Quiz 2 - Block 1

Geodäte auf Torus

Parametrisierung der Oberfläche:

$$x = (R + r\cos\varphi)\cos\theta$$
$$y = (R + r\cos\varphi)\sin\theta$$
$$z = r\sin\varphi$$

a) Infinitesimale Elemente:

$$dx = -(R + r\cos\varphi)\sin\theta\, d\theta - r\cos\theta\sin\varphi\, d\varphi \qquad \checkmark$$
$$dy = (R + r\cos\varphi)\cos\theta\, d\theta - r\sin\theta\sin\varphi\, d\varphi \qquad \checkmark$$
$$dz = r\cos\varphi\, d\varphi \qquad \checkmark$$

b) Länge auf Torus:

$$L = \int_1^2 \overset{\checkmark}{ds} = \int_1^2 \sqrt{(dx)^2 + (dy)^2 + (dz)^2} \qquad \checkmark$$

$$= \int_1^2 \Big[(R + r\cos\varphi)^2 \sin^2\theta (d\theta)^2 + r^2\cos^2\theta \sin^2\varphi (d\varphi)^2$$
$$+ 2(R + r\cos\varphi)\sin\theta\, r\cos\theta\sin\varphi\, d\theta\, d\varphi$$
$$+ (R + r\cos\varphi)^2\cos^2\theta (d\theta)^2 + r^2\sin^2\theta\sin^2\varphi (d\varphi)^2$$
$$- 2(R + r\cos\varphi)\cos\theta\, r\sin\theta\sin\varphi\, d\theta\, d\varphi$$
$$+ r^2\cos^2\varphi (d\varphi)^2 \Big]^{1/2} \qquad \checkmark\checkmark\checkmark$$

$$= \int_1^2 \left[(R + r\cos\varphi)^2 (d\theta)^2 + r^2 (d\varphi)^2 \right]^{1/2} \qquad \checkmark$$

$$= \int_1^2 d\varphi \sqrt{r^2 + (R + r\cos\varphi)^2 \left(\frac{d\theta}{d\varphi}\right)^2} \qquad \checkmark$$

c) $\Rightarrow F = \sqrt{r^2 + (R + r\cos\varphi)^2\, \theta'^2}$ ✓

Euler-Lagrange-Gleichungen: $\frac{d}{d\varphi}\frac{\partial F}{\partial \theta'} - \frac{\partial F}{\partial \theta} = 0$ ✓

F hängt nicht von θ ab: $\frac{\partial F}{\partial \theta} = 0$ ✓

$\Rightarrow \frac{\partial F}{\partial \theta'} = \text{const.}$ ✓

$$\frac{\cancel{2}\theta'(R+r\cos\varphi)^2}{\cancel{2}\sqrt{r^2+(R+r\cos\varphi)^2\theta'^2}} = c$$ ✓

$$\theta'^2 (R+r\cos\varphi)^4 = c^2\left(r^2 + (R+r\cos\varphi)^2\theta'^2\right)$$

$$\theta'^2\left((R+r\cos\varphi)^4 - c^2(R+r\cos\varphi)^2\right) = c^2 r^2$$

$$\frac{d\theta}{d\varphi} = \theta' = \pm c r \cdot \left((R+r\cos\varphi)^4 - c^2(R+r\cos\varphi)^2\right)^{-1/2}$$ ✓✓✓

d) $$\int_{\theta_S}^{\theta} d\tilde{\theta} = \pm\int_{\varphi_S}^{\varphi} d\tilde{\varphi}\; cr\left((R+r\cos\tilde{\varphi})^4 - c^2(R+r\cos\tilde{\varphi})^2\right)^{-1/2}$$ ✓✓

$= \theta - \theta_S$