Blatt 9 Tobias

95/100

Mittwoch, 13. Januar 2021 12:21

/1a)

$$U = \frac{d\phi}{dt}$$

mit $\phi = (A \cdot \vec{n}) \cdot \vec{B} = AB$

sei h die y-Koordinate des waagerechten Drahts

Parabel: $f_1(x) = \alpha x^2$

Gerade: $f_2(x) = h$

Schnittpunkt $h = \alpha x^2 \Rightarrow x = \pm\sqrt{\frac{h}{\alpha}}$

Fläche A:

$$A = \int_{-\sqrt{\frac{h}{\alpha}}}^{\sqrt{\frac{h}{\alpha}}} (f_2 - f_1)\, dx = \int_{-\sqrt{\frac{h}{\alpha}}}^{\sqrt{\frac{h}{\alpha}}} (h - \alpha x^2)\, dx$$

$$= \left[hx - \frac{\alpha}{3} h^3\right]_{-\sqrt{\frac{h}{\alpha}}}^{\sqrt{\frac{h}{\alpha}}} = 2\, h\sqrt{\frac{h}{\alpha}} - \frac{2\alpha}{3}\sqrt{\frac{h}{\alpha}}^3$$

$$= 2h\sqrt{\frac{h}{\alpha}}\left(1 - \frac{1}{3}\right)$$

$$= \frac{4}{3} h\sqrt{\frac{h}{\alpha}} = \frac{4}{3\sqrt{\alpha}} h^{\frac{3}{2}}$$

$$h(t) = \frac{1}{2}\omega t^2$$

$$A(h(t)) = \frac{4}{3\sqrt{\alpha}} \left(\frac{1}{2}\omega t^2\right)^{\frac{3}{2}}$$

$$= \frac{4}{3}\sqrt{\frac{\omega^3}{8\alpha}}\, t^3$$

$$U(t) = \frac{d\phi}{dt} = 0 + B\frac{dA}{dt}$$

$$= B \cdot \sqrt{\frac{2\omega^3}{\alpha}}\, t^2 \checkmark = By\sqrt{\frac{8\omega}{a}}$$

-5

b)

$$\phi = \int \vec{B}\, d\vec{A} = \int_0^a\int_0^a k y^3 t^2\, \vec{e}_z \cdot \vec{e}_z\, dx\, dy$$

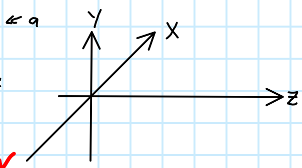
$$= \frac{k}{4} a^5 t^2$$

$$U = \frac{d\phi}{dt} = \frac{k}{2} a^5 t \checkmark$$

/2

$\sigma = \frac{It}{\pi a^2}$

Im Spalt verhält sich das E-Feld wie in einem Plattenkondensator, $\omega \ll a$



In Zylinderkoordinaten:

$\vec{E} = \frac{\sigma}{\varepsilon_0} \vec{e}_z = \frac{It}{\pi a^2 \varepsilon_0} \vec{e}_z$ ✓

Dann gilt nach dem Durchflutungsgesetz:

$$\oint_\gamma \vec{H}\, d\vec{s} = \iint_A \frac{\partial \vec{D}}{\partial t}\, d\vec{A} + I$$

Annahme $\vec{H} = H(r)\, \vec{e}_\varphi$

Mit $\varepsilon_r = \mu_r = 1$

und $\gamma = \begin{pmatrix} r\sin\varphi \\ r\cos\varphi \\ 0 \end{pmatrix}$ $\varphi \in [0, 2\pi]$

und $\frac{\partial \vec{E}}{\partial t} = \frac{I}{\pi a^2 \varepsilon_0} \vec{e}_z\, \Theta(a-r)$

folgt:

$$\frac{1}{\mu_0} \oint_{\gamma} B\, \vec{e}_\varphi\, d\vec{s} = \varepsilon_0 \iint_A \frac{I}{\pi a^2 \varepsilon_0} \vec{e}_z \cdot \vec{e}_z\, \Theta(a-r')\, r' dr' d\varphi$$

$$\frac{1}{\mu_0} \int_0^{2\pi} r B\, d\varphi = 2 \frac{I}{a^2} \int_0^r \Theta(a-r')\, r'\, dr'$$

$$B = \frac{\mu_0 I}{r \pi a^2} \left[\frac{1}{2} r'^2 \right]_0^{\min\{a, r\}}$$

Fall $r \leq a$:

$$B = \frac{\mu_0 I r}{2\pi a^2}$$

Fall $r > a$:

$$B = \frac{\mu_0 I}{2 r \pi}$$

$\vec{B} = \frac{\mu_0 I r}{2\pi a^2} \vec{e}_\varphi$ im Spalt ✓

b) $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{I^2 r t}{2\pi^2 a^4 \varepsilon_0} \vec{e}_z \times \vec{e}_\varphi$

$= -\frac{I^2 r t}{2\pi^2 a^4 \varepsilon_0} \vec{e}_r$ ✓

$w_{em} = \frac{1}{2} (\varepsilon_0 \vec{E}^2 + \frac{1}{\mu_0} \vec{B}^2)$ $\quad E = \frac{It}{\pi a^2 \varepsilon_0} \vec{e}_z$

$= \frac{1}{2} \frac{I^2 t^2}{\pi^2 a^4 \varepsilon_0} + \frac{\mu_0 I^2 r^2}{8\pi^2 a^4}$

$= \frac{I^2}{2\pi^2 a^4} \left(\frac{t^2}{\varepsilon_0} + \frac{\mu_0 r^2}{4} \right)$ ✓

c) $\partial_t w_{em} + \vec{\nabla} \cdot \vec{S}$ $\quad \vec{\nabla} = \vec{e}_r \frac{\partial}{\partial r} + \frac{1}{r} \vec{e}_\varphi \frac{\partial}{\partial \varphi} + \vec{e}_z \frac{\partial}{\partial z}$

$= \frac{I^2 t}{\pi^2 a^4 \varepsilon_0} + \left(-\frac{I^2 t}{2\pi^2 a^4 \varepsilon_0}\right)$

$= 0$ ✓ V

d) $W_S = \iiint w_{em} \, dV$

$= \int_0^w \int_0^a \int_0^{2\pi} \frac{I^2}{2\pi^2 a^4} \left(\frac{t^2}{\varepsilon_0} + \frac{\mu_0 r^2}{4} \right) r \, d\varphi \, dr \, dz$

$= \frac{I^2 w}{\pi a^4} \int_0^a r \left(\frac{t^2}{\varepsilon_0} + \frac{\mu_0 r^2}{4} \right) dr$

$= \frac{I^2 w}{\pi} \left(\frac{t^2}{2a^2 \varepsilon_0} + \frac{\mu_0}{16} \right)$

$\frac{dW_S}{dt} = \frac{I^2 w t}{\pi a^2 \varepsilon_0}$

Energiefluss durch Poynting vektor

$\frac{dW_P}{dt} = \int_A \vec{S} \, d\vec{A}$

$= \int_0^w \int_0^{2\pi} -\vec{S} \; r \, dz \, d\varphi \Big|_{r=a}$

$= \frac{I^2 t w}{2\pi^2 a^2 \varepsilon_0} \int_0^{2\pi} d\varphi$

$= \frac{I^2 t w}{\pi a^2 \varepsilon_0}$

entspricht der Zunahme der Gesamtenergie ✓

/3

$$\vec{\Pi} = \varepsilon_0 \vec{S} = \varepsilon_0 (\vec{E} \times \vec{B})$$
$$= \varepsilon_0 E B \vec{e}_y$$

$$\vec{P} = \int_V \vec{\pi}\, dV = \varepsilon_0 A B d E \vec{e}_y$$ ✓

b)

$\sigma = \varepsilon_0 E \qquad Q = A \varepsilon_0 E$

$$I = \frac{dQ}{dt}$$

$$\vec{F} = -I d \vec{e}_z \times \vec{B}$$
$$= -I d B \vec{e}_y$$

$$\vec{p} = \int \vec{F}\, dt = \int_{A\varepsilon_0 E}^{0} -dB e_y\, dQ$$
$$= \varepsilon_0 A d B E \vec{e}_y$$ ✓