Energieerhaltung:

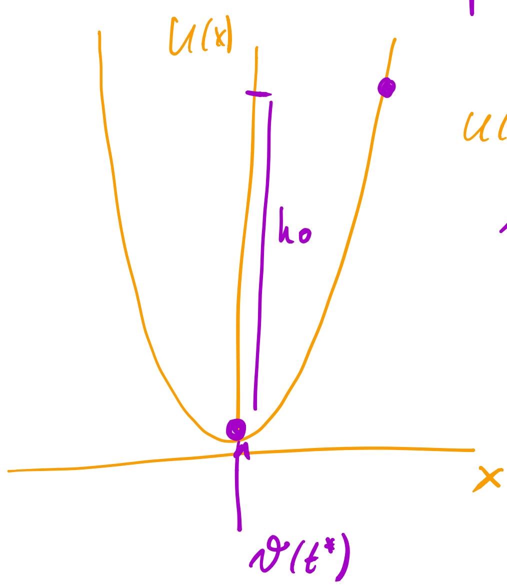
$$m\frac{d^2x}{dt^2} = F(x,t) \qquad \text{mit } F(x,t) = F(x)$$

$$\Rightarrow \quad E(t) = \frac{m}{2} v^2(t) + U(x(t))$$

$$F = -\frac{dU(x)}{dx}$$

$$\frac{dE}{dt} = 0$$

$$E(t) = E(t_0)$$



$U(x) = A\,x^{328}$

Am Anfang in Ruhe

$v(t=0) = 0$

$E(t=0) = A\,h_0^{328}$

$t^* : \; U(t^*) = 0$

$E(t^*) = \frac{m}{2} v^2(t^*)$

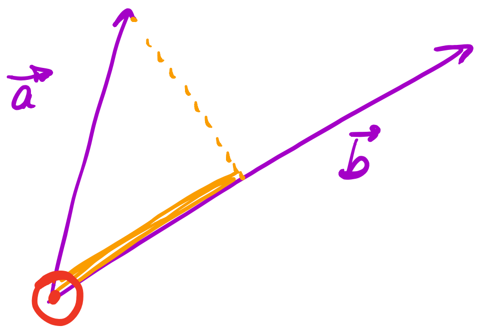
$$\Rightarrow \quad v^2(t^*) = \frac{2A}{m} h_0^{328}$$

Vektoren

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

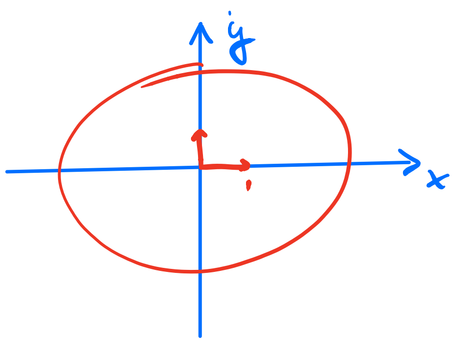
$$\vec{a} \cdot \vec{b} = \sum_{i=1}^{3} a_i b_i$$

$\vec{a} \cdot \vec{b} = 0$
wenn $\perp$



$$\vec{c} = \vec{a} \times \vec{b}$$
$$= \sum_{ijk} \varepsilon_{ijk} a_j b_k \vec{e}_i$$

$\vec{c} = \vec{0}$ wenn $\parallel$



Bewegung auf dem Kreis

$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad |\vec{r}| = R = \sqrt{\vec{r} \cdot \vec{r}}$
const.

$\vec{e}_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad \vec{e}_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad \omega(t+T) = \omega t + 2\pi$

$$\vec{r}(t) = R\cos(\omega t)\,\vec{e}_x + R\sin(\omega t)\,\vec{e}_y$$

$\boxed{T = \frac{2\pi}{\omega}}$
Umlaufzeit
ω: Kreisfrequenz

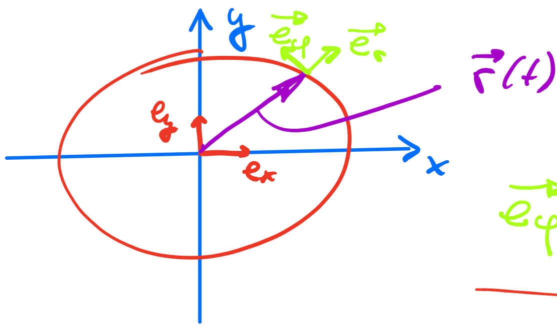
$$\vec{r}^{\,2} = \vec{r} \cdot \vec{r} = \left(R\cos(\omega t)\,\vec{e}_x + R\sin(\omega t)\,\vec{e}_y\right) \cdot \left(R\cos(\omega t)\,\vec{e}_x + R\sin(\omega t)\,\vec{e}_y\right)$$
$$= R^2\cos^2(\omega t) + R^2\sin^2(\omega t)$$
$$= R^2$$

$\vec{e}_x \cdot \vec{e}_x = 1$

$\vec{e}_x \cdot \vec{e}_y = 0$

$\cos^2\alpha + \sin^2\alpha = 1$

$$\vec{r}(t) = R\cos(\omega t)\,\vec{e}_x + R\sin(\omega t)\,\vec{e}_y$$



$$\vec{e}_r = \cos(\omega t)\,\vec{e}_x + \sin(\omega t)\,\vec{e}_y$$

$$\vec{e}_\varphi = -\sin(\omega t)\,\vec{e}_x + \cos(\omega t)\,\vec{e}_y$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \left(-R\sin(\omega t)\,\vec{e}_x + R\cos(\omega t)\,\vec{e}_y\right)\omega$$

$$\vec{r}(t) = R\,\vec{e}_r(t)$$

$$\vec{v}(t) = R\omega\,\vec{e}_\varphi(t)$$

$$\frac{d\vec{e}_\varphi}{dt} = \left[-\cos(\omega t)\,\vec{e}_x - \sin(\omega t)\,\vec{e}_y\right]\omega$$

$$\frac{d^2\vec{r}}{dt^2} = \frac{d\vec{v}}{dt} = R\omega\,\frac{d}{dt}\vec{e}_\varphi(t)$$

$$= -R\omega^2\,\vec{e}_r = \frac{\vec{F}}{m}$$

partielle Ableitungen

$f(x) \qquad f'(x) \longrightarrow \frac{d f(x)}{dx}$

Funktion von mehreren Variablen

$u(x, y, z, t) \qquad x = x(t), \quad y = y(t)$

$$\frac{du}{dt} = \frac{d}{dt} u(x(t), y(t), z(t), t)$$

$$= \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial u}{\partial t}$$

$$f(g(t)) \qquad \frac{df}{dt} = \frac{df}{dg}\frac{dg}{dt}$$

$$\frac{\partial u(x, y, z, t)}{\partial y} = \lim_{\Delta \to 0} \frac{u(x, y+\Delta, z, t) - u(x, y, z, t)}{\Delta}$$

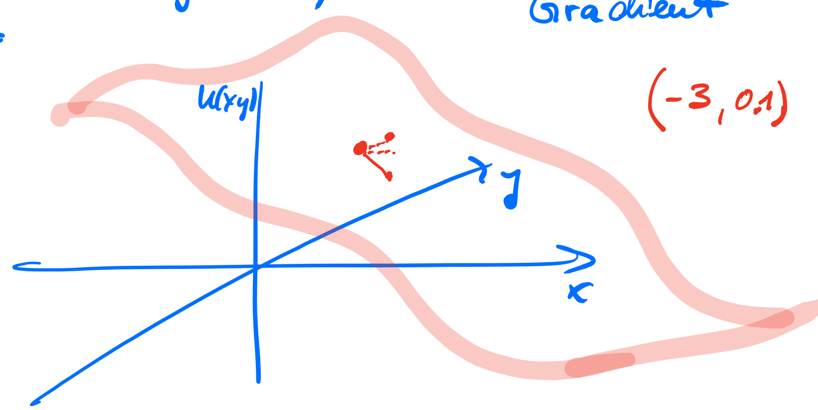
$$\nabla u := \left(\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$$

Nabla

Gradient

$\text{grad } u =$

$\frac{\partial u}{\partial \vec{s}}$



$$\boxed{\frac{du}{dt} = \nabla u \cdot \vec{v} + \frac{\partial u}{\partial t}}$$

Energieerhaltung in drei Dimensionen

$$m \frac{d^2\vec{r}(t)}{dt^2} = \vec{F}(\vec{r}, t) \qquad \vec{r} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\frac{dx_i}{dt} \cdot \Big| \quad m \frac{d^2 x_i}{dt^2} = F_i(x_1, x_2, x_3, t)$$

$$\frac{d}{dt}\left(\frac{dx_i}{dt}\right)^2 = 2 \frac{dx_i}{dt} \frac{d^2 x_i}{dt^2}$$

$$\int_{t_0}^{t} dt' : \qquad \frac{m}{2} \frac{d}{dt} v_i^2 = F_i \frac{dx_i}{dt}$$

$$\sum_{i=1}^{3} : \frac{m}{2}\left(v_i^2(t) - v_i^2(t_0)\right) = \int_{t_0}^{t} F_i \frac{dx_i}{dt'} dt'$$

$$\sum_{i=1}^{3} v_i^2(t) = \vec{v}(t) \cdot \vec{v}(t)$$

$$\frac{m}{2}\left(\vec{v}^2(t) - \vec{v}^2(t_0)\right) = \int_{t_0}^{t} \underline{\vec{F} \cdot \frac{d\vec{x}}{dt'}} dt' = -\int_{t_0}^{t} \frac{dU}{dt'} dt'$$

Annahme:

i) $\vec{F} = -\nabla U$

ii) $U(x, y, z, t) = U(x, y, z)$

$$\frac{dU}{dt} = \nabla U \cdot \vec{v} = -\vec{F} \cdot \vec{v} = -\vec{F} \cdot \frac{d\vec{r}}{dt}$$

$$\int_{t_0}^{t} \frac{dU(t')}{dt'} dt' = U(\vec{r}(t)) - U(\vec{r}(t_0))$$

$$E = \frac{m}{2}\vec{v}^2(t) + U(\vec{r}(t)) = \frac{m}{2}\vec{v}^2(t_0) + U(\vec{r}(t_0))$$

$$\boxed{\frac{dE}{dt} = 0}$$

$$\frac{m}{2}\left(\vec{v}^2(t) - \vec{v}^2(t_0)\right) = -\int_{t_0}^{t} \frac{dU}{dt'}\,dt'$$

$$= -\left[U(t) - U(t_0)\right]$$

$$= -\left(U(\vec{r}(t)) - U(\vec{r}(t_0))\right)$$

$$E = \frac{m}{2}\vec{v}^2(t) + U(\vec{r}(t)) = \frac{m}{2}\vec{v}^2(t_0) + U(\vec{r}(t_0))$$

$$\boxed{\vec{F} = -\nabla U}$$ konservative Kräfte

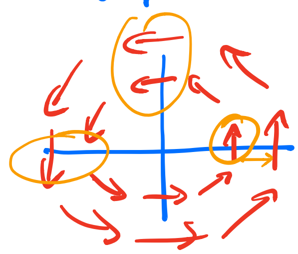
Vektoranalysis

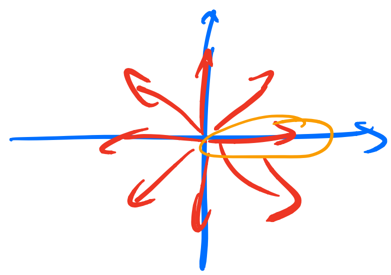
Gradient: $\vec{\nabla} u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z} \right)$

Rotation: $(\vec{\nabla} \times \vec{f})_i = \sum_{jk} \varepsilon_{ijk} \frac{\partial}{\partial x_j} f_k$

$$(\nabla \times \vec{f})_3 = \varepsilon_{321} \frac{\partial}{\partial x_2} f_1 + \varepsilon_{312} \frac{\partial}{\partial x_1} f_2$$

$$= \frac{\partial f_2}{\partial x_1} - \frac{\partial f_1}{\partial x_2}$$



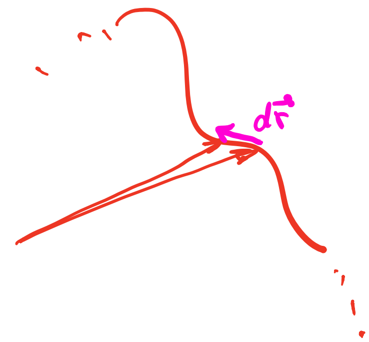


Der Stokes'sche Satz

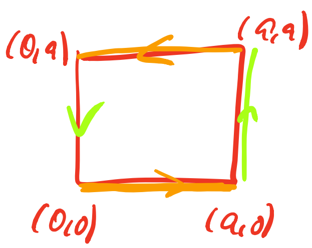
Fläche A

Kurve die den
Rand der
Fläche A beschreibt
C

$$\oint_{C} d\vec{r}\cdot\vec{f}(\vec{r}) = \int_{A} d\vec{S}\cdot(\vec{\nabla}\times\vec{f})$$



$\vec{f}$



$$\oint d\underline{\underline{\vec{r}}}\cdot\vec{f}(\vec{r}) = \int_{0}^{a} dx\left(f_x(x,0) - f_x(x,a)\right) + \int_{0}^{a} dy\left(f_y(a,y) - f_y(0,y)\right)$$

— $d\vec{r} = (dx, 0)$

$$\int_{a}^{b} dx\, f(x) = -\int_{b}^{a} dx\, f(x)$$

| $d\vec{r} = (0, dy)$