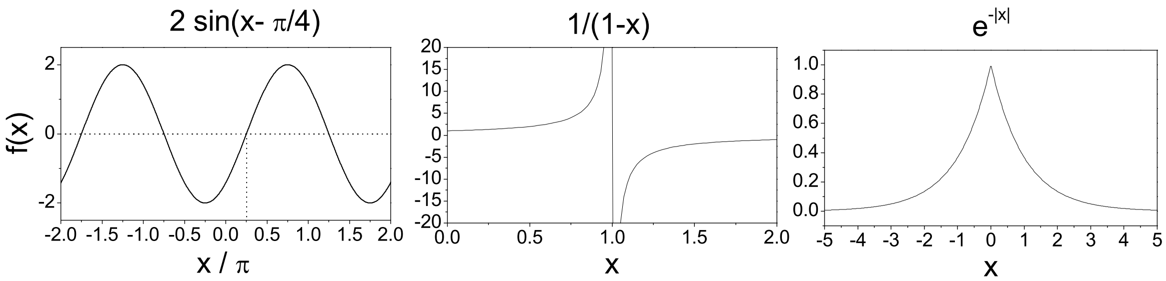
Lösungsvorschlag 0. Übungsblatt Theorie A **WS 2009/2010**
Karlsruhe Institute of Technology

Prof. Dr. Gerd Schön — Dr. G. Metalidis www.tfp.uni-karlsruhe.de/Lehre

Aufgabe 1



Aufgabe 2

- $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{ax+3} = a\mathrm{e}^{ax+3}$
- $\frac{\mathrm{d}}{\mathrm{d}x}\sin(ax^3) = 3ax^2\cos(ax^3)$
- $\frac{\mathrm{d}}{\mathrm{d}a}\frac{1}{x+a^2} = \frac{-2a}{(x+a^2)^2}$
- $\frac{\mathrm{d}}{\mathrm{d}\theta}(\tan\theta\cos\theta) = \frac{\mathrm{d}}{\mathrm{d}\theta}\sin\theta = \cos\theta$
- $\frac{\mathrm{d}}{\mathrm{d}x}\left(2x^3+4\right)^{3/2} = \frac{3}{2}\left(2x^3+4\right)^{1/2}6x^2$
- $\int_0^\pi \cos x\,\mathrm{d}x = \sin x|_0^\pi = 0$
- $\int_0^a \sqrt{x+a}\,\mathrm{d}x = \frac{2}{3}(x+a)^{3/2}|_0^a = \frac{2}{3}a^{3/2}\left(2\sqrt{2}-1\right)$
- $\int_{-\pi}^{\pi} x\cos x\,\mathrm{d}x = 0$ (ungerade Funktion)
- $\int_{-\ln 2}^{\ln 2}\mathrm{e}^x\mathrm{d}x = \mathrm{e}^x|_{-\ln 2}^{\ln 2} = \frac{3}{2}$
- $\int_0^\pi x\cos x^2\mathrm{d}x = \frac{1}{2}\int_0^{\pi^2}\mathrm{d}y\cos y = \frac{1}{2}\sin y|_0^{\pi^2} = \frac{1}{2}\sin\pi^2$ (Substitution $y = x^2$)

Aufgabe 3

Für $n=0$: $2^0 = 1 = 2^1 - 1 = 1 \rightarrow$ OK

Für $n+1$: $\sum_{i=0}^{n+1} 2^i = \sum_{i=0}^{n} 2^i + 2^{n+1} = 2^{n+1} - 1 + 2^{n+1} = 2\cdot 2^{n+1} - 1 = 2^{n+2} - 1 \rightarrow$ OK

Aufgabe 4

- $\frac{-1+5\mathrm{i}}{2+3\mathrm{i}} = \frac{(-1+5\mathrm{i})(2-3\mathrm{i})}{(2+3\mathrm{i})(2-3\mathrm{i})} = \frac{13+\mathrm{i}13}{4+9} = 1+\mathrm{i}$
- $2\mathrm{e}^{\mathrm{i}\pi/3} = 2\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}\right) = 1 + \mathrm{i}\sqrt{3}$

Aufgabe 5

- $z = 1 + \mathrm{i}\sqrt{3} \Rightarrow |z| = \sqrt{1^2 + \sqrt{3}^2} = 2,\ \text{and} \quad \cos\phi = \frac{1}{2},\ \sin\phi = \frac{\sqrt{3}}{2}$
  $\Rightarrow |z| = 2,\ \text{and}\ \phi = \frac{\pi}{3}$
  $\Rightarrow z = 2\,\mathrm{e}^{\mathrm{i}\pi/3}$
- $z = -1 - \mathrm{i} \Rightarrow |z| = \sqrt{1^2 + 1^2} = \sqrt{2},\ \text{and}\ \cos\phi = \frac{-1}{\sqrt{2}},\ \sin\phi = \frac{-1}{\sqrt{2}}$
  $\Rightarrow |z| = \sqrt{2},\ \text{and}\ \phi = \frac{5\pi}{4}$
  $\Rightarrow z = \sqrt{2}\,\mathrm{e}^{\mathrm{i}\frac{5\pi}{4}}$