$$\vec{F} = m \frac{d^2\vec{r}}{dt^2}$$

$$\vdots$$

$$\frac{m}{2}\vec{v}(t)^2 - \frac{m}{2}\vec{v}(t_0)^2 = \int_{t_0}^{t} \vec{F}\cdot\frac{d\vec{r}}{dt'}\,dt' = -\int \frac{dU}{d\vec{r}}\frac{d\vec{r}}{dt}\,dt$$

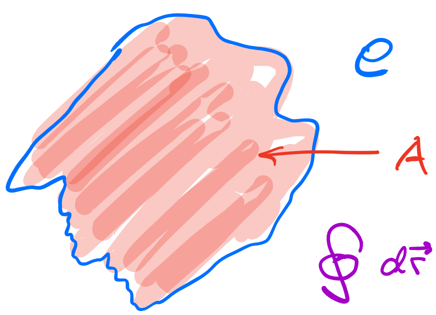
$$= -\int dU$$

$$= -U(t) + U(t_0)$$

$$\vec{F}(\vec{r}) = -\vec{\nabla} U(\vec{r})$$

$$\vec{\nabla} U(\vec{r}) = \left(\frac{\partial U(x,y,z)}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right)$$

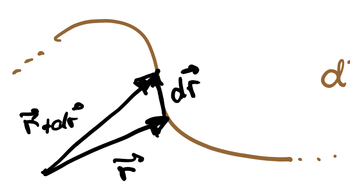
Stokes'scher Satz



$$\oint_{C} d\vec{r}\cdot\vec{f}(\vec{r}) = \int d\vec{s}\cdot(\vec{\nabla}\times\vec{f})$$

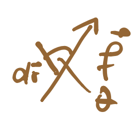
i) $\oint$ Integration entlang einer geschlossenen Kurve

ii) $d\vec{r}\cdot\vec{f}(\vec{r})$

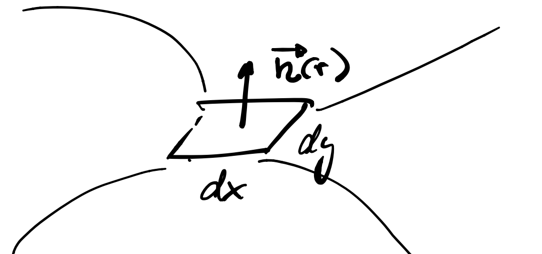


$$d\vec{r}\cdot\vec{f}(\vec{r}) = dr\,|\vec{f}(\vec{r})|\cos\theta(\vec{r})$$

$$dr = |d\vec{r}|$$



iii) $\int_A d\vec{s}\cdot\vec{\theta}(\vec{r})$

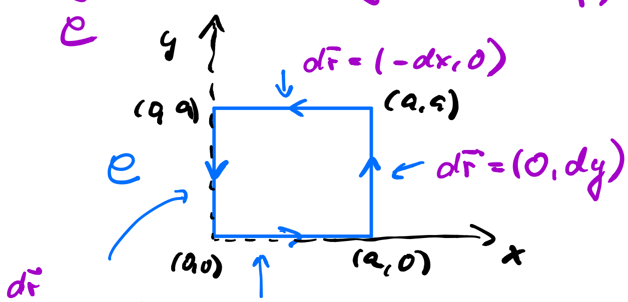


$d\vec{s} \parallel \vec{n}$

• bestimme an jedem Punkt $\vec{n}(\vec{r})$ den ($\vec{n}^2 = 1$) lokalen Normalenvektor der Fläche

$$\int_A d\vec{s}\cdot\vec{\theta} = \int dx \int dy \ \vec{n}\cdot\vec{\theta}$$

$$\oint_C d\vec{r}\cdot\vec{f}(\vec{r}) = \int d\vec{s}\cdot(\vec{\nabla}\times\vec{f})$$



$d\vec{r} = (dx, 0)$: $\quad d\vec{r}\,\vec{f}(\vec{r}) = (dx, 0)\begin{pmatrix} f_x(x,0) \\ f_y(x,0) \end{pmatrix}$

$$= f_x(x,0)\,dx$$

$$\oint_C d\vec{r}\cdot\vec{f}(\vec{r}) = \int_0^a dx\,\big(f_x(x,0) - f_x(x,a)\big) + \int_0^a dy\,\big(f_y(a,y) - f_y(0,y)\big)$$

$$\boxed{\oint d\vec{r}\cdot\vec{f}(\vec{r}) = -\oint d\vec{r}\cdot\vec{f}(\vec{r})}$$

$$\int_A d\vec{s} = \vec{e}_z \int_0^a dx \int_0^a dy \qquad \vec{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\int d\vec{s} \cdot (\vec{\nabla} \times \vec{f}) = \int_0^a dx \int_0^a dy \, (\vec{\nabla} \times \vec{f})_z$$

$$= \int_0^a dx \int_0^a dy \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \right)$$

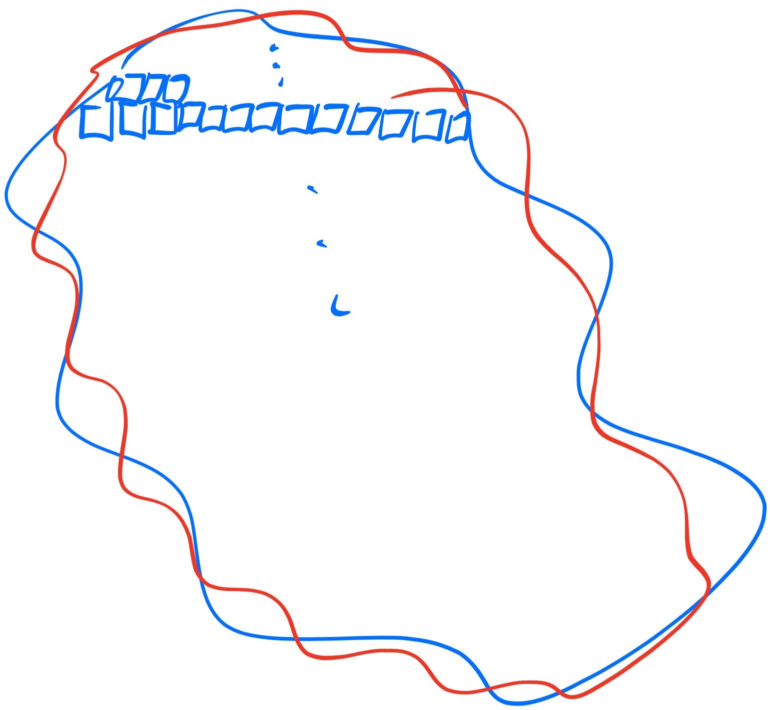
$$\int_0^a dx \, \frac{\partial f_y(x,y)}{\partial x} = f_y(a,y) - f_y(0,y)$$

$$\int_0^a dy \, \frac{\partial f_x(x,y)}{\partial y} = f_x(x,a) - f_x(x,0)$$

$$= \int_0^a dy \left(\underline{f_y(a,y)} - \underline{f_y(0,y)} \right)$$

$$- \int_0^a dx \left(\underline{f_x(x,a)} - \underline{f_x(x,0)} \right)$$

$$\oint_{\mathcal{C}} d\vec{r}\cdot\vec{f}(\vec{r}) = \int_0^a dx\,\Big(\underline{f_x(x,0)} - \underline{f_x(x,a)}\Big) + \int_0^a dy\,\Big(\underline{f_y(a,y)} - \underline{f_y(0,y)}\Big)$$



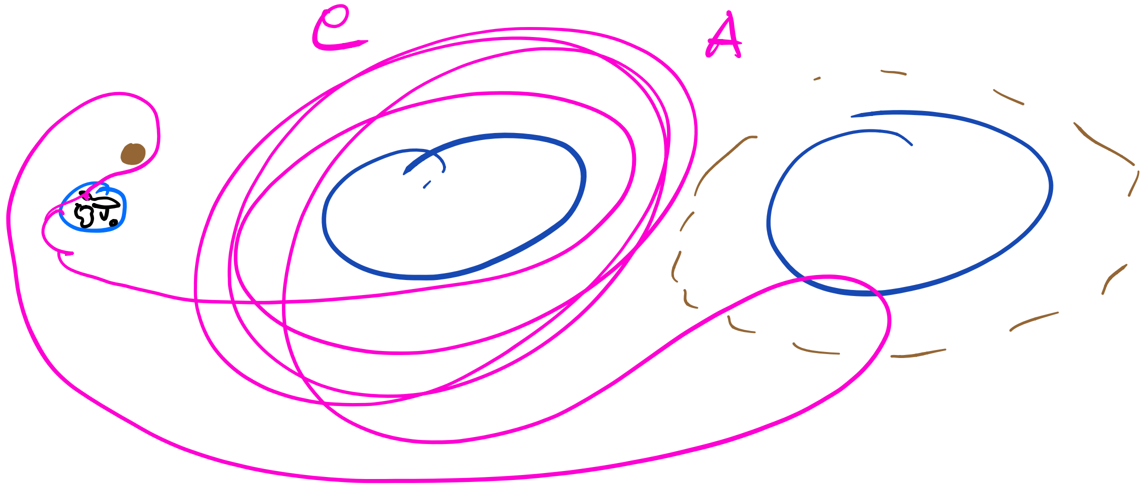
$$\frac{m}{2}\,\vec{v}(t)^2 - \frac{m}{2}\,\vec{v}(t_0)^2 = \int_{t_0}^{t} \vec{F}\cdot\frac{d\vec{r}}{dt'}\,dt'$$

$$= \int_{\vec{r}(t_0)}^{\vec{r}(t)} \vec{F}\cdot d\vec{r} = \text{Arbeit}$$

$\vec{F} = -\vec{\nabla} U$

Kraft entlang einer geschlossenen Kurve

$$W = \oint_{C} \vec{F}\cdot d\vec{r} = \int_{A} d\vec{s} \cdot \vec{\nabla}\times\vec{F} = 0$$



$$\vec{\nabla}\times\vec{F} = \sum_{ijk} \vec{e}_i \frac{\partial}{\partial x_j} F_k \,\varepsilon_{ijk} \qquad F_k = -\frac{\partial U}{\partial x_k}$$

$$= -\sum_{ijk} \vec{e}_i \frac{\partial}{\partial x_j}\frac{\partial}{\partial x_k} \varepsilon_{ijk}\, U(\vec{r})$$

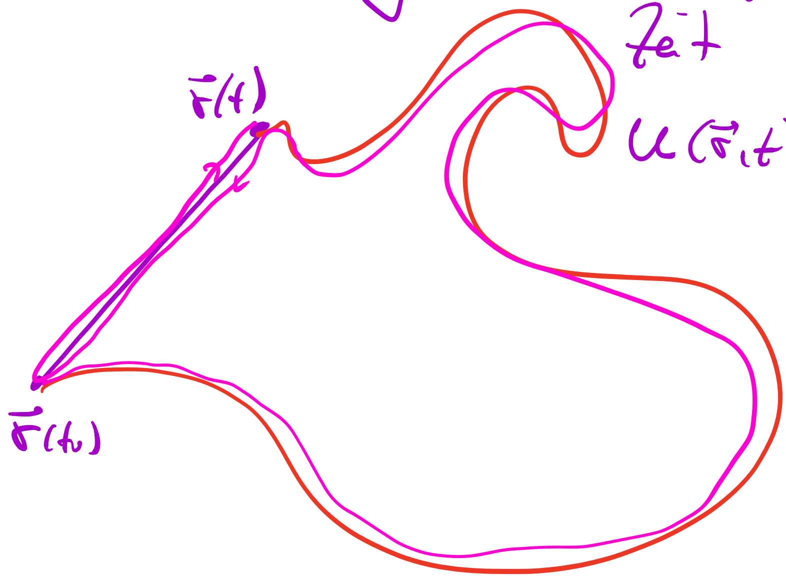
$$= -\sum_{ijk} \vec{e}_i \frac{\partial}{\partial x_k}\frac{\partial}{\partial x_j} \overset{= -\varepsilon_{ijk}}{\varepsilon_{ikj}}\, U(\vec{r})$$

$$= \sum_{ijk} \vec{e}_i \frac{\partial}{\partial x_j}\frac{\partial}{\partial x_k} \varepsilon_{ijk}\, U(\vec{r}) = -\vec{\nabla}\times\vec{F}$$

$\Longrightarrow \vec{0} = \vec{\nabla}\times\vec{F}$ wenn $\vec{F} = -\vec{\nabla} U$

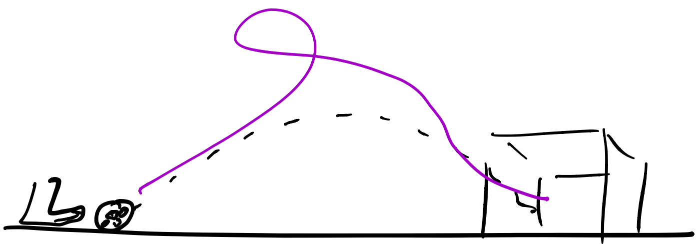
Energieerhaltung $\Leftrightarrow$ Homogenität d. Zeit

$$U(\vec{r}, t) = U(\vec{r})$$



Arbeit ist Wegunabhängig

Das Variationsprinzip der Mechanik



Welches Prinzip entscheidet über die richtige Trajektorie?

Annahme:

$$S[\vec{r}(t)] = \int_{t_0}^{t} dt' \, L(\vec{r}(t'), \vec{v}(t'), t')$$

physikalische Trajektorien minimieren S

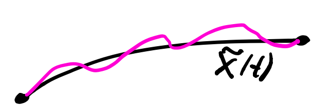
betrachten eine räumliche Dimension

$$S[x(t)] = \int_{t_0}^{t} L(x(t'), \dot{x}(t'), t') \, dt'$$

Wir kennen $x(t_0) = x_0 \quad x(t) = x_1$

$\tilde{x}(t)$ sei die physikalisch realisierte Trajektorie

$\tilde{x}(t_0) = x_0 \quad \tilde{x}(t) = x_1$



$$x(t') = \tilde{x}(t') + \delta x(t')$$

$\delta x(t_0) = \delta x(t) = 0$

$$\Delta S = \int_{t_0}^{t} L(\tilde{x}+\delta x, \dot{\tilde{x}}+\delta\dot{x}, t')\,dt' - \int_{t_0}^{t} L(\tilde{x}, \dot{\tilde{x}}, t')\,dt'$$

$$L(\tilde{x}(t')+\delta x(t'), \dot{\tilde{x}}(t')+\delta\dot{x}(t'), t')$$

$$\approx L(\tilde{x}, \dot{\tilde{x}}, t') + \frac{\partial L}{\partial \tilde{x}}\delta x + \underline{\underline{\frac{\partial L}{\partial \dot{\tilde{x}}}\delta\dot{x}}}$$

$$\int_{t_0}^{t} \frac{\partial L}{\partial \dot{\tilde{x}}} \frac{d}{dt'}\delta x(t')\,dt'$$

$$= -\int_{t_0}^{t} \left(\frac{d}{dt'}\frac{\partial L}{\partial \dot{\tilde{x}}}\right)\delta x(t')\,dt'$$

$$\Delta S = \int_{t_0}^{t} \underbrace{\left(\frac{\partial L}{\partial \tilde{x}} - \frac{d}{dt'}\frac{\partial L}{\partial \dot{\tilde{x}}}\right)}_{=0}\delta x(t')\,dt'$$

Euler-Lagrange Gleichungen!