Blatt 1 Tobias

Freitag, 6. November 2020 11:48

45/50

/1 SI:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(1{,}602\cdot 10^{-19}C)^2}{(10^{-9}m)^2} = 2{,}31\cdot 10^{-10}N$$ ✓

cgs:

$$F = \frac{q^2}{r^2} = \frac{(4{,}803\cdot 10^{-10}Fr)}{(10^{-7}cm)} = 2{,}31\cdot 10^{-5}dyn$$ ✓

/2 a)

a) $$\int E(r)\, dA = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int \rho(r)\, dV$$

$$\rho(r) = \frac{Q}{4\pi R^2}\,\delta(r-R)$$

$$E(r)\cdot 4\pi r^2 = \frac{Q}{4\pi R^2\varepsilon_0}\int \delta(r-R)\, dV$$

$$= \frac{Q}{4\pi R^2\varepsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^{\underline{r}} \delta(r-R)\,\underline{r^2}\sin\theta\,d\underline{r}\,d\theta\,d\varphi$$

Notation...

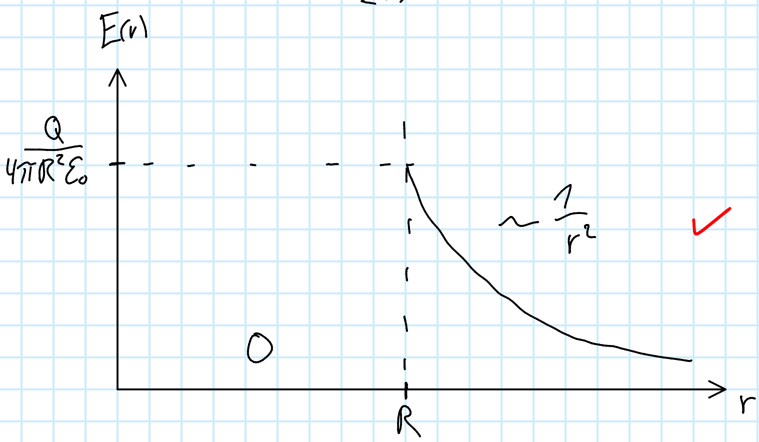
$$E(r) = \frac{Q}{16\pi^2R^2r^2\varepsilon_0}\cdot 2\pi\cdot 2\cdot\int_0^r \delta(r-R)\,r^2\,dr$$

Fall $r \geq R$:

$$E(r) = \frac{Q}{4\pi r^2\varepsilon_0}$$

Fall $r < R$:

$$E(r) = 0$$



✓

b) Fall $r \leq R$

$$\rho(r) = \frac{3Q}{4\pi R^3}$$

$$\int E(r)\, dA = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho(r)\, dV$$

$$E(r) \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \frac{4}{3}\pi r^3 \cdot \frac{3Q}{4\pi R^3}$$

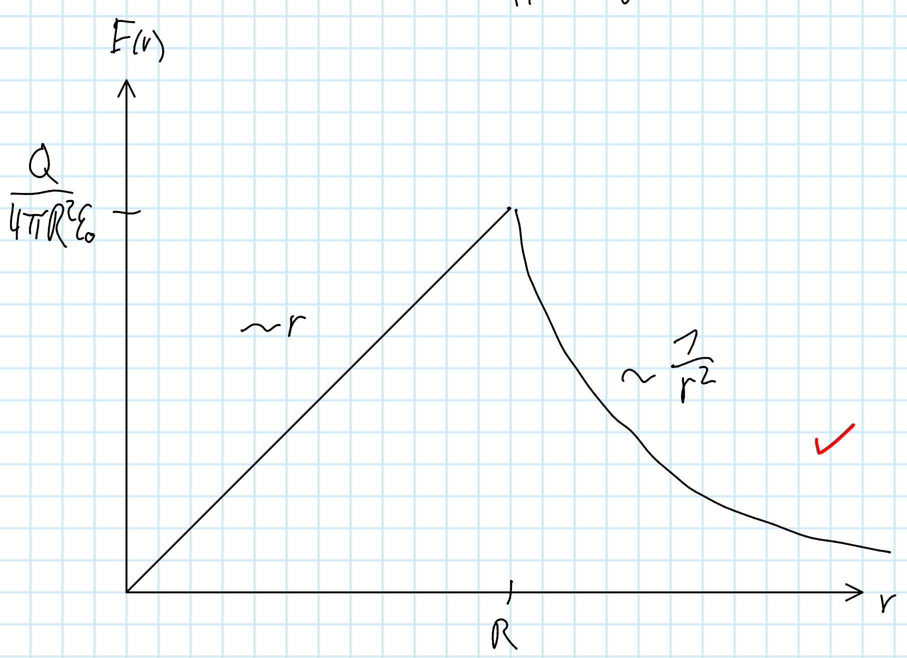
$$E(r) = \frac{Qr}{4\varepsilon_0 \pi R^3}$$

Fall $r > R$

$$\int E(r)\, dA = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho(r)\, dV$$

$$E(r) \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E(r) = \frac{Q}{4\pi r^2 \varepsilon_0}$$



c) $\rho \sim r^n$

Annahme: $\rho(r,n) = \rho_n \cdot r^n$

$$Q = \int \rho_n r^n \, dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho_n r^n \, r^2 \sin\theta \, d\varphi \, d\theta \, dr$$

$$Q = 4\pi \int_0^R \rho_n r^{n+2} \, dr$$

$$\frac{Q}{4\pi} = \rho_n \left[\frac{r^{n+3}}{n+3} \right]_0^R$$

$$\rho_n = \frac{(n+3)\,Q}{4\pi R^{n+3}}$$

$$\rho(r,n) = \frac{(n+3)\,Q}{4\pi R^{n+3}} \, r^n \quad \text{für } r \in [0,R]$$

Fall $r \leq R$

$$\int E \, dA = \frac{q}{\varepsilon_0}$$

$$4\pi r^2 \, E = \frac{1}{\varepsilon_0} \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \frac{(n+3)\,Q}{4\pi R^{n+3}} \, r^n \cdot r^2 \sin\theta \, dr \, d\theta \, d\varphi$$

$$E = \frac{(n+3)\,Q}{4\pi r^2 R^{n+3} \varepsilon_0} \int_0^r r^{n+2} \, dr$$

$$E(r) = \frac{Q \, r^{n+1}}{4\pi R^{n+3} \varepsilon_0}$$

Fall $r > R$

$$\int E(r) \, dA = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho(r) \, dV$$

$$E(r) \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E(r) = \frac{Q}{4\pi r^2 \varepsilon_0}$$