Institut für Theoretische Teilchenphysik

Theoretische Physik A

WS 2000/01

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Lösungsvorschläge

1. Ameisenspaziergang (5 Punkte)

(a)

$$s(x) = \int_a^x dx \sqrt{1+y'^2} = \int_a^x dx \sqrt{1+\sinh^2 x} = \int_a^x dx \cosh x = \sinh x - \sinh a$$

(b)

$$x(s) = \operatorname{Arsinh}(s+\sinh a)$$

$$y(s) = \cosh x(s) = \sqrt{1+\sinh^2[x(s)]} = \sqrt{1+(s+\sinh a)^2}$$

(c)

$$\sinh' t = \cosh t \quad \Rightarrow \quad \operatorname{Arsinh}' t = \frac{1}{\cosh(\operatorname{Arsinh} t)} = \frac{1}{\sqrt{1+t^2}}$$

$$\frac{dx}{ds}(s) = \frac{1}{\sqrt{1+(s+\sinh a)^2}}, \qquad \frac{dy}{ds}(s) = \frac{s+\sinh a}{\sqrt{1+(s+\sinh a)^2}}$$

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 = \frac{1}{1+(s+\sinh a)^2} + \frac{(s+\sinh a)^2}{1+(s+\sinh a)^2} = 1$$

2. Wir planen eine Achterbahn (15 Punkte)

(a)

$$a_x(t) = -a_h \sin \omega t$$
$$a_y(t) = a_h \cos \omega t \begin{cases} \cdot(+1) & \text{für } 0 \leq t \leq \frac{1}{2}T \\ \cdot(-1) & \text{für } \frac{1}{2}T \leq t \leq T \end{cases}$$
$$a_z(t) = a_v \begin{cases} \cdot(+1) & \text{für } 0 \leq t \leq \frac{1}{4}T \\ \cdot(-1) & \text{für } \frac{1}{4}T \leq t \leq \frac{3}{4}T \\ \cdot(+1) & \text{für } \frac{3}{4}T \leq t \leq T \end{cases}$$

(b)

$$v_x(t) = v_0 - \frac{a_h}{\omega}(1 - \cos \omega t)$$
$$v_y(t) = \frac{a_h}{\omega} \sin \omega t \begin{cases} \cdot(+1) & \text{für } 0 \leq t \leq \frac{1}{2}T \\ \cdot(-1) & \text{für } \frac{1}{2}T \leq t \leq T \end{cases}$$
$$v_z(t) = a_v \begin{cases} \cdot t & \text{für } 0 \leq t \leq \frac{1}{4}T \\ \cdot\left(\frac{1}{2}T - t\right) & \text{für } \frac{1}{4}T \leq t \leq \frac{3}{4}T \\ \cdot(t - T) & \text{für } \frac{3}{4}T \leq t \leq T \end{cases}$$

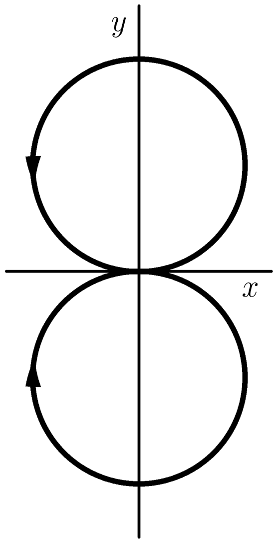
(c)

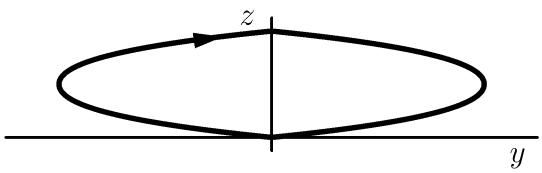
$$x(t) = v_0 t - \frac{a_h}{w}\left(t - \frac{1}{\omega} \sin \omega t\right)$$
$$y(t) = \frac{a_h}{w^2}(1 - \cos \omega t) \begin{cases} \cdot(+1) & \text{für } 0 \leq t \leq \frac{1}{2}T \\ \cdot(-1) & \text{für } \frac{1}{2}T \leq t \leq T \end{cases}$$
$$z(t) = \frac{1}{2}a_v \begin{cases} \cdot t^2 & \text{für } 0 \leq t \leq \frac{1}{4}T \\ \cdot\left[\frac{1}{8}T^2 - 1\left(t - \frac{1}{2}T\right)^2\right] & \text{für } \frac{1}{4}T \leq t \leq \frac{3}{4}T \\ \cdot(t - T)^2 & \text{für } \frac{3}{4}T \leq t \leq T \end{cases}$$

(d)

$$z\left(\frac{1}{2}T\right) = \frac{1}{16}a_v T^2 = z_{\max} \quad \Rightarrow \quad a_v = \frac{16 z_{\max}}{T^2}$$
$$x(T) = v_0 T - \frac{a_h T}{\omega} = 0 \quad \Rightarrow \quad a_h = v_0 \omega$$
$$x\left(\frac{1}{8}T\right) = \frac{v_0}{\omega} = \frac{1}{2}x_{\text{tot}} \quad \Rightarrow \quad v_0 = \frac{1}{2}\omega x_{\text{tot}}$$

(e)





(f)

$$v^2(t) = v_x^2(t) + v_y^2(t) + v_z^2(t) = v_0^2 \cos^2 \omega t + v_0^2 \sin^2 \omega t + a_v^2 t^2 = v_0^2 + a_v^2 t^2$$

$$\begin{aligned} s(t) &= \int_0^t dt'\, v^2(t') = \int_0^t dt' \sqrt{v_0^2 + a_v^2 t'^2} \\ &= \frac{v_0^2 a_v^2}{2a_v^3} \operatorname{Arsinh} \frac{a_v^2 t}{v_0 a_v} + \frac{a_v^2 t}{2a_v^2} \sqrt{v_0^2 + a_v^2 t^2} \\ &= \frac{v_0^2}{2a_v} \operatorname{Arsinh} \frac{a_v t}{v_0} + \frac{t}{2} \sqrt{v_0^2 + a_v^2 t^2} \end{aligned}$$

$$\begin{aligned} s(T) = 4s\left(\frac{1}{4}T\right) &= \frac{\pi^2 x_{\text{tot}}^2}{2z_{\text{max}}} \operatorname{Arsinh} \frac{2z_{\text{max}}}{\pi x_{\text{tot}}} + \sqrt{\pi^2 x_{\text{tot}}^2 + 4z_{\text{max}}^2} \\ &= \pi x_{\text{tot}} \left(\frac{1}{\epsilon} \operatorname{Arsinh} \epsilon + \sqrt{1+\epsilon^2}\right) \qquad \text{mit } \epsilon := \frac{2z_{\text{max}}}{\pi x_{\text{tot}}} = \frac{1}{\pi} \\ &\approx \pi x_{\text{tot}} \,(0.984 + 1.049) \approx 128m \end{aligned}$$

Dieses Ergebnis ist erstaunlich nahe am Umfang von zwei Kreisen, $2\pi x_{\text{tot}} \approx 126m$. Die Höhe von $10m$ bewirkt also nur eine kleine Korrektur.