Institut für Theoretische Teilchenphysik

Theoretische Physik A

WS 2000/01

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Lösungsvorschläge

1. Integrationsmethoden (4 Punkte)

(a)

$$\frac{d}{dx}\left(\frac{1}{2}(x+\sinh x\cosh x)\right) = \frac{1}{2}(1+\cosh^2 x+\sinh^2 x) = \cosh^2 x$$

(b)

$$\int dx\,\cosh^2 x = \int dx\,\frac{e^{2x}+2+e^{-2x}}{4} = \frac{1}{2}x+\frac{e^{2x}-e^{-2x}}{8} = \frac{1}{2}x+\frac{(e^x+e^{-x})(e^x-e^{-x})}{8}$$
$$= \frac{1}{2}x+\frac{\cosh x\sinh x}{2}$$

(c)

$$\int dx\,\cosh^2 x = \sinh x\cosh x - \int dx\,\sinh^2 x = \sinh x\cosh x - \int dx\,\left(\cosh^2 x - 1\right)$$
$$= \sinh x\cosh x + x - \int dx\,\cosh^2 x \quad \left(\text{Auflösen nach } \int dx\,\cosh^2 x\right)$$

2. <u>Bahnkurve II</u> (5 Punkte)

(a)

$$\mathbf{v}(t) = \begin{pmatrix} c \\ r\omega \cos \omega t \end{pmatrix}$$

(b) $v(t) = \sqrt{c^2 + r^2\omega^2 \cos^2 \omega t}$

(c)

$$\mathbf{a}(t) = \begin{pmatrix} 0 \\ -r\omega^2 \sin \omega t \end{pmatrix}$$

(d) $a(t) = r\omega^2 |\sin \omega t|$

3. <u>Bahnkurve III</u> (7 Punkte)

(a)

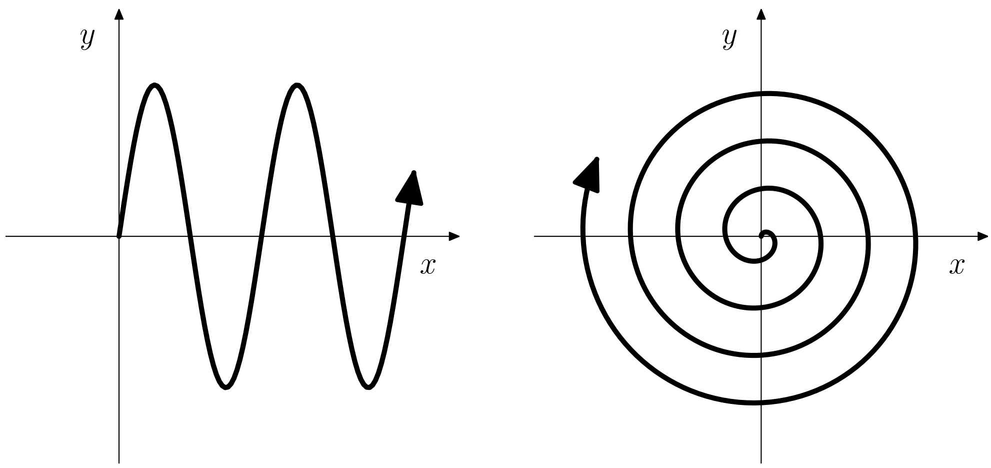
$$\mathbf{v}(t) = \begin{pmatrix} ct\omega \cos \omega t + c \sin \omega t \\ -ct\omega \sin \omega t + c \cos \omega t \end{pmatrix}$$

(b) $v(t) = c\sqrt{1 + \omega^2 t^2}$

(c)

$$\mathbf{a}(t) = \begin{pmatrix} -ct\omega^2 \sin \omega t + 2c\omega \cos \omega t \\ -ct\omega^2 \cos \omega t - 2c\omega \sin \omega t \end{pmatrix}$$

(d) $a(t) = c\omega\sqrt{4 + \omega^2 t^2}$



4. Integral (4 Punkte)

Aus Koeffizientenvergleich in

$$ax^2 + 2bx + c = a(y^2 + y_0^2)$$

folgt mit $y = x - x_0$: $x_0 = -b/a$ und $y_0 = \sqrt{ac - b^2}/a$. Damit lautet das Integral:

$$\int dx \, \sqrt{ax^2 + 2bx + c} = \sqrt{a} \int dy \, \sqrt{y^2 + y_0^2}$$

Substitution:

$$y = y_0 \sinh \phi, \qquad dy = y_0 \cosh \phi \, d\phi,$$

also

$$\begin{aligned}
\int dy \, \sqrt{y^2 + y_0^2} = \int y_0^2 \cosh^2 \phi \, d\phi &= \frac{y_0^2}{2}(\phi + \sinh \phi \cosh \phi) \\
&= \frac{y_0^2}{2} \left(\operatorname{Arsinh} \frac{y}{y_0} + \frac{y}{y_0} \sqrt{1 + \frac{y^2}{y_0^2}} \right) \\
&= \frac{y_0^2}{2} \operatorname{Arsinh} \frac{y}{y_0} + \frac{y}{2} \sqrt{y^2 + y_0^2}
\end{aligned}$$

Einsetzen ergibt

$$\int dx \, \sqrt{ax^2 + 2bx + c} = \frac{ac - b^2}{2a^{3/2}} \operatorname{Arsinh} \frac{ax + b}{\sqrt{ac - b^2}} + \frac{ax + b}{2a} \sqrt{ax^2 + 2bx + c}.$$