$$V_{eff}(r) = \frac{mL}{2r^2} - \frac{mMG}{r}$$

Erde • ——— R ——— × (in Ruhe am Anfang)

Wie lange dauert es bis der Satellit auf die Erde trifft: Impact time T

gegebenen Größen: m, M, G, R

$$E_{pot} \simeq \frac{mMG}{R} \qquad E_{kin} = m\left(\frac{R}{T}\right)^2$$

$$\frac{mMG}{R} \cong \frac{m}{2}\left(\frac{R}{T}\right)^2$$

$$T^2 \sim \frac{1}{MG} R^3$$

$$T = C\frac{1}{\sqrt{MG}} R^{3/2}$$

$$E = \frac{m}{2}\vec{v}^2 + V(\vec{r}) \qquad V(\vec{r}) = -\frac{GmM}{r}$$

• ——— × $\quad \vec{v}$

$$\vec{v} = \frac{dr}{dt}\vec{e}_r$$

$$E = \frac{m}{2}\left(\frac{dr}{dt}\right)^2 + V(r) \qquad t=0 \quad E = -\frac{GmM}{R}$$

$$\frac{dr}{dt} = \pm\sqrt{\frac{2}{m}\left(E - V(r)\right)}$$

$\underbrace{\bullet\text{———}\times}_{r}$

$$\frac{dr}{dt} = -\sqrt{\frac{2}{m}\left(\frac{GmM}{r} - \frac{GmM}{R}\right)}$$

$$= -\sqrt{2MG}\left(\frac{1}{r} - \frac{1}{R}\right)^{1/2}$$

$$\int_R^0 \frac{dr}{\sqrt{\frac{1}{r} - \frac{1}{R}}} = -\sqrt{2MG}\int_0^T dt$$

$$\int_a^b dx = -\int_b^a dx$$

$$\int_0^R \frac{dr}{\sqrt{\frac{1}{r} - \frac{1}{R}}} = \sqrt{2MG}\; T$$

$$x = r/R$$

$$dr = R\,dx$$

$$\int_0^1 \frac{R\,dx}{\sqrt{\frac{1}{Rx} - \frac{1}{R}}} = \sqrt{2MG}\; T$$

$$T = \frac{1}{\sqrt{2MG}} R^{3/2} \underbrace{\int_0^1 dx \frac{1}{\sqrt{\frac{1}{x} - 1}}}_{= \int_0^1 dx \sqrt{\frac{x}{1-x}}}$$

$$\int_0^1 dx \frac{1}{\sqrt{1-x}} = -2(1-x)^{1/2}\Big|_0^1$$

$$x = \sin^2 t$$

$$\sqrt{\frac{x}{1-x}} = \tan t$$

$$\frac{dx}{dt} = 2\sin t\cos t$$

$$\int_0^1 dx \sqrt{\frac{x}{1-x}} = 2\int_0^{\pi/2} \frac{\sin t}{\cos t}\, \sin t \cos t \, dt$$

$$= 2\int_0^{\pi/2} \sin^2 t \, dt = \frac{\pi}{2}$$

$$T = \frac{1}{\sqrt{2MG}}\, R^{3/2}\, \frac{\pi}{2} = C\, \frac{R^{3/2}}{\sqrt{MG}}$$

$$C = \frac{\pi}{2^{3/2}}$$

man könnte doch denken:

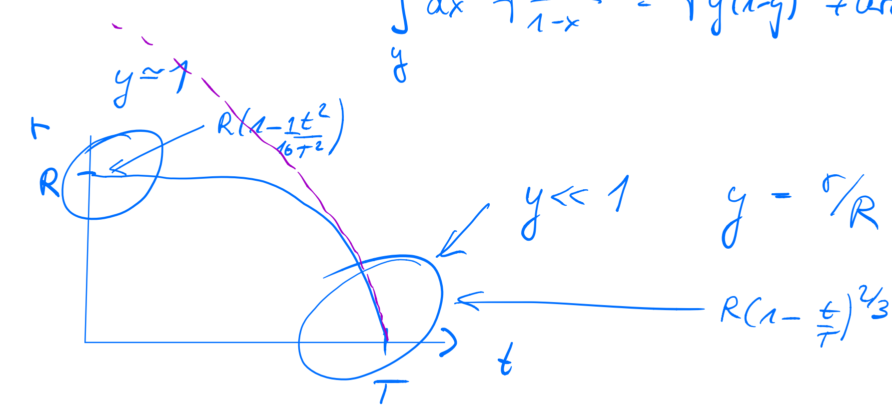
$$\boxed{r(t) = R\left(1-\frac{t}{T}\right)^{2/3}}$$

Vermutung (stellt sich als falsch heraus)

$$\int_R^{r(t)} \frac{dr'}{\sqrt{2\left(\frac{GM}{r'}-\frac{GM}{R}\right)}} = -\int_0^t dt'$$

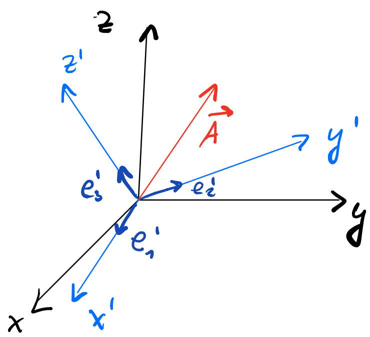
$$\int_{r(t)/R}^1 \frac{dx}{\sqrt{\frac{1}{x}-1}} = \sqrt{2MG}\; t$$

$$\int_y^1 dx \sqrt{\frac{x}{1-x}} = \sqrt{y(1-y)} + \arccos(\sqrt{y})$$



Scheinkräfte

Wir betrachten die Bewegungsgleichung in einem rotierenden Koordinatensystem



$$\vec{A} = A_1'\vec{e}_1' + A_2'\vec{e}_2' + A_3'\vec{e}_3'$$

$$\left.\frac{d\vec{A}}{dt}\right|_L = \sum_{i=1}^{3}\left(\frac{dA_i'}{dt}\,\vec{e}_i' + A_i'\,\frac{d\vec{e}_i'}{dt}\right)$$

$$= \left.\frac{d\vec{A}}{dt}\right|_B + \sum_{i=1}^{3} A_i'\,\frac{d\vec{e}_i'}{dt}$$

$$\frac{d}{dt}\left(\vec{e}_i'\cdot\vec{e}_j' = \delta_{i,j}\right.$$

$$\dot{\vec{e}}_i'\cdot\vec{e}_j' + \vec{e}_i'\cdot\dot{\vec{e}}_j' = 0$$

$i = j$ $\qquad \dot{\vec{e}}_i'\cdot\vec{e}_i' = 0$ $\qquad \vec{e}_i \perp \dot{\vec{e}}_i$ $\quad (*)$

$i \neq j$ $\qquad \dot{\vec{e}}_i'\cdot\vec{e}_j' = -\vec{e}_i'\cdot\dot{\vec{e}}_j'$ $\quad (**)$

$(*) \Longrightarrow$

$$\dot{\vec{e}}_1' = a_1\vec{e}_2' + a_2\vec{e}_3'$$

$$\dot{\vec{e}}_2' = a_3\vec{e}_1' + a_4\vec{e}_3'$$

$$\dot{\vec{e}}_3' = a_5\vec{e}_1' + a_6\vec{e}_2'$$

$$\vec{e}_2'\cdot\dot{\vec{e}}_1' = a_1 \qquad \vec{e}_1'\cdot\dot{\vec{e}}_2' = a_3$$

$$\Longrightarrow a_1 = -a_3$$

analog: $a_6 = -a_4$ $\qquad a_5 = -a_2$

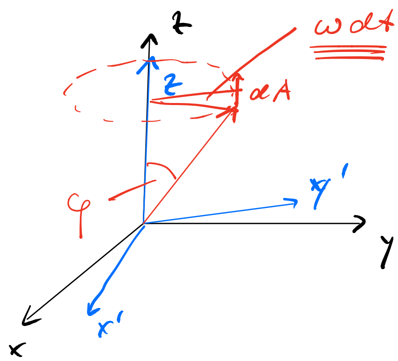
$$\left.\frac{d\vec{A}}{dt}\right|_L = \left.\frac{d\vec{A}}{dt}\right|_B + \vec{e}_1{}'\left(-a_1 A_2' - a_2 A_3'\right) + \vec{e}_2{}'\left(a_1 A_1' - a_4 A_3'\right) + \vec{e}_3{}'\left(a_2 A_1' + a_4 A_2'\right)$$

$$\vec{\omega} \equiv (a_4, -a_2, a_1) = (\omega_1, \omega_2, \omega_3)$$

$$\left.\frac{d\vec{A}}{dt}\right|_L = \left.\frac{d\vec{A}}{dt}\right|_B + \vec{\omega} \times \vec{A}$$

$$(\vec{\omega} \times \vec{A}')_1 = \varepsilon_{1ij}\, \omega_i A_j' = \omega_2 A_3' - \omega_3 A_2'$$

Interpretation von $\vec{\omega}$



$$dA = \omega\, dt\, \sin\varphi \cdot A$$

$$\frac{d\vec{A}}{dt} = \vec{\omega} \times \vec{A}$$

Winkelgeschwindigkeit