Institute of Theoretical Particle Physics

Classical Theoretical Physics I
WS 2023

Prof. Dr. U. Nierste
Dr. L. Chen, Tim Kretz

Exercise Sheet 3
Start: 17.11.2023
Due: 24.11.2023

Please put your name on each sheet of your solution. Please put your tutoring session information on the front page (location, time, name of the tutor, etc.)

Problem 7: We consider an object with mass m moving under the influence of air friction and an external force γm. The velocity $v(t)$ satisfies the following differential equation:

$$\dot{v} = \gamma - \alpha v - \beta v^2, \qquad \alpha, \beta, \gamma > 0. \tag{1}$$

Examples of this situation include an object (dropped from a not too great height) in the Earth's atmosphere (where γ corresponds to the Earth's acceleration g) or the vehicle from Problem 6, where $m\gamma$ is the force with which the engine drives the vehicle. Instead of proceeding analogously to Problem 6, we choose a solution approach that is physically more intuitive and leads to shorter expressions in the intermediate steps.
a) (1 Point) Which of the following properties apply to eq:7: i) linear, ii) homogeneous, or iii) first order?

b) (1 Point) Determine a time-constant solution $v(t) = v_c$ with a positive constant v_c. What can you say about the sum of external force and frictional force for this particular solution?
c) (1 Point) Write $v(t) = w(t) + v_c$ with the v_c determined in (b) and derive a differential equation for w from eq:7. Eliminate γ in this equation using v_c.
d) (2 Points) Determine $w(t)$, where the integration constant is expressed as $w_0 := w(0)$. Distinguish the cases i) $w_0 > 0$, ii) $w_0 = 0$, and iii) $-v_c \le w_0 < 0$. Specify the limit speed $\lim_{t\to\infty} v(t)$ in all cases. In which cases is the object accelerated or decelerated?
Hints: Due to $v \ge 0$, $w \ge -v_c$. What do you know about the sign of $w + \frac{\alpha}{\beta} + 2v_c$? Distinguish already during integration the cases $w > 0$, $w = 0$, and $w < 0$.

Problem 8: Horizontal Throw: A stone is thrown horizontally from a height h at time $t = 0$ with a speed $v_0 \ge 0$. This means that the components of its velocity are $v_x = v_0$ and $v_y = -gt$, where g denotes the acceleration due to gravity. Neglect air friction.
a) (1 Point) At what time T does the stone reach the ground?

b) (4 Points) Calculate the distance traveled

$$s = \int_0^T dt \sqrt{v_0^2 + g^2 t^2}.$$

To verify, take the limit $v_0 \to 0$.
Hint: $\operatorname{arsinh} y = \ln(y + \sqrt{1 + y^2})$.