

Übungen zur Theoretischen Physik A WS 02/03

PROF. P. WÖLFLE

Musterlösung

DR. M. GREITER

Blatt 2**1. Integrationsmethoden**

(a)

$$\frac{d}{dx} \left(\frac{1}{2} (x + \sinh x \cosh x) \right) = \frac{1}{2} (1 + \cosh^2 x + \sinh^2 x) = \cosh^2 x$$

(b)

$$\begin{aligned} \int dx \cosh^2 x &= \int dx \frac{e^{2x} + 2 + e^{-2x}}{4} = \frac{1}{2}x + \frac{e^{2x} - e^{-2x}}{8} = \frac{1}{2}x + \frac{(e^x + e^{-x})(e^x - e^{-x})}{8} \\ &= \frac{1}{2}x + \frac{\cosh x \sinh x}{2} \end{aligned}$$

(c)

$$\begin{aligned} \int dx \cosh^2 x &= \sinh x \cosh x - \int dx \sinh^2 x = \sinh x \cosh x - \int dx (\cosh^2 x - 1) \\ &= \sinh x \cosh x + x - \int dx \cosh^2 x \quad (\text{Auflösen nach } \int dx \cosh^2 x) \end{aligned}$$

2. Bahnkurve II

(a)

$$\mathbf{v}(t) = \begin{pmatrix} c \\ r\omega \cos \omega t \end{pmatrix}$$

$$(b) \quad v(t) = \sqrt{c^2 + r^2 \omega^2 \cos^2 \omega t}$$

(c)

$$\mathbf{a}(t) = \begin{pmatrix} 0 \\ -r\omega^2 \sin \omega t \end{pmatrix}$$

$$(d) \quad a(t) = r\omega^2 |\sin \omega t|$$

3. Bahnkurve III

(a)

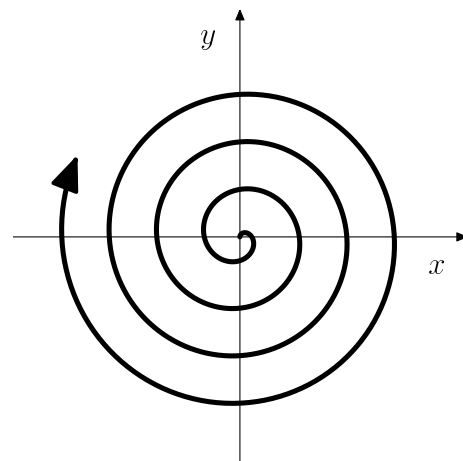
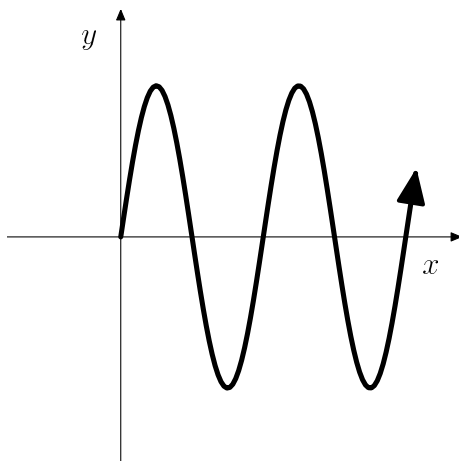
$$\mathbf{v}(t) = \begin{pmatrix} ct\omega \cos \omega t + c \sin \omega t \\ -ct\omega \sin \omega t + c \cos \omega t \end{pmatrix}$$

(b) $v(t) = c\sqrt{1 + \omega^2 t^2}$

(c)

$$\mathbf{a}(t) = \begin{pmatrix} -ct\omega^2 \sin \omega t + 2c\omega \cos \omega t \\ -ct\omega^2 \cos \omega t - 2c\omega \sin \omega t \end{pmatrix}$$

(d) $a(t) = c\omega\sqrt{4 + \omega^2 t^2}$



4. Integral

Aus Koeffizientenvergleich in

$$ax^2 + 2bx + c = a(y^2 + y_0^2)$$

folgt mit $y = x - x_0$: $x_0 = -b/a$ und $y_0 = \sqrt{ac - b^2}/a$. Damit lautet das Integral:

$$\int dx \sqrt{ax^2 + 2bx + c} = \sqrt{a} \int dy \sqrt{y^2 + y_0^2}$$

Substitution:

$$y = y_0 \sinh \phi, \quad dy = y_0 \cosh \phi d\phi,$$

also

$$\begin{aligned} \int dy \sqrt{y^2 + y_0^2} &= \int y_0^2 \cosh^2 \phi d\phi = \frac{y_0^2}{2} (\phi + \sinh \phi \cosh \phi) \\ &= \frac{y_0^2}{2} \left(\operatorname{Arsinh} \frac{y}{y_0} + \frac{y}{y_0} \sqrt{1 + \frac{y^2}{y_0^2}} \right) \\ &= \frac{y_0^2}{2} \operatorname{Arsinh} \frac{y}{y_0} + \frac{y}{2} \sqrt{y^2 + y_0^2} \end{aligned}$$

Einsetzen ergibt

$$\int dx \sqrt{ax^2 + 2bx + c} = \frac{ac - b^2}{2a^{3/2}} \operatorname{Arsinh} \frac{ax + b}{\sqrt{ac - b^2}} + \frac{ax + b}{2a} \sqrt{ax^2 + 2bx + c}.$$