsruhe, 2021