a) z: $\vec{\nabla} \times (\vec{\nabla}\phi) = \vec{0}$

$$(\vec{\nabla} \times (\vec{\nabla}\phi))_i = \varepsilon_{ijk}\,\partial_j \partial_k \phi = \frac{1}{2}(\varepsilon_{ijk}\,\partial_j\partial_k + \varepsilon_{ijk}\,\partial_j\partial_k)\phi$$

$$= \frac{1}{2}(\varepsilon_{ijk}\partial_j\partial_k - \varepsilon_{ikj}\partial_j\partial_k)\phi = \frac{1}{2}(\varepsilon_{ijk}\partial_j\partial_k - \varepsilon_{ijk}\partial_k\partial_j)\phi = \frac{1}{2}(\varepsilon_{ijk}\partial_j\partial_k - \varepsilon_{ijk}\partial_j\partial_k)\phi = 0$$

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b) z: $\vec{\nabla}(\vec{\nabla} \times \vec{A}) = 0$

$$\vec{\nabla}(\vec{\nabla} \times \vec{A})_i = \vec{\nabla}(\varepsilon_{ijk}\partial_j A_k) = \partial_i \varepsilon_{ijk}\partial_j A_k = \varepsilon_{ijk}\partial_i\partial_j A_k = -\varepsilon_{jik}\partial_j\partial_i A_k$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla} \times \vec{A}) = 0$$

2

hier fehlt ein Argument → warum folgt jetzt die Null?

c) $\varepsilon_{ijk}\varepsilon_{kmn} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}$ (1)

z $[\vec{\nabla} \times (\vec{\nabla} \times \vec{A})]_i = \partial_i(\partial_j A_j) - \partial_j^2 A_i = \vec{\nabla}(\vec{\nabla}\vec{A}) - \Delta\vec{A}$

$$[\vec{\nabla} \times (\vec{\nabla} \times \vec{A})]_i = \varepsilon_{ijk}\partial_j(\vec{\nabla} \times \vec{A})_k = \varepsilon_{ijk}\partial_j\varepsilon_{kmn}\partial_m A_n$$

$$= \varepsilon_{ijk}\varepsilon_{kmn}\partial_j\partial_m A_n \overset{(1)}{=} (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm})\partial_j\partial_m A_n$$

$$= \delta_{im}\delta_{jn}\partial_j\partial_m A_n - \delta_{in}\delta_{jm}\partial_j\partial_m A_n$$

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$$= \partial_n\partial_i A_n - \partial_m\partial_m A_i = \partial_n\partial_i A_n - \partial_m^2 A_i = \vec{\nabla}(\vec{\nabla}\vec{A} - \Delta\vec{A})$$

(2) $x = \rho\cos\varphi$, $y = \rho\sin\varphi$, $z = z$

$\vec{V}_i = \partial_i \vec{r}$, $\vec{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, $i \in \{\rho, \varphi, z\}$

a) $\vec{V}_i = \partial_i(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z) = \partial_i(\rho\cos\varphi\,\hat{e}_x + \rho\sin\varphi\,\hat{e}_y + z\hat{e}_z)$

$\vec{V}_\rho = \cos\varphi\,\hat{e}_x + \sin\varphi\,\hat{e}_y$, $\quad |\vec{V}_\rho| = \sqrt{\cos^2\varphi + \sin^2\varphi} = 1$

$\vec{V}_\varphi = -\rho\sin\varphi\,\hat{e}_x + \rho\cos\varphi\,\hat{e}_y$, $\quad |\vec{V}_\varphi| = \sqrt{\rho^2\sin^2\varphi + \rho^2\cos^2\varphi} = \rho$

$\vec{V}_z = \hat{e}_z$, $\quad |\vec{V}_z| = 1$

$$\Rightarrow \hat{e}_\rho = \frac{\vec{V}_\rho}{|\vec{V}_\rho|} = \cos\varphi\, \hat{e}_x + \sin\varphi\, \hat{e}_y$$

$$\hat{e}_\varphi = \frac{\vec{V}_\varphi}{|\vec{V}_\varphi|} = -\sin\varphi\, \hat{e}_x + \cos\varphi\, \hat{e}_y$$

$$\hat{e}_z = \hat{e}_z$$

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b) • $\hat{e}_x = \cos\varphi\, \hat{e}_\rho - \sin\varphi\, \hat{e}_\varphi$ naja, also das sollst du ausrechnen und herleiten

$$= \cos^2\varphi\, \hat{e}_x + \sin\varphi\cos\varphi\, \hat{e}_y + \sin^2\varphi\, \hat{e}_x - \sin\varphi\cos\varphi\, \hat{e}_y = \hat{e}_x$$

• $\hat{e}_y = \sin\varphi\, \hat{e}_\rho + \cos\varphi\, \hat{e}_\varphi = \sin\varphi\cos\varphi\, \hat{e}_x + \sin^2\varphi\, \hat{e}_y - \cos\varphi\sin\varphi\, \hat{e}_x + \cos^2\varphi\, \hat{e}_y = \hat{e}_y$

• $\hat{e}_z = \hat{e}_z$

c) z: $\partial_x \phi = \left(\cos\varphi\, \partial_\rho - \frac{\sin\varphi}{\rho}\partial_\varphi\right)\phi$

$\partial_y \phi = \left(\sin\varphi\, \partial_\rho + \frac{\cos\varphi}{\rho}\partial_\varphi\right)\phi$

$$\partial_i \phi = \frac{\partial\phi}{\partial x}\frac{\partial x}{\partial_i} + \frac{\partial\phi}{\partial y}\frac{\partial y}{\partial_i}, \quad i \in \{\rho, \varphi\}$$

(1) $\partial_\rho \phi = \frac{\partial\phi}{\partial x}\frac{\partial x}{\partial_\rho} + \frac{\partial\phi}{\partial y}\frac{\partial y}{\partial\rho} = \frac{\partial\phi}{\partial x}\frac{\partial(\rho\cos\varphi)}{\partial\rho} + \frac{\partial\phi}{\partial y}\frac{\partial(\rho\sin\varphi)}{\partial\rho}$

$= \frac{\partial\phi}{\partial x}\cos\varphi + \frac{\partial\phi}{\partial y}\sin\varphi$

(2) $\partial_\varphi \phi = \frac{\partial\phi}{\partial x}\frac{\partial x}{\partial\varphi} + \frac{\partial\phi}{\partial y}\frac{\partial y}{\partial\varphi} = \frac{\partial\phi}{\partial x}\frac{\partial(\rho\cos\varphi)}{\partial\varphi} + \frac{\partial\phi}{\partial y}\frac{\partial(\rho\sin\varphi)}{\partial\varphi}$

$= -\frac{\partial\phi}{\partial x}\rho\sin\varphi + \frac{\partial\phi}{\partial y}\rho\cos\varphi$

Aus (1) folgt: $(\partial_\rho\phi - \partial_y\phi\sin\varphi)\cdot\frac{1}{\cos\varphi} = \partial_x\phi$

Aus (2) folgt: $(\partial_\varphi\phi + \partial_x\phi\,\rho\sin\varphi)\cdot\frac{1}{\rho\cos\varphi} = \partial_y\phi$

(1) in (2): $\left(\partial_\varphi \phi + \left((\partial_\rho \phi - \partial_y \phi \sin\varphi)\frac{1}{\cos\varphi}\rho\sin\varphi\right)\right)\frac{1}{\rho\cos\varphi} = \partial_y \phi$

$$\Leftrightarrow \frac{\partial_\varphi \phi}{\rho\cos\varphi} + \partial_\rho \phi \frac{\rho\sin\varphi}{\rho\cos^2\varphi} - \partial_y \phi \frac{\rho\sin^2\varphi}{\rho\cos^2\varphi} = \partial_y \phi$$

$$\Leftrightarrow \frac{\partial_\varphi \phi}{\rho\cos\varphi} + \partial_\rho \phi \frac{\sin\varphi}{\cos^2\varphi} = \partial_y \phi \left(\frac{\sin^2\varphi}{\cos^2\varphi} + 1 \right)$$

$$\Leftrightarrow \frac{\partial_\varphi \phi}{\rho\cos\varphi} \cdot \frac{1}{\frac{\sin^2\varphi + \cos^2\varphi}{\cos^2\varphi}} + \partial_\rho \phi \frac{\sin\varphi}{\cos^2\varphi} \cdot \frac{1}{\frac{\sin^2\varphi + \cos^2\varphi}{\cos^2\varphi}} = \partial_y \phi$$

$$\Leftrightarrow \frac{\partial_\varphi \phi}{\rho\cos\varphi} \frac{\cos^2\varphi}{\underbrace{\sin^2\varphi + \cos^2\varphi}_{=1}} + \partial_\rho \phi \frac{\sin\varphi}{\cos^2\varphi} \cdot \frac{\cos^2\varphi}{\underbrace{\sin^2\varphi + \cos^2\varphi}_{=1}} = \partial_y \phi$$

$$\Leftrightarrow \partial_\varphi \phi \frac{\cos\varphi}{\rho} + \partial_\rho \phi \sin\varphi = \partial_y \phi$$

$$\Leftrightarrow \left(\sin\varphi \, \partial_\rho + \frac{\cos\varphi}{\rho}\partial_\varphi\right)\phi$$ ✓

Einsetzen in (1):

$$(\partial_\rho \phi - \partial_y \phi \sin\varphi) \cdot \frac{1}{\cos\varphi} = \partial_x \phi$$

$$\Leftrightarrow \left(\partial_\rho \phi - \left(\partial_\varphi \phi \frac{\cos\varphi}{\rho} + \partial_\rho \phi \sin\varphi\right)\sin\varphi\right)\frac{1}{\cos\varphi} = \partial_x \phi$$

$$\Leftrightarrow \frac{\partial_\rho \phi}{\cos\varphi} - \left(\partial_\varphi \phi \frac{\cos\varphi \sin\varphi}{\rho\cos\varphi} + \frac{\partial_\rho \phi \sin^2\phi}{\cos\varphi}\right) = \partial_x \phi$$

$$\Leftrightarrow \frac{\partial_\rho \phi}{\cos\varphi} - \partial_\varphi \phi \frac{\sin\varphi}{\rho} - \partial_\rho \phi \frac{\sin^2\phi}{\cos\varphi} = \partial_x \phi$$

$$\Leftrightarrow \partial_\rho \phi \left(\frac{1 - \sin^2\phi}{\cos\varphi}\right) - \partial_\varphi \phi \frac{\sin\varphi}{\rho} = \partial_x \phi \qquad \Big| \quad 1 - \sin^2\varphi = \cos^2\varphi$$

$$\Leftrightarrow \partial_\rho \phi \cos\varphi - \partial_\varphi \phi \frac{\sin\varphi}{\rho} = \partial_x \phi$$

$$\Leftrightarrow \left(\cos\varphi\, \partial_\rho - \frac{\sin\varphi}{\rho} \partial_\varphi\right) \phi = \partial_x \phi$$ ✓ 5

d) z: $\vec{\nabla} \phi = (\hat{e}_x \partial_x + \hat{e}_y \partial_y + \hat{e}_z \partial_z) \phi$

$$\vec{\nabla} \phi = ((\cos\varphi\, \hat{e}_\rho - \sin\varphi\, \hat{e}_\varphi) \partial_x + (\sin\varphi\, \hat{e}_\rho + \cos\varphi\, \hat{e}_\varphi) \partial_y + \hat{e}_z \partial_z) \phi$$

$$= (\cos\varphi\, \hat{e}_\rho - \sin\varphi\, \hat{e}_\varphi)\left(\cos\varphi\, \partial_\rho - \frac{\sin\varphi}{\rho} \partial_\varphi\right) \phi$$
$$+ (\sin\varphi\, \hat{e}_\rho + \cos\varphi\, \hat{e}_\varphi)\left(\sin\varphi\, \partial_\rho + \frac{\cos\varphi}{\rho} \partial_\varphi\right) \phi + (\hat{e}_z \partial_z) \phi$$

$$= \Big(\cos^2\varphi\, \hat{e}_\rho \partial_\rho - \cancel{\frac{\cos\varphi \sin\varphi}{\rho} \hat{e}_\rho \partial_\varphi} - \cancel{\sin\varphi \cos\varphi\, \hat{e}_\varphi \partial_\rho} + \frac{\sin^2\varphi}{\rho} \hat{e}_\varphi \partial_\varphi$$
$$+ \sin^2\varphi\, \hat{e}_\rho \partial_\rho + \cancel{\frac{\sin\varphi \cos\varphi}{\rho} \hat{e}_\rho \partial_\varphi} + \cancel{\cos\varphi \sin\varphi\, \hat{e}_\varphi \partial_\rho}$$
$$+ \frac{\cos^2\varphi}{\rho} \hat{e}_\varphi \partial_\varphi + \hat{e}_z \partial_z \Big) \phi$$

$$= \left(\hat{e}_\rho \partial_\rho + \frac{\hat{e}_\varphi \partial_\varphi}{\rho} + \hat{e}_z \partial_z\right) \phi$$ ✓ 5

e) z: $\vec{\nabla} \cdot \vec{A} = \frac{1}{\rho} \partial_\rho (\rho A_\rho) + \frac{1}{\rho} \partial_\varphi A_\varphi + \partial_z A_z$, $A_\alpha = \hat{e}_\alpha \vec{A}$

$$\vec{\nabla} \cdot \vec{A} = \partial_x A_x + \partial_y A_y + \partial_z A_z$$

$$= \left(\cos\varphi\, \partial_\rho - \frac{\sin\varphi}{\rho} \partial_\varphi\right) \cdot \left(\left(\cos\varphi\, \hat{e}_\rho - \sin\varphi\, \hat{e}_\varphi\right) \cdot \vec{A}\right)$$

$$+ \left(\sin\varphi\, \partial_\rho + \frac{\cos\varphi}{\rho} \partial_\varphi\right) \left(\left(\sin\varphi\, \hat{e}_\rho + \cos\varphi\, \hat{e}_\varphi\right) \vec{A}\right) + \partial_z (\hat{e}_z \vec{A})$$

$$= \left(\cos\varphi\, \partial_\rho - \frac{\sin\varphi}{\rho} \partial_\varphi\right) \left(\cos\varphi\, A_\rho - \sin\varphi\, A_\varphi\right)$$

$$+ \left(\sin\varphi\, \partial_\rho + \frac{\cos\varphi}{\rho} \partial_\varphi\right) \left(\sin\varphi\, A_\rho + \cos\varphi\, A_\varphi\right) + \partial_z A_z$$

$$= \cos^2 \partial_\rho A_\rho - \cancel{\cos\varphi \sin\varphi\, \partial_\rho A_\varphi} - \cancel{\frac{\sin\varphi \cos\varphi}{\rho} \partial_\varphi A_\rho} + \frac{\sin^2\varphi}{\rho} \partial_\varphi A_\varphi$$

$$+ \sin^2\varphi\, \partial_\rho A_\rho + \cancel{\sin\varphi \cos\varphi\, \partial_\rho A_\varphi} + \cancel{\frac{\cos\varphi \sin\varphi}{\rho} \partial_\varphi A_\rho}$$

$$+ \frac{\cos^2\varphi}{\rho} \partial_\varphi A_\varphi + \partial_z A_z$$

$$= \partial_\rho A_\rho + \frac{1}{\rho} \partial_\varphi A_\varphi + \partial_z A_z$$

$$= \frac{1}{\rho} \partial_\rho (\rho A_\rho) + \frac{1}{\rho} \partial_\varphi A_\varphi + \partial_z A_z$$

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f) $(\vec{\nabla} \times \vec{A})_z = \partial_x A_y - \partial_y A_x$

z: $(\vec{\nabla} \times \vec{A})_z = \frac{1}{\rho} \left[\partial_\rho (\rho A_\varphi) - \partial_\varphi A_\rho \right]$

$$\partial_x A_y - \partial_y A_x = \left(\cos\varphi\, \partial_\rho - \frac{\sin\varphi}{\rho} \partial_\varphi\right) A_y - \left(\sin\varphi\, \partial_\rho + \frac{\cos\varphi}{\rho} \partial_\varphi\right) A_x$$

$$= \left(\cos\varphi\, \partial_\rho - \frac{\sin\varphi}{\rho} \partial_\varphi\right) \left(\left(\sin\varphi\, \hat{e}_\rho + \cos\varphi\, \hat{e}_\varphi\right) \vec{A}\right) - \left(\sin\varphi\, \partial_\rho + \frac{\cos\varphi}{\rho} \partial_\varphi\right)$$

$$\cdot \left(\left(\cos\varphi\, \hat{e}_\rho - \sin\varphi\, \hat{e}_\varphi\right) \cdot \vec{A}\right)$$

$$= \left(\cos\varphi\,\partial_\rho - \frac{\sin\varphi}{\rho}\partial_\varphi\right)\left(\sin\varphi A_\rho + \cos\varphi A_\varphi\right) - \left(\sin\varphi\,\partial_\rho + \frac{\cos\varphi}{\rho}\partial_\varphi\right)$$
$$\cdot\left(\cos\varphi A_\rho - \sin\varphi A_\varphi\right)$$

$$= \cancel{\cos\varphi\sin\varphi\,\partial_\rho A_\rho} + \cos^2\varphi\,\partial_\rho A_\varphi - \frac{\sin^2\varphi}{\rho}\partial_\varphi A_\rho - \cancel{\frac{\sin\varphi\cos\varphi}{\rho}\partial_\varphi A_\varphi}$$
$$- \cancel{\sin\varphi\cos\varphi\,\partial_\rho A_\rho} + \sin^2\varphi\,\partial_\rho A_\varphi - \frac{\cos^2\varphi}{\rho}\partial_\varphi A_\rho + \cancel{\frac{\cos\varphi\sin\varphi}{\rho}\partial_\varphi A_\varphi}$$

$$= \underbrace{(\cos^2\varphi + \sin^2\varphi)}_{=1}\partial_\rho A_\varphi - \frac{1}{\rho}\underbrace{(\sin^2\varphi + \cos^2\varphi)}_{=1}\partial_\varphi A_\rho$$

$$= \partial_\rho A_\varphi - \frac{1}{\rho}\partial_\varphi A_\rho = \frac{1}{\rho}\left[\partial_\rho(\rho A_\varphi) - \partial_\varphi A_\rho\right]$$ ✓

5

g) $$\phi(\vec{r}) = \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{\rho^2\cos^2\varphi + \rho^2\sin^2\varphi}} = \frac{1}{\sqrt{\rho^2\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{=1}}} = \frac{1}{\rho}$$ ✓

$$\vec{\nabla}\phi \overset{(3)}{=} \left(\hat{e}_\rho\partial_\rho + \frac{\hat{e}_\varphi}{\rho}\partial_\varphi + \hat{e}_z\partial_z\right)\frac{1}{\rho}$$
$$= \hat{e}_\rho\partial_\rho\frac{1}{\rho} = -\hat{e}_\rho\cdot\frac{1}{\rho^2}$$ ✓

$$\vec{\nabla}(\vec{\nabla}\phi) = \vec{\nabla}\left(-\hat{e}_\rho\frac{1}{\rho^2}\right) \overset{(4)}{=} \frac{1}{\rho}\partial_\rho\left(\rho\,\hat{e}_\rho\left(-\hat{e}_\rho\frac{1}{\rho^2}\right)\right) + \frac{1}{\rho}\partial_\varphi\left(\hat{e}_\varphi\left(-\hat{e}_\rho\frac{1}{\rho^2}\right)\right)$$
$$+ \partial_z\left(\hat{e}_z\cdot\left(-\hat{e}_\rho\frac{1}{\rho^2}\right)\right)$$

$$= \frac{1}{\rho}\partial_\rho\left(-\frac{1}{\rho}\right) = \frac{1}{\rho}\cdot\frac{1}{\rho^2} = \frac{1}{\rho^3}$$ ✓

$$\left(\vec{\nabla}\times(\vec{\nabla}\phi)\right)_z = \frac{1}{\rho}\left[\partial_\rho\left(\rho\,\hat{e}_\varphi\left(-\hat{e}_\rho\cdot\frac{1}{\rho^2}\right)\right) - \partial_\varphi\left(\hat{e}_\rho\left(-\hat{e}_\rho\cdot\frac{1}{\rho^2}\right)\right)\right]$$

$$= \frac{1}{\rho}\partial_\varphi\frac{1}{\rho^2} = 0$$ ✓

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h) $$\vec{A}(\vec{r}) = (x^2+y^2)^{\frac{n}{2}}\left(x\,\hat{e}_y - y\,\hat{e}_x\right)$$

$$= \left(\rho^2\cos^2\varphi + \rho^2\sin^2\varphi\right)^{\frac{n}{2}}\left(\rho\cos\varphi\left(\sin\varphi\,\hat{e}_\rho + \cos\varphi\,\hat{e}_\varphi\right)\right.$$
$$\left.- \rho\sin\varphi\left(\cos\varphi\,\hat{e}_\rho - \sin\varphi\,\hat{e}_\varphi\right)\right)$$

$$= (\rho^2)^{\frac{n}{2}} \cdot (\rho\cos\varphi\sin\varphi\, \hat{e}_\rho + \rho\cos^2\varphi\, \hat{e}_\varphi$$
$$- \rho\sin\varphi\cos\varphi\, \hat{e}_\rho + \rho\sin^2\varphi\, \hat{e}_\varphi)$$

$$= \rho^{n+1}\, \hat{e}_\varphi$$

$$\vec{\nabla}\vec{A} = \frac{1}{\rho}\partial_\rho(\rho(\hat{e}_\rho \cdot \rho^{n+1}\hat{e}_\varphi)) + \frac{1}{\rho}\partial_\varphi(\hat{e}_\varphi \cdot \rho^{n+1}\hat{e}_\varphi) + \partial_z(\hat{e}_z\, \rho^{n+1}\hat{e}_\varphi)$$

$$= \frac{1}{\rho}\partial_\varphi \rho^{n+1} = 0$$

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$$(\vec{\nabla}\times\vec{A})_z = \frac{1}{\rho}\left[\partial_\rho(\rho\,\hat{e}_\varphi\, \rho^{n+1}\hat{e}_\varphi) - \partial_\varphi \hat{e}_\rho\, \rho^{n+1}\hat{e}_\varphi\right]$$

$$= \frac{1}{\rho}\, \partial_\rho \rho^{n+2} = \begin{cases} \frac{1}{\rho}(n+2)\cdot\rho^{n+1} = (n+2)\,\rho^n & n \neq -2) \\ 0 & \text{für } n=-2 \end{cases}$$

③ a) $\vec{A}(\vec{r}) = \rho^n(\cos\alpha\,\hat{e}_\rho + \sin\alpha\,\hat{e}_\varphi)$

$$\oint_{\partial F} \vec{A}\, d\vec{r} = \oint_{\partial F} R^n(\cos\alpha\,\hat{e}_\rho + \sin\alpha\,\hat{e}_\varphi)\,\hat{e}_\varphi\, R\, d\varphi$$

$$= \oint_{\partial F} R^n \sin\alpha\, R\, d\varphi = 2\pi R^{n+1}\sin\alpha \quad \checkmark$$

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b) $$\int_F (\vec{\nabla}\times\vec{A})\, d\vec{f} = \int_0^R\int_0^{2\pi} (\vec{\nabla}\times\vec{A})\,\hat{e}_z\,\rho\, d\varphi\, d\rho = \int_0^R\int_0^{2\pi} [\vec{\nabla}\times\vec{A}]_z\,\rho\, d\varphi\, d\rho$$

$$\stackrel{(5)}{=} \int_0^R\int_0^{2\pi} \frac{1}{\rho}\left[\partial_\rho(\rho A_\varphi) - \partial_\varphi A_\rho\right]\rho\, d\varphi\, d\rho$$

$$= \int_0^R\int_0^{2\pi} \frac{1}{\rho}\left[\partial_\rho(\rho\,\rho^n\sin\alpha) - \partial_\varphi(\rho\,\rho^n\cos\alpha)\right]\rho\, d\varphi\, d\rho = \begin{cases} \ldots & \text{für } n\neq -1 \\ 0 & \text{für } n=-1 \end{cases}$$

$$= \int_0^R \int_0^{2\pi} \sin\alpha\,(n+1)\rho^n \, d\varphi \, d\rho = \int_0^R 2\pi \sin\alpha\,(n+1)\,\rho^n \, d\rho$$

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$$= \left[2\pi \sin\alpha \; \rho^{n+1} \right]_0^R = 2\pi \sin\alpha \, R^{n+1}$$ für $n \neq -1$

$= 0$ für $n = -1$

$\Rightarrow$ Gleiches Ergebnis wie in a) für $n \neq -1$

c) $$\oint_{\partial V} \vec{A} \, d\vec{S} = \oint_{\partial V} R^n (\cos\alpha \, \hat{e}_\rho + \sin\alpha \, \hat{e}_\varphi) \, \hat{e}_\rho \, R \, d\varphi$$

$$= \oint_{\partial V} R^{n+1} \cos\alpha \, d\varphi = 2\pi R^{n+1} \cos\alpha$$ ✓

4

d) $$\int_V (\vec{\nabla} \cdot \vec{A}) \, dV = \int_V \left(\frac{1}{\rho} \partial_\rho (\rho A_\rho) + \frac{1}{\rho} \partial_\varphi A_\varphi + \partial_z A_z \right) \rho \, d\rho \, d\varphi$$

$$= \int_0^{2\pi} \int_0^R \left(\partial_\rho \rho^{n+1} \cos\alpha + \frac{1}{\rho} \partial_\varphi \rho^n \sin\alpha \right) d\rho \, d\varphi = \begin{cases} \dots & \text{für } n \neq -1 \\ 0 & \text{für } n = -1 \end{cases}$$

$$= \int_0^{2\pi} \int_0^R (n+1)\rho^n \cos\alpha \, d\rho \, d\varphi = \int_0^R 2\pi (n+1) \rho^n \cos\alpha \, d\rho$$

$$= \left[\rho^{n+1} \cos\alpha \right]_0^R = 2\pi R^{n+1} \cos\alpha$$

$= 0$ für $n = -1$

8

$\Rightarrow$ gleiches Ergebnis wie in c) für $n \neq -1$ wie oben