Quiz 3 – Block 2

Teilchen in zylinderförmigem Potential

$$U = U(x^2+y^2, z)$$
$$T = \frac{m}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2) \quad \checkmark$$

$$\rightarrow L = T - U = \frac{m}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2) - U(x^2+y^2, z) \quad \checkmark$$

Betrachte Transformation

$$\left.\begin{aligned} x \rightarrow x^* &= x + \varepsilon y \\ y \rightarrow y^* &= y - \varepsilon x \\ z \rightarrow z^* &= z \\ t \rightarrow t^* &= t \end{aligned}\right\} \quad \begin{aligned} \psi_x &= y \quad \checkmark \\ \psi_y &= -x \quad \checkmark \\ \psi_z &= 0 \quad \checkmark \\ \varphi &= 0, \ \frac{dt^*}{dt} = 1 \quad \checkmark \end{aligned}$$

a) Teste Anwendbarkeit des Noether-Theorems:

$$\frac{d}{d\varepsilon}\left(L^* \cdot \frac{dt^*}{dt}\right)\Big|_{\varepsilon=0} \quad \checkmark$$

$$= \frac{d}{d\varepsilon}\left(\frac{m}{2}\left((\dot{x}+\varepsilon\dot{y})^2+(\dot{y}-\varepsilon\dot{x})^2+\dot{z}^2\right) - U(x^{*2}+y^{*2}, z^*)\right)\Big|_{\varepsilon=0}$$

$$= \frac{d}{d\varepsilon}\left(\frac{m}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2 + 2\varepsilon\underbrace{(\dot{x}\dot{y}-\dot{y}\dot{x})}_{=0} + \varepsilon^2(\dot{y}^2+\dot{x}^2)\right. \quad \checkmark\checkmark \text{ (T-Teil)}$$

$$\left. - U(x^{*2}+y^{*2}, z)\right)\Big|_{\varepsilon=0}$$

$$= \left(-\frac{\partial U}{\partial(x^{*2}+y^{*2})} \cdot \underbrace{\frac{d(x^{*2}+y^{*2})}{d\varepsilon}} - \frac{\partial U}{\partial z^*}\underbrace{\frac{dz^*}{d\varepsilon}}_{=0}\right)\Big|_{\varepsilon=0}$$

$$= \frac{d}{d\varepsilon}(x^2+y^2+2\varepsilon(xy-xy)+\varepsilon^2(y^2+x^2))$$

$$= 2\varepsilon(x^2+y^2) \quad \checkmark\checkmark\checkmark \text{ (U-Teil)}$$

$= 0$ ✓ $\Rightarrow$ Noether-Theorem erfüllt ✓ (nicht erwartetes) ✓

b) Berechne Noetherladung:

$$Q = \sum_i \frac{\partial L}{\partial \dot{x}_i} \psi_i + \Big(L - \sum_i \frac{\partial L}{\partial \dot{x}_i} \dot{x}_i\Big) \underbrace{\varphi}_{=0}$$ ✓

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

$$\frac{\partial L}{\partial \dot{y}} = m\dot{y}$$ ✓

$$\frac{\partial L}{\partial \dot{z}} = m\dot{z}$$

$$\Rightarrow Q = m\dot{x}y + m\dot{y}(-x) = m(\dot{x}y - x\dot{y})$$ ✓✓

vgl. Drehimpuls $L_z = m(\vec{r} \times \dot{\vec{r}})_z$

$$= m(x\dot{y} - y\dot{x}) = -Q$$ ✓