Practice Exam: Classical Theoretical Physics I
Winter Semester 2025/26

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Sheet 15
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This exercise sheet is a practice exam. The problems correspond in type, scope, and difficulty level to the actual final exam. Please note that the exam is to be completed without aids and you have 90 minutes (1:30 hours).

Surname:

Matriculation No.:

First Name:

Tutor / Tutorial Group:

Major:

Exam Regs (PO):

Important Instructions:

- **Please have your student ID visible.**
- **Hand in this cover sheet with your solutions.**
- **Write your name and matriculation number on every page.**
- **Please use a new sheet of paper for each problem.**
- **Do not use red ink or pencils.**
- **Smartphones and other electronic devices must be turned off and stored away.**

Problem	1	2	3	4	Σ
Points					
Out of	4	5	2	4	$100\% = 15$

Problem 1. Warm-up **[4 Points]**

Answer the following questions briefly (bullet points are sufficient):

(a) **[1 Point]** What are the properties of orthogonal matrices? Is a rotation matrix orthogonal?

(b) **[1 Point]** By which expression do you determine the associated potential $U(\mathbf{r})$ for a given potential force field $\mathbf{F}(\mathbf{r})$?

(e) **[2 Points]** A one-dimensional harmonic oscillator with Stokes friction is described by the equation of motion:

$$m\ddot{x} = -r\dot{x} - kx.$$

What is the natural frequency of the undamped oscillator? Which qualitatively different types of motion are observed, and for which values of the friction coefficient r?

Problem 2. Conservative Force Field and Potential **[5 Points]**

Given the force field:

$$\mathbf{F}(\mathbf{r}) = \begin{pmatrix} ax+by \\ cx+dy \\ ez \end{pmatrix}$$

(a) [1 Point] What condition must the constants $a, b, c, d, e \neq 0$ satisfy for $\mathbf{F}(\mathbf{r})$ to be a conservative force field?

(b) [1.5 Points] Determine the associated potential $U(\mathbf{r})$. As a check, calculate the force field resulting from your potential.

(c) [2 Points] Determine the work done on a mass point by explicitly calculating the line integrals along the following paths:

(i) C_1: Directly from $(x,y,z) = (0,0,0)$ to $(1,0,1)$.

(ii) C_2: Along a semicircle in the xy-plane from $(1,0,1)$ via $(0,-1,1)$ to $(-1,0,1)$.

(iii) C_3: Directly from $(-1,0,1)$ to $(0,0,0)$.

(d) [0.5 Points] Now use the property that the force field is conservative to verify your results from part (c).

Problem 3. Particular Solution **[2 Points]**

We want to derive a general formula for calculating a particular solution to the differential equation:

$$\ddot{x} + \omega^2 x = f(t)\,,$$

(a) [2 Points] Decompose the left side of this equation into linear factors:

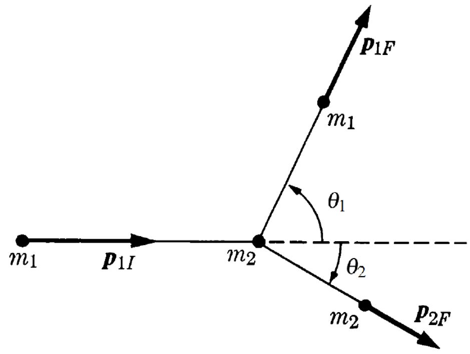
$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + i\omega\right)\left(\frac{\mathrm{d}}{\mathrm{d}t} - i\omega\right)x(t) = f(t)$$

and let $y(t) = \left(\frac{\mathrm{d}}{\mathrm{d}t} - i\omega\right)x(t)$. You obtain an inhomogeneous linear first-order ODE for $y(t)$. Solve this ODE using the method of variation of constants, i.e., using the ansatz:

$$y(t) = u(t)e^{-i\omega t}\,.$$

Problem 4. Elastic Collision **[4 Points]**

Consider a "projectile" particle of mass m_1 and initial momentum p_{1I}, which collides with a "target" particle of mass m_2 that is initially at rest. After the collision, particle m_1 is scattered by an angle θ_1 with final momentum p_{1F}. Particle m_2 recoils with momentum p_{2F} at an angle θ_2 relative to the original direction of p_{1I}.



(a) [1 point] Set up the equations for the conservation of linear momentum along the direction of p_{1I} and perpendicular to it.

(b) [1 point] Use your equations from (a) to show that:

$$p_{2F}^2 = p_{1I}^2 + p_{1F}^2 - 2p_{1I}p_{1F}\cos\theta_1$$

(c) [2 points] Given the energy conservation relationship:

$$\frac{p_{1I}^2 - p_{1F}^2}{m_1} = \frac{p_{2F}^2}{m_2}$$

Substitute the expression from (b) to derive the quadratic equation for the ratio $\frac{p_{1F}}{p_{1I}}$. In particular, show that:

$$\frac{p_{1F}}{p_{1I}} = \frac{m_1}{m_1+m_2}\cos\theta_1 \pm \left[\left(\frac{m_1}{m_1+m_2}\right)^2 \cos^2\theta_1 + \frac{m_2-m_1}{m_1+m_2}\right]^{1/2}$$

Formula sheet

$$\sin^2\theta + \cos^2\theta = 1,$$

$$\mathrm{e}^{\pm i\theta} = \cos\theta \pm i\sin\theta,$$

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right),$$

$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right),$$

$$\int_0^t e^{i\alpha s}\,ds = \frac{e^{i\alpha t} - 1}{i\alpha},$$

$$\int_0^\pi \sin\theta\cos\theta\,d\theta = \int_0^\pi \frac{1}{2}\sin(2\theta)\,d\theta = \left[-\frac{1}{4}\cos(2\theta)\right]_0^\pi = 0,$$

$$\int_0^\pi \sin^2\theta\,d\theta = \int_0^\pi \frac{1-\cos(2\theta)}{2}\,d\theta = \left[\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right]_0^\pi = \frac{\pi}{2},$$

$$\int_0^\pi \cos^2\theta\,d\theta = \int_0^\pi \frac{1+\cos(2\theta)}{2}\,d\theta = \left[\frac{\theta}{2} + \frac{\sin(2\theta)}{4}\right]_0^\pi = \frac{\pi}{2}.$$