Classical Theoretical Physics II

Lecture: Prof. Dr. K. Melnikov – Exercises: Dr. A. Behring

Exercise Sheet 13

Issue: 16.07. – Submission: 23.07. @ 10:00 Uhr – Discussion: —

Remark: This exercise sheet is a "bonus sheet" that does not count towards the maximum number of points that can be obtained, but that still allows you collect some additional points. You can submit it through the usual ILIAS page and it will be graded by your tutors in that case. It will be returned at the beginning of the last week of July. There will be no Saalübung or tutorials for this sheet, but we will publish a solution. In case of questions you can ask them in the forum and we will try to answer them there.

Exercise 1: Moments of inertia — 4 points

Calculate the moment of inertia

(a) of a football (outer radius $R_a = 15\,\mathrm{cm}$, thickness of the homogeneous material $d \ll R_a$, mass $M = 0.4\,\mathrm{kg}$) for a rotation around an axis through the centre of the ball,

(b) of a water molecule (m and M are the masses of the hydrogen (H) and oxygen (O) atoms) with respect to an axis which goes through the centre of mass S of the molecule and which is perpendicular to the plane in which the molecule lies. You are given the distances $\overline{HH} = 2a$ and $\overline{OH} = b$.

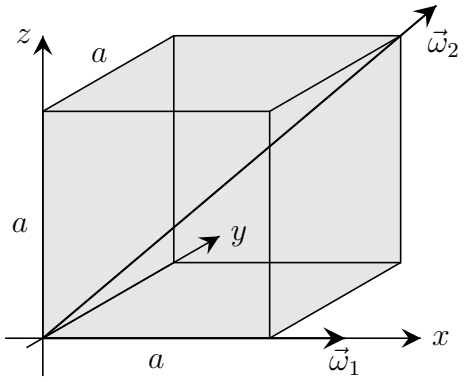
Hints: For (a): Start by calculating the moment of inertia I and the volume of a hollow sphere with inner and outer radii $R_a - d$ and R_a and then find an approximation for I in the limit $d \ll R_a$.
For (b): The atoms are to be regarded as point-like. The plane of the molecule is spanned by the difference vectors of the positions of the atoms.

Exercise 2: Inertia tensor and principal axes — 8 points

Consider a cube with side length a and mass density

$$\rho(\vec{r}) = \rho_0 \frac{r^2}{a^2}\,, \tag{2.1}$$

where the origin of the reference frame coincides with the lower left edge of the cube (see sketch).



(a) Calculate the inertia tensor of the cube.

(b) The cube rotates with an angular velocity of $\vec{\omega}_1$ around the x-axis (see sketch). Find the angular momentum of the cube. Are the angular momentum and the axis of rotation parallel to each other?

(c) Find the principal axes of the cube and its moments of inertia around these axes.

(d) Now the cube rotates with an angular velocity of $\vec{\omega}_2$ around its main diagonal (see sketch). Again, find the angular momentum of the cube. Are the angular momentum and the axis of rotation parallel to each other? What is the difference to (b)?

Exercise 3: Rolling cone **8 points**

(a) Show that the inertia tensor of a circular cone (mass M, height h, opening angle 2α, homogeneous mass density) with respect to the body frame shown in Fig. 1 is diagonal. Calculate the principal moments of inertia.

(b) The cone rolls in the xy-plane of the fixed frame (x, y, z), where the tip of the cone stays fixed (see Fig. 2). Calculate the kinetic energy of the cone as a function of the angular velocity $\dot{\phi}$.

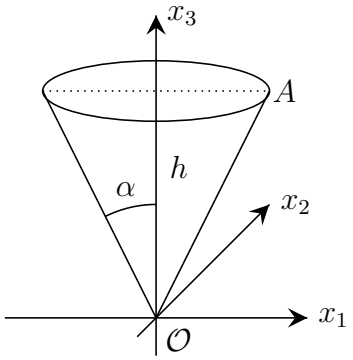


Figure 1: Cone in a body frame

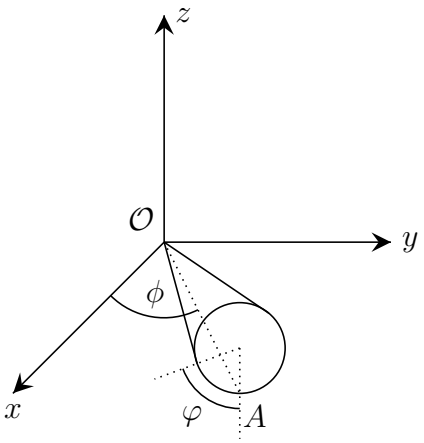


Figure 2: Rolling cone