Institute for Theoretical Particle Physics

Classical Theoretical Physics I

WS 2023

Exercise Sheet 1

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Start: 24.10.2023, 12:00
Due: 2.11.2023, 12:00

Please upload your solution on

https://ilias.studium.kit.edu/goto.php?target=crs_2215325&client_id=produktiv.

Please put your name on each sheet of your solution. Please put your tutoring session information on the first page (location, time, name of the tutor, etc.).

Problem 1: Calculate the first derivative $f'(x) \equiv \frac{df}{dx}$ of the following functions (for any $\alpha \in \mathbb{R}$):

a) (1 point) $f(x) = x^\alpha \sin x$,

b) (1 point) $f(x) = \sin x^\alpha := \sin(x^\alpha)$,

c) (1 point) $f(x) = \sin^\alpha x := (\sin x)^\alpha$,

d) (2 points) $f(x) = x^\alpha \sin \dfrac{1}{x}$. for $\alpha > 1$ and $x \to 0$.

Problem 2: An *antiderivative* F of a given function f is understood to be a function that satisfies $\frac{dF}{dx} = f(x)$. Two different antiderivatives only differ by an additive constant.

a) (1 point) Calculate all antiderivatives of $f(x) = x^\alpha$ for $\alpha \in \mathbb{R}$, $\alpha \neq -1$.

b) (1 point) Calculate all antiderivatives of $f(x) = x^2 \cos x$.

c) (3 points) For $x > 0$ we define the function $L(x)$ by the following properties:

$$\frac{dL}{dx} = \frac{1}{x} \qquad \text{and} \quad L(1) = 0. \tag{1}$$

Use only eq. (1) to derive the following properties:

i) (0,5 point) $L(x) + L(y) = L(xy)$.

ii) (0,5 point) $L(x^\alpha) = \alpha L(x)$ for $\alpha \in \mathbb{R}$.

The inverse function f^{-1} of a function f satisfies the properties $f^{-1}(f(x)) = x$ and $f(f^{-1}(y)) = y$. (for example, for $x > 0$: $f(x) = \sqrt{x}$ has the inverse function $f^{-1}(y) = y^2$.)

iii) (1 point) Show that $\dfrac{df^{-1}}{dy} = \dfrac{1}{df/dx}$, where $y = f(x)$.

iv) (1 point) Express $\dfrac{dL^{-1}}{dy}$ by $L^{-1}(y)$.

Do you recognize L and L^{-1} from the Mathematik-Vorkurs?