Classical Theoretical Physics II

Lecture: Prof. Dr. K. Melnikov – Exercises: Dr. A. Behring

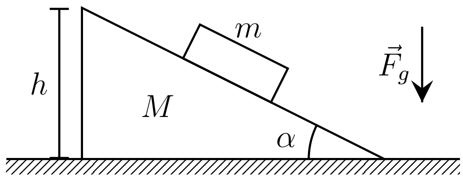
Exercise Sheet 1

Issue: 16.04. – Submission: 23.04. @ 10:00 Uhr – Discussion: 26./27.04.

Exercise 1: Sliding Wedge

8 points

Under the influence of gravity a block of mass m slides without friction on a wedge of mass M with an inclination of α. The edge of the wedge has height h. The wedge itself can slide frictionlessly on a horizontal surface.



(a) Derive the Newtonian equations of motion by identifying and using all forces that act on the block and the wedge. In particular, the block and wedge of course exert forces on each other.
Hint: Remember that, due to the lack of friction, forces can only be transmitted orthogonally to the contact surfaces.

(b) Solve the equations of motion that you just derived. Use as the initial conditions that both the wedge and the block are at rest at time t_0 and that the block is put at the upper edge of the wedge. How long does it take for the block to reach the surface?

Hint: Later on, we will return to this example and see that it becomes much easier to solve using the methods that we will discuss in the lecture.

Exercise 2: Falling object

4 points

We want to consider a simple case, where we can explicitly use the principle of least action to determine the trajectory of a particle.
As an example, we use the situation where a mass m falls vertically in the Earth's gravitational field. At time $t = 0$ the mass should be at height $z(0) = h$ and at time $t = T$ it should have reached the ground, $z(T) = 0$.
In order to parametrise different trajectories of the particle, we make the ansatz $z(t) = c_0 + c_1 t + c_2 t^2$, where c_0, c_1 and c_2 are constants that parametrise different paths and that still have to be determined.

(a) Some constants can be determined from the boundary conditions: Determine the constants c_0 and c_1 from the conditions $z(0) = h$ and $z(T) = 0$.

(b) Write down the Lagrangian that describes this situation.

(c) Compute the action,

$$S = \int_0^T \mathrm{d}t \; L(z(t), \dot{z}(t)) \,, \tag{2.1}$$

as a function of m, g, h, T, c_2.

(d) Use the principle of least action to derive that $c_2 = -g/2$. Is this the result you expected?

(e) *(optional)* Repeat the calculation with the Ansatz $z(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$. What do you find for c_3?

Exercise 3: Lagrangian with velocity-dependent potential — 4 points

Consider the Lagrangian for a particle with mass m in a velocity-dependent potential U,

$$L = \frac{1}{2} m \dot{\vec{r}}^{\,2} - U(\vec{r}, \dot{\vec{r}}, t) \,. \tag{3.1}$$

The velocity-dependence of the potential is parametrised by

$$U(\vec{r}, \dot{\vec{r}}, t) = e\,\phi(\vec{r}, t) - \frac{e}{c} \vec{A}(\vec{r}, t) \cdot \dot{\vec{r}} \,, \tag{3.2}$$

where e and c are constants of Nature, $\phi(\vec{r}, t)$ is an electric potential and $\vec{A}(\vec{r}, t)$ is a vector potential.

(a) Write down the Euler-Lagrange equations.

(b) Apply the following vector identity

$$\vec{\nabla}\,(\vec{x} \cdot \vec{y}\,) = (\vec{x} \cdot \vec{\nabla}\,)\,\vec{y} + (\vec{y} \cdot \vec{\nabla}\,)\,\vec{x} + \vec{y} \times (\vec{\nabla} \times \vec{x}\,) + \vec{x} \times (\vec{\nabla} \times \vec{y}\,) \tag{3.3}$$

to rewrite the term $\frac{\partial}{\partial \vec{r}}\,(\vec{A} \cdot \dot{\vec{r}}) \equiv \vec{\nabla}\,(\vec{A} \cdot \dot{\vec{r}})$ in the Euler-Lagrange equations.

(c) Show that the Euler-Lagrange equations can be brought into the form

$$m \ddot{\vec{r}} = -\frac{e}{c} \frac{\partial \vec{A}}{\partial t} - e \vec{\nabla} \phi + \frac{e}{c}\, \dot{\vec{r}} \times (\vec{\nabla} \times \vec{A}) \,. \tag{3.4}$$

(d) In Maxwell's theory of electrodynamics, the electric and magnetic fields are expressed in terms of an electric potential ϕ and a vector potential $\vec{A}$. We have that

$$\vec{E} = -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \phi \,, \qquad \vec{B} = \vec{\nabla} \times \vec{A} \,. \tag{3.5}$$

Use these formulas to rewrite Eq. (3.4) in terms of $\vec{E}$ and $\vec{B}$. Can you identify the force on the right-hand side of Eq. (3.4)?