Classical Theoretical Physics I

WS 2014

Exercise Sheet 2

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Please put your name on each sheet of your solution. Please put your tutoring session information on the front page (location, time, name of the tutor, etc.)

Problem 3: This problem is about applying integration by parts (Equation (3) from the lecture). Consider the function $I_n(x) = \int_0^x dy y^n \exp(y)$, where $n \in \mathbb{N}_0$

a) (1 Point) Calculate $I_0(x)$.

b) (2 Points) Express (for $n \geq 1$) $I_n(x)$ by $I_{n-1}(x)$. (Such an equation is called *Recursion formula*.)

c) (1 Point) Calculate $I_1(x)$, $I_2(x)$ and $I_3(x)$.

d) (1 Point) Calculate $I_n(x)$ (i.e. solve the recursion). Hint: The so-called *Pochhammer-Symbol* will become handy. It is defined as $(a)_n := a \cdot (a+1) \cdot \ldots \cdot (a+n-1)$, where $(a)_0 := 1$. Guess the solution for $I_n(x)$ and show that the recursion formula holds for $I_0(x)$, where $n = 0$. This proof method is called *mathematical induction*.

Problem 4: We are looking for the solution $y(x)$ of the following equation: $\frac{dy}{dx} = f(x)y(x)$, where $f(x)$ is an arbitrary real continuous function. We also limit ourselves to solutions in which $y(x)$ is real (this type of equation is called *differential equation*).

a) (2 Points) Assume that $y(x)$ is non-zero and (strictly) monotonic on the interval $[x_0, x_1]$. Simplify the left-hand-side of

$$\int_{x_0}^{x_1} dx \frac{1}{y(x)} \frac{dy}{dx} = \int_{x_0}^{x_1} dx f(x)$$

by finding a suitable substitution such that the integration can be carried out. Hint: Consider the transformation from Eq.(10) in the lecture.

b) (1 Point) Express $y(x)$ through an antiderivative $F(x)$ of $f(x)$.

c) (1 Point) Which $y(x)$ fulfills the equation $\frac{dy}{dx} = \lambda x^\alpha y(x)$, where $\alpha, \lambda \in \mathbb{R}$ and $x > 0$? Hints: Don't forget the integration constant!

d) (1 Point) Which $y(x)$ fulfills the equation $\frac{dy}{dx} = \exp(\alpha x) y(x)$, where $\alpha \in \mathbb{R}$ and $x > 0$?