① $\int_{-\infty}^{\infty} dx\ \delta(x-x_0) f(x) = f(x_0)$, $\int_{-\infty}^{\infty} dx\ \delta(x-x_0) = 1$

a) (i) z: $\delta(-x) = \delta(x)$

$$\int_{-\infty}^{\infty} dx\ \delta(-x) f(x) = -\int_{-\infty}^{\infty} dy\ \delta(x) f(-y) \qquad \Big|\ \text{subst. } y = -x$$

$$= \int_{\infty}^{-\infty} dy\ \delta(x)\ f(-y) = f(0) = \int_{-\infty}^{\infty} dx\ \delta(x) f(x)$$ ✓

(ii) z: $x\,\delta(x) = 0$

$$\int_{-\infty}^{\infty} dx\ \delta(x) f(x) \quad \text{mit } f(x) = x: \int_{-\infty}^{\infty} dx\ \delta(x)\ x = 0 \Leftrightarrow \delta(x) x = 0$$ ✓

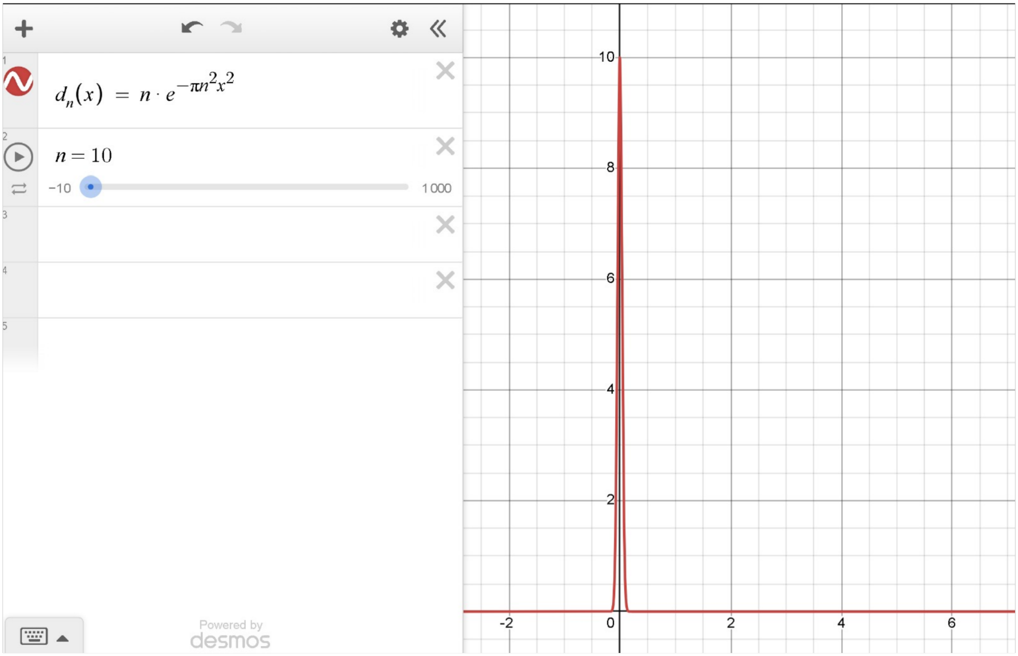
(iii) z: $\delta(ax) = \frac{1}{|a|}\delta(x)$

$$\int_{-\infty}^{\infty} dx\ \delta(ax) f(x) = \int_{-\infty}^{\infty} dy\ \frac{1}{|a|}\delta(y) f\left(\frac{y}{a}\right) = \frac{1}{|a|} f(0) \qquad \Big|\ \text{subst. } ax = y,\ x = \frac{y}{a},\ dx = \frac{dy}{a}$$

$$= \int_{-\infty} dx\ \frac{1}{|a|}\delta(x) f(x)$$ ✓

5) $\delta(x) = \lim\limits_{n\to\infty} \delta_n(x)$ mit $\delta_n(x) = n e^{-\pi n^2 x^2}$

Für $n = 10$:



$$\int_{-\infty}^{\infty} dx\, \delta_n(x) = \int_{-\infty}^{\infty} dx\, n e^{-\pi n^2 x^2} = \int_{-\infty}^{\infty} du\, \frac{e^{-u^2}}{\sqrt{\pi}} \quad \Big| \text{ Subst. } u = \sqrt{\pi}\, n x,\ du = \sqrt{\pi}\, n\, dx \Leftrightarrow dx = \frac{du}{\sqrt{\pi}\, n}$$

Über die Definition des Gaußintegrals $\int_{-\infty}^{\infty} du\, e^{-u^2} = \sqrt{\pi}$ erhalten wir:

$$\int_{-\infty}^{\infty} dx\, \delta_n(x) = 1$$

$$\int_{-\infty}^{\infty} \delta_n(x - x_0) f(x)\, dx$$

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2}$$

$$\int_{-\infty}^{\infty} \delta_n(x-x_0) f(x)\, dx = \int_{-\infty}^{\infty} dx\, \delta_n(x-x_0) \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)(x-x_0)^k}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!} \int_{-\infty}^{\infty} dx\, \delta_n(x-x_0)(x-x_0)^k$$

$\underline{k=0:}$ $\int_{-\infty}^{\infty} dx\, \delta_n(x-x_0) = 1$

$\underline{k=1:}$ $\int_{-\infty}^{\infty} dx\, \delta_n(x-x_0)(x-x_0) = 0$

$\underline{k=2:}$ $\int_{-\infty}^{\infty} dx\, \delta_n(x-x_0)(x-x_0)^2$

$$= \int_{-\infty}^{\infty} dx\, n e^{-\pi n^2 (x-x_0)^2} (x-x_0)^2$$

| Subst. $y = n(x-x_0)$
$dy = n\,dx \Leftrightarrow dx = \frac{dy}{n}$
$y^2 = n^2(x-x_0)^2$

$$= \int_{-\infty}^{\infty} \frac{dy}{\cancel{n}} \cdot \cancel{n}\, e^{-\pi y^2} \frac{y^2}{n^2} = \int_{-\infty}^{\infty} dy\, e^{-\pi y^2} \frac{y^2}{n^2}$$

$$= \sqrt{\frac{\pi}{\pi}} \cdot \frac{1!!}{2\pi n^2} = \frac{1}{2\pi n^2}$$ ✓

$$\int_{-\infty}^{\infty} y^{2b} e^{-\alpha y^2} dy = \sqrt{\frac{\pi}{\alpha}} \frac{(2b-1)!!}{(2\alpha)^b}$$

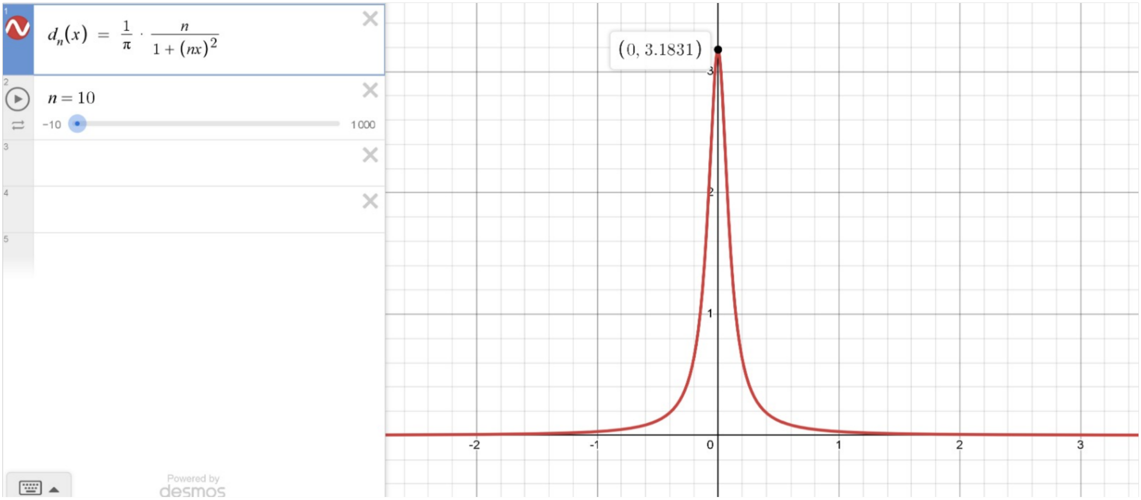
(nachgeschlagen) mit $b=1$ und $\alpha = \pi$

Damit ergibt sich: ✓

$$f(x_0) + f''(x_0) \cdot \frac{1}{4\pi n^2} + \mathcal{O}\left(\frac{1}{n^3}\right) \underset{n\to\infty}{=} f(x_0)$$ ✓

Die Taylorentwicklung kann auf wenige Glieder beschränkt werden, weil höhere Ordnungen den Faktor $\frac{1}{n^a}$ mit a = Ordnung haben, die für $n \gg 1$ vernachlässigbar sind.

c) $\delta_n(x) = \frac{1}{\pi} \frac{n}{1+(nx)^2}$



$$\int_{-\infty}^{\infty} dx\, \delta_n(x) = \int_{-\infty}^{\infty} dx\, \frac{1}{\pi} \frac{n}{1+(nx)^2} \quad \Big| \text{Subst. } u = nx, \; du = n\,dx \Leftrightarrow dx = \frac{du}{n}$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} du\, \frac{n}{\cancel{n}(1+u^2)} = \frac{1}{\pi} \Big[\arctan(u) \Big]_{-\infty}^{\infty} = \frac{1}{\pi} \Big[\arctan(nx) \Big]_{-\infty}^{\infty}$$

$$= \frac{1}{\pi} \Big[\lim_{b \to \infty} \arctan(n \cdot b) - \lim_{a \to -\infty} \arctan(n \cdot a) \Big] = \frac{1}{\pi} \Big[\frac{\pi}{2} - \Big(-\frac{\pi}{2}\Big) \Big] = 1$$ ✓

$$\int_{-\infty}^{\infty} \delta_n(x-x_0) f(x)\, dx = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{n}{1+n^2(x-x_0)^2} f(x)\, dx = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{n}{1+n^2(x-x_0^2)} \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!} (x-x_0)^k\, dx$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} \frac{n(x-x_0)^k}{1+n^2(x-x_0)^2} f^{(k)}(x_0) \frac{1}{k!}\, dx \quad , \text{ subst. } y = n(x-x_0) \Rightarrow dx = \frac{1}{n} dy$$

$$\int_{-\infty}^{\infty} \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!} \frac{y^k}{1+y^2} \frac{1}{n^k}\, dy$$

. Die Funktion $T(y) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{n^k} \frac{1}{k!} \frac{y^k}{1+y^2}$ ist nicht i.A. nicht analytisch lösbar, da sie sich nicht als Potenzreihe der Form $\sum_{k=0}^{\infty} a_k y^k$ schreiben ✓ lässt. Wir können also nicht wie in b) vorgehen.

(Integrale divergieren)

d)

$z: \vec{\nabla} \cdot \frac{\vec{r}}{3} = 4\pi \delta(\vec{r})$ mit $\delta(\vec{r}) = \delta(x)\delta(y)\delta(z)$

(i) $4\pi \int_{\mathbb{R}^3} \delta(x)\delta(y)\delta(z)\, f(r)\, dx\,dy\,dz = f(0)$, $\delta(\vec{r})$ liefert also außerhalb $r=0$ keine Beiträge und es gilt

$4\pi\delta(\vec{r}) = 0$ für alle $r \neq 0$ ✓

(ii)

$$\int_V \vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3}\right) dV \overset{\text{Satz v. Gauß}}{=} \oint_{\partial V} \frac{\vec{r}}{r^3}\, \vec{dA} = \oint_{\partial V} \frac{\hat{e}_r}{r^2}\, \hat{e}_r\, dA$$

$$= \oint_{\partial V} \frac{1}{R^2}\, dA = \frac{1}{\cancel{R^2}} 4\pi \cancel{R^2} = 4\pi$$ ✓

(iii) Integriert man $\vec{\nabla}\left(\frac{\vec{r}}{r^3}\right)$ über eine beliebige, also insbesondere auch eine infinitesimale Kugel, die den Ursprung enthält, so erhält man immer den Wert 4π. Integriert man über ein Volumen, das den Ursprung nicht enthält, so erhält man Null. Dieses Verhalten wird durch eine Deltafunktion im Ursprung beschrieben:

$\vec{\nabla} \cdot \frac{\vec{r}}{3} = 4\pi\delta(\vec{r})$ ✓

e) $z: \vec{\nabla} \times \left(\frac{\hat{e}_z \times \vec{\rho}}{\rho^2}\right) = 2\pi\,\delta(x)\,\delta(y)\,\hat{e}_z$, $\vec{\rho} = x\hat{e}_x + y\hat{e}_y$

$$= \vec{\nabla} \times \left(\frac{\hat{e}_z \times (x\hat{e}_x + y\hat{e}_y)}{\rho^2}\right) = \vec{\nabla} \times \left(\frac{x\hat{e}_y - y\hat{e}_x}{\rho^2}\right)$$

Vektorfeld aus 2h): $A_n(\vec{r}) = (x^2+y^2)^{\frac{n}{2}}(x\hat{e}_y - y\hat{e}_x)$

Das Vektorfeld entspricht also $A_{-2}(\vec{r}) = (x^2+y^2)^{-1}(x\hat{e}_y - y\hat{e}_x)$

$$= \frac{(x\hat{e}_y - y\hat{e}_x)}{\rho^2}$$

(i) $$\vec{\nabla} \times \left(\frac{x\hat{e}_y - y\hat{e}_x}{x^2+y^2}\right) = \vec{\nabla} \times \left(\frac{\hat{e}_y \cdot x}{x^2+y^2}\right) - \vec{\nabla} \times \left(\frac{\hat{e}_x\, y}{x^2+y^2}\right)$$

$$= (\partial_x \hat{e}_x + \partial_y \hat{e}_y + \partial_z \hat{e}_z) \times \left(\frac{\hat{e}_y \cdot x}{x^2+y^2}\right) - (\partial_x \hat{e}_x + \partial_y \hat{e}_y + \partial_z \hat{e}_z)\left(\frac{\hat{e}_x\, y}{x^2+y^2}\right)$$

$$= \partial_x \frac{x}{x^2+y^2}\hat{e}_z - \underbrace{\partial_z \frac{x}{x^2+y^2}}_{=0}\hat{e}_x - \left(-\partial_y \frac{y}{x^2+y^2}\hat{e}_z + \underbrace{\partial_z \frac{y}{x^2+y^2}}_{=0}\hat{e}_y\right)$$

$$= \left(\partial_x \frac{x}{x^2+y^2} + \partial_y \frac{y}{x^2+y^2}\right)\hat{e}_z$$

$$= \left(\frac{y^2-x^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2}\right)\hat{e}_z = \vec{0} \quad \text{für } \rho = \sqrt{x^2+y^2} \neq 0$$

(ii)

$$\int_F d\vec{f}\left(\vec{\nabla} \times \left(\frac{x\hat{e}_y - y\hat{e}_x}{x^2+y^2}\right)\right) = \int_{\partial F} d\vec{s}\, \frac{x\hat{e}_y - y\hat{e}_x}{x^2+y^2}$$

$$= \int_{\partial F} ds\, \hat{e}_\varphi \frac{\cos\varphi(\sin\varphi\hat{e}_\rho + \cos\varphi\hat{e}_\varphi) - \sin\varphi(\cos\varphi\hat{e}_\rho - \sin\varphi\hat{e}_\varphi)}{\rho}$$

$$= \int_{\partial F} ds\, \frac{\cos^2\varphi + \sin^2\varphi}{\rho} = \frac{2\pi R}{R} = 2\pi \quad \Big| \text{ mit } \rho = R$$

(iii) Wählt man eine infinitesimale Kreisscheibe um die z-Achse, so erhält man ebenfalls den Wert 2π. Es muss also für jeden Punkt auf der z-Achse ein Delta-Peak existieren (mit Vorfaktor 2π)

$$\Rightarrow \vec{\nabla} \times \left(\frac{\hat{e}_z \times \vec{\rho}}{\rho^2} \right) = 2\pi\, \delta(x)\, \delta(y)\, \hat{e}_z$$

② $\vec{A}(\vec{r}) = \psi \vec{\nabla} \phi$

a) z: $\int_V dV \left[(\vec{\nabla}\psi)(\vec{\nabla}\phi) + \psi \vec{\nabla}^2 \phi \right] = \oint_{\partial V} d\vec{f}\; \psi \vec{\nabla} \phi$

$$= \int_V dV\, \vec{\nabla} \vec{A} = \int_V dV\, \vec{\nabla}(\psi \vec{\nabla} \cdot \phi) \overset{\text{Produkt-regel}}{=} \int_V dV\, (\vec{\nabla} \cdot \psi)(\vec{\nabla}\phi)\, \vec{\nabla}^2 \phi$$

Satz v. Gauß $= \oint_{\partial V} d\vec{f}\, \vec{A} = \oint_{\partial V} d\vec{f}\; \psi \vec{\nabla} \phi$

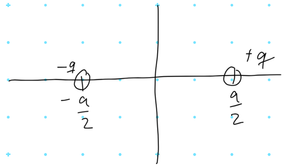
b) $\vec{B}(\vec{r}) = \phi \vec{\nabla} \psi$, z: $\int_V dV \left[\psi \vec{\nabla}^2 \phi - \phi \vec{\nabla}^2 \psi \right] = \oint_{\partial V} d\vec{f} \left[\psi \vec{\nabla} \phi - \phi \vec{\nabla} \psi \right]$

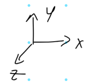
$$\int_V dV \left[\vec{\nabla} (\vec{A} - \vec{B}) \right] = \int_V dV \left[\vec{\nabla}(\psi \vec{\nabla} \phi) - \vec{\nabla}(\phi \vec{\nabla} \psi) \right]$$

Produktregel
$$= \int_V dV \left[\cancel{(\vec{\nabla}\psi)(\vec{\nabla}\phi)} (\psi \vec{\nabla}^2 \phi) - \cancel{(\vec{\nabla}\phi)(\vec{\nabla}\psi)} (\phi \vec{\nabla}^2 \psi) \right] = \int_V dV \left[\psi \vec{\nabla}^2 \phi - \phi \vec{\nabla}^2 \psi \right]$$

Satz v. Gauß
$$= \oint_{\partial V} d\vec{f} \left[\vec{A} - \vec{B} \right] = \oint_{\partial V} d\vec{f} \left[\psi \vec{\nabla} \phi - \phi \vec{\nabla} \psi \right]$$

(3) $\vec{E}(\vec{r}) = \frac{q_i}{4\pi\varepsilon_0} \frac{\vec{r}-\vec{r}_i{}'}{|\vec{r}-\vec{r}_i{}'|^3}$





$$\vec{E}_{ges}(\vec{r}) = \vec{E}_{q+}(\vec{r}) + \vec{E}_{q-}(\vec{r})$$

Ladung q am Ort $\vec{r}_+ = \frac{a}{2}\hat{e}_x \Rightarrow \vec{r}-\vec{r}_+ = (x-\frac{a}{2})\hat{e}_x + y\hat{e}_y + z\hat{e}_z$

Ladung $-q$ am Ort $\vec{r}_- = -\frac{a}{2}\hat{e}_x \Rightarrow \vec{r}-\vec{r}_- = (x+\frac{a}{2})\hat{e}_x + y\hat{e}_y + z\hat{e}_z$

$$\Rightarrow \vec{E}_{ges}(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\left[\frac{(x-\frac{a}{2})\hat{e}_x + y\hat{e}_y + z\hat{e}_z}{\sqrt{(x-\frac{a}{2})^2+y^2+z^2}^{\,3}} - \frac{(x+\frac{a}{2})\hat{e}_x + y\hat{e}_y + z\hat{e}_z}{\sqrt{(x+\frac{a}{2})^2+y^2+z^2}^{\,3}}\right] ✓$$

b) $|\vec{a}| \ll |\vec{r}|$ Hinweis: $|\vec{a}+\vec{b}|^2 = a^2 + 2ab\cos\phi + b^2$ *

$$\frac{1}{|\vec{r}-\vec{r}_\pm|^3} = \frac{1}{|\vec{r} \mp \frac{\vec{a}}{2}\hat{e}_x|^3} = \frac{1}{\sqrt{|\vec{r}\mp\frac{\vec{a}}{2}\hat{e}_x|^2}^{\,3}} \overset{(*)}{=} \frac{1}{(r^2+\frac{a^2}{4}\mp ra\cos\phi)^{\frac{3}{2}}}$$

$$= \frac{1}{r^3\left(\sqrt{1+\frac{1}{4}\frac{a^2}{r^2}\mp\frac{a}{r}\cos\phi}\right)^3}$$

Die Taylorentwicklung ergibt:

$$\frac{1}{r^3\left(\sqrt{1+\frac{1}{4}\frac{a^2}{r^2}\mp\frac{a}{r}\cos\phi}\right)^3} = \frac{1}{r^3} \pm \frac{3\cos\phi}{2}\frac{|a|}{r^4} + O\left(\frac{|\vec{a}|^2}{|\vec{r}|^2}\right) ✓$$

c) z.z.: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{3\vec{r}(\vec{r}\cdot\vec{p})}{|\vec{r}|^5} - \frac{\vec{p}}{|\vec{r}|^3}\right] + O\left(\frac{|\vec{a}|^3}{|\vec{r}|^5}\right)$

mit dem E-Feld aus a): $\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{r}-\frac{a}{2}\hat{e}_x}{\sqrt{(x-\frac{a}{2})^2+y^2+z^2}^{\,3}} - \frac{\vec{r}+\frac{a}{2}\hat{e}_x}{\sqrt{(x+\frac{a}{2})^2+y^2+z^2}^{\,3}}\right]$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r-\frac{\vec{a}}{2}}{|\vec{r}-\vec{r}'|^3} - \frac{\vec{r}+\frac{\vec{a}}{2}}{|\vec{r}+\vec{r}'|^3}\right]$$

b)

$$\approx \frac{q}{4\pi\varepsilon_0}\left[\left(\vec{r}-\frac{\vec{a}}{2}\right)\left(\frac{1}{r^3}+\frac{3}{2}\cos\phi\,\frac{a}{r^4}+O\left(\frac{a^2}{r^2}\right)\right)\right.$$

$$\left.+\left(-\vec{r}-\frac{\vec{a}}{2}\right)\left(\frac{1}{r^3}-\frac{3}{2}\cos\phi\,\frac{a}{r^4}+O\left(\frac{a^2}{r^2}\right)\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{r}}{r^3}+\frac{3}{2}\cos\phi\,\frac{\vec{r}}{r^4}a-\frac{\vec{a}}{2}\cdot\frac{1}{r^3}-\frac{3}{4}\cos\phi\,\frac{\vec{a}}{r^4}a\right.$$

$$\left.-\frac{\vec{r}}{r^3}+\frac{3}{2}\cos\phi\,\frac{\vec{r}}{r^4}a-\frac{\vec{a}}{2}\cdot\frac{1}{r^3}+\frac{3}{4}\cos\phi\,\frac{\vec{a}}{r^4}a\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[a\cos\phi\, 3q\,\frac{\vec{r}}{r^4}-\frac{\vec{a}}{r^3}q\right.$$

$$\boxed{* \;\; \frac{\vec{r}\cdot\vec{a}}{r\cdot a}=\cos\phi}$$

$$\overset{*}{=} \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{r}\,\vec{a}}{r}\,3q\,\frac{\vec{r}}{r^4}-\frac{\vec{a}\,q}{r^3}\right]=\frac{1}{4\pi\varepsilon_0}\left[\frac{3\vec{r}(\vec{r}\,\vec{p})}{r^5}-\frac{\vec{p}}{r^3}\right]+O\left(\frac{a^3}{r^5}\right)$$

✓

schön!

96/100

:)

d)

Im Folgenden wird das E-Feld des Dipols exakt und genähert grafisch betrachtet. Dabei wurde die Ladung q, die Permitivitat ϵ_0 auf 1 und der Vektor $\vec{a}$ auf (1,0,0) gesetzt.

In[306]:=

```
a := {1, 0, 0}
aBetrag := 1
ϵ := 1
q := 1
```

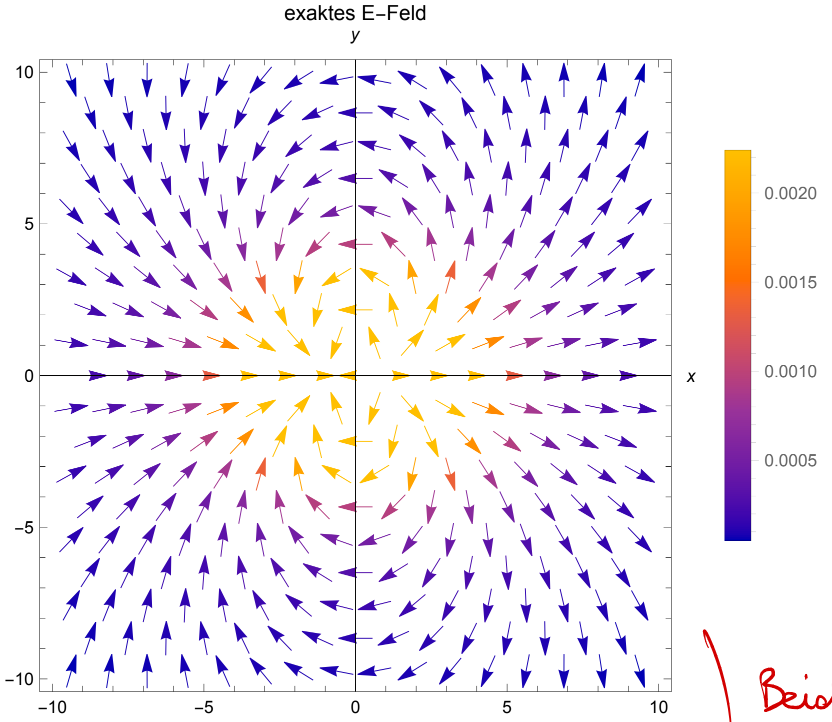
```
VectorPlot[{1 / (4 * Pi * ϵ) * ((x - 0.5 * 1) / (Sqrt[(x - 0.5 * 1) ^ 2 + y ^ 2]) ^ 3 -
      (x + 0.5 * 1) / (Sqrt[(x + 0.5 * 1) ^ 2 + y ^ 2]) ^ 3), 1 / (4 * Pi * ϵ) *
    ((y) / (Sqrt[(x - 0.5 * 1) ^ 2 + y ^ 2]) ^ 3 - (y) / (Sqrt[(x + 0.5 * 1) ^ 2 + y ^ 2]) ^ 3)},
  {x, -10, 10}, {y, -10, 10}, PlotLabel → "exaktes E-Feld",
  PlotLegends → Automatic, Axes → True, AxesLabel → {x, y}]

VectorPlot[{1 / (4 * Pi * ϵ) *
    ((3 * x * x * q) / ((Sqrt[(x - 0.5) ^ 2 + y ^ 2]) ^ 5) - 1 / ((Sqrt[(x - 0.5) ^ 2 + y ^ 2]) ^ 3)),
   1 / (4 * Pi * ϵ) * ((3 * y * x * q) / ((Sqrt[(x - 0.5) ^ 2 + y ^ 2]) ^ 5))},
  {x, -10, 10}, {y, -10, 10}, PlotLabel → "approximiertes E-Feld",
  PlotLegends → Automatic, Axes → True, AxesLabel → {x, y}]
```

```
DensityPlot[
  (((1 / (4 * Pi * ϵ) * ((x - 0.5 * 1) / (Sqrt[(x - 0.5 * 1) ^ 2 + y ^ 2]) ^ 3 - (x + 0.5 * 1) /
             (Sqrt[(x + 0.5 * 1) ^ 2 + y ^ 2]) ^ 3)) -
         (1 / (4 * Pi * ϵ) * ((3 * x * x * q) / ((Sqrt[(x - 0.5) ^ 2 + y ^ 2]) ^ 5) -
             1 / ((Sqrt[(x - 0.5) ^ 2 + y ^ 2]) ^ 3)))) ^ 2 + ((1 / (4 * Pi * ϵ) *
           ((y) / (Sqrt[(x - 0.5 * 1) ^ 2 + y ^ 2]) ^ 3 - (y) / (Sqrt[(x + 0.5 * 1) ^ 2 + y ^ 2]) ^ 3)) -
         (1 / (4 * Pi * ϵ) * ((3 * y * x * q) / ((Sqrt[(x - 0.5) ^ 2 + y ^ 2]) ^ 5)))) ^ 2) ^
   0.5, {x, -10, 10}, {y, -10, 10}, PlotLabel →
   "absoluter Fehler",
  FrameLabel →
   {x, y},
  PlotLegends →
   BarLegend[Automatic,
    LegendLabel → "|E_approx-E_exact|"],
  ColorFunction → "Rainbow",
  PlotRange →
```

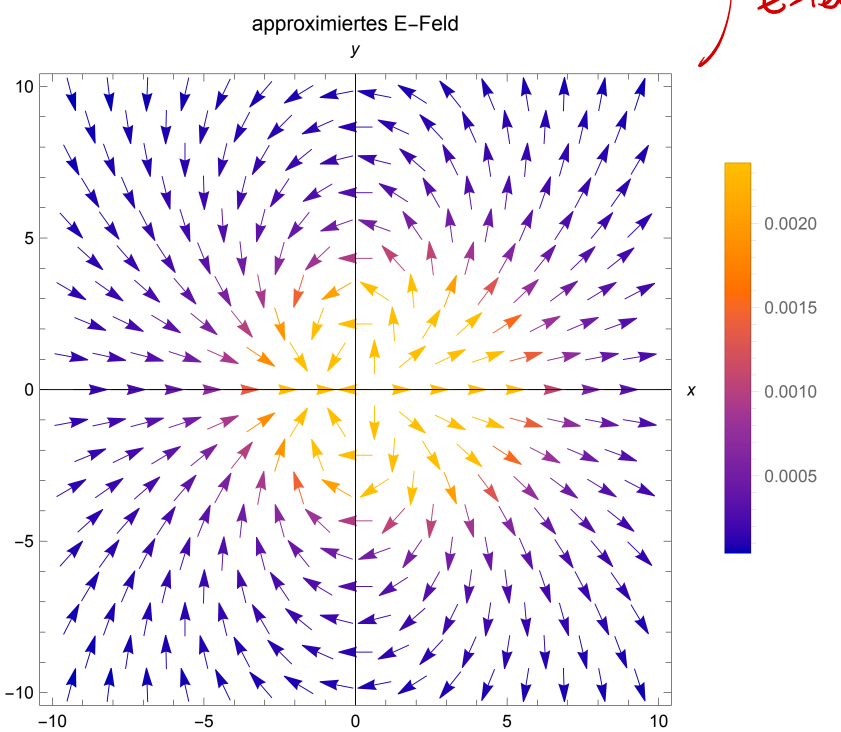
```
  {0, 0.01}]

DensityPlot[
 (((1 / (4 * Pi * ϵ) * ((x - 0.5 * 1) / (√((x - 0.5 * 1) ^2 + y^2)) ^3 - (x + 0.5 * 1) /
            (√((x + 0.5 * 1) ^2 + y^2)) ^3)) -
        (1 / (4 * Pi * ϵ) * ((3 * x * x * q) / ((√((x - 0.5) ^2 + y^2)) ^5) -
            1/ ((√((x - 0.5) ^2 + y^2)) ^3)))) ^2 + ((1 / (4 * Pi * ϵ) *
          ((y) / (√((x - 0.5 * 1) ^2 + y^2)) ^3 - (y) / (√((x + 0.5 * 1) ^2 + y^2)) ^3)) -
        (1 / (4 * Pi * ϵ) * ((3 * y * x * q) / ((√((x - 0.5) ^2 + y^2)) ^5) ))) ^2) ^0.5/
   ((1 / (4 * Pi * ϵ) * ((x - 0.5 * 1) / (√((x - 0.5 * 1) ^2 + y^2)) ^3 -
          (x + 0.5 * 1) / (√((x + 0.5 * 1) ^2 + y^2)) ^3)) ^2 +
     (1 / (4 * Pi * ϵ) * ((y) / (√((x - 0.5 * 1) ^2 + y^2)) ^3 -
          (y) / (√((x + 0.5 * 1) ^2 + y^2)) ^3)) ^2) ^0.5
 , {x, -10, 10}, {y, -10, 10}, PlotLabel → "relativer Fehler", FrameLabel → {x, y},
 PlotLegends → BarLegend[Automatic, LegendLabel → "| E_approx - E_exact | / | E_exact |"],
 ColorFunction → "Rainbow", PlotRange → {0, 1}]

Plot[(((1 / (4 * Pi * ϵ) * ((t - 0.5 * 1) / (√((t - 0.5 * 1) ^2 + 0^2)) ^3 -
          (t + 0.5 * 1) / (√((t + 0.5 * 1) ^2 + 0^2)) ^3)) -
      (1 / (4 * Pi * ϵ) * ((3 * t * t * q) / ((√((t - 0.5) ^2 + 0^2)) ^5) -
          1/ ((√((t - 0.5) ^2 + 0^2)) ^3)))) ^2) ^0.5,
 {t, 0.1, 10}, PlotLabel → "Absoluter Fehler für r=(t,0,0)",
 PlotRange →
  {0, 100},
 AxesLabel → {"|r|", "|E_approx-E_exact|"}]

Plot[(((1 / (4 * Pi * ϵ) * ((t - 0.5 * 0) / (√((0 - 0.5 * 1) ^2 + t^2)) ^3 -
          (t + 0.5 * 1) / (√((0 + 0.5 * 1) ^2 + t^2)) ^3)) -
      (1 / (4 * Pi * ϵ) * ((3 * t * 0 * q) / ((√((0 - 0.5) ^2 + t^2)) ^5) -
          1/ ((√((0 - 0.5) ^2 + t^2)) ^3)))) ^2) ^0.5,
 {t, 0.1, 10}, PlotLabel → "Absoluter Fehler für r=(0,t,0)",
 PlotRange →
  {0, 1},
```

```
AxesLabel → {"|r|", "|E_approx-E_exact|"}]
```

Out[310]=



Out[311]=



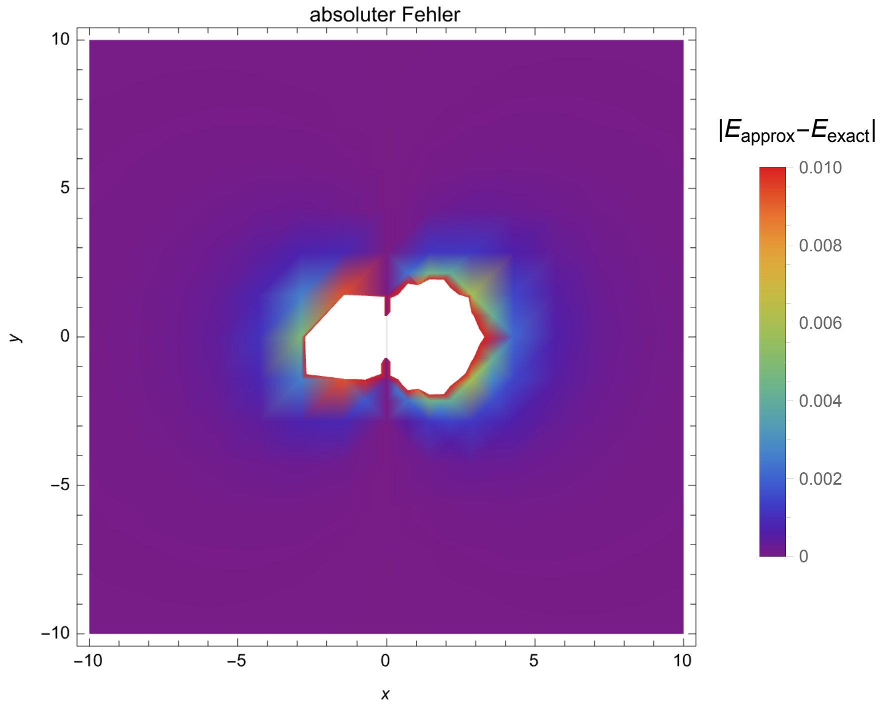
Beides
E-Feld der Näherung

haha keine Ahnung
warum das so cursed
aussieht

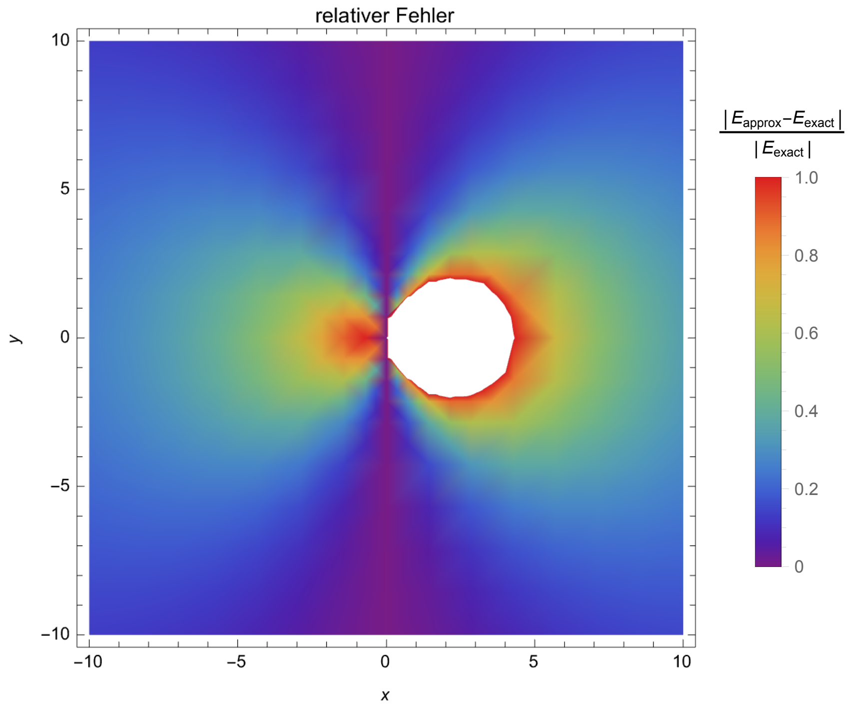
Sollte aber beides
achsensymmetrisch sein

Unten mal die
Musterplots + Code

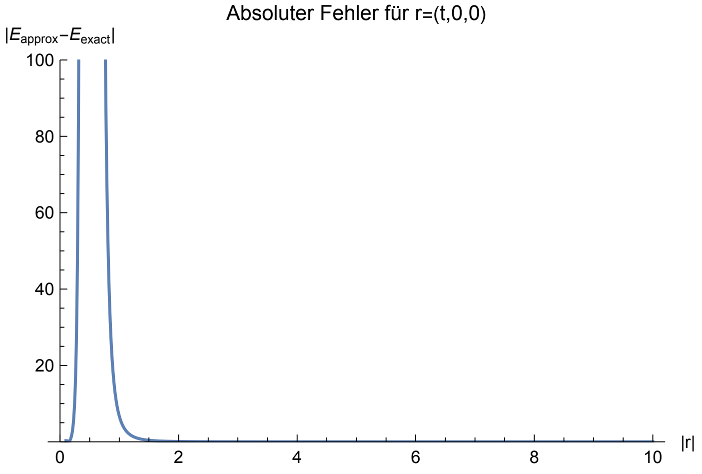
Out[312]=



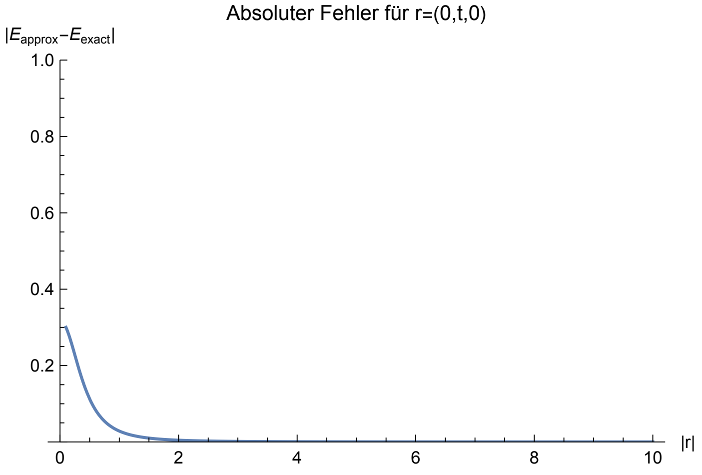
Out[313]=



Out[314]=



Out[315]=



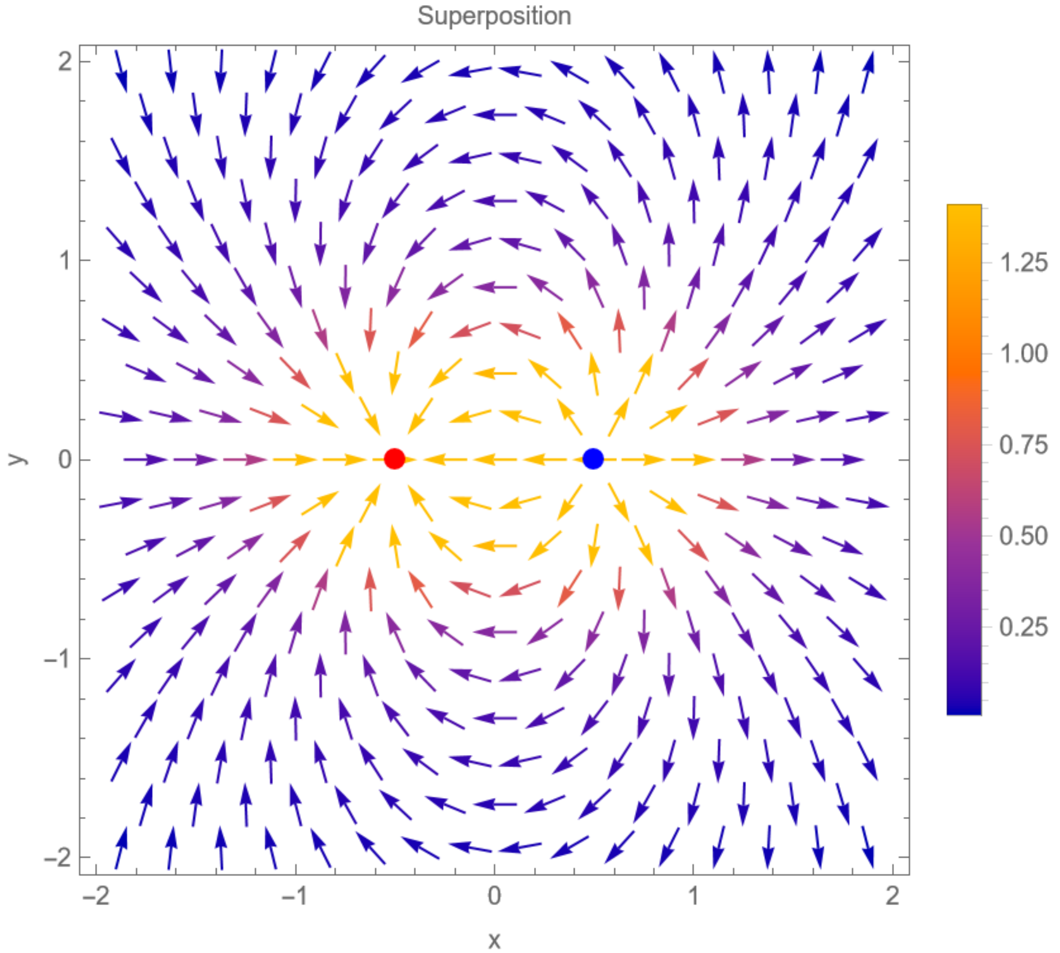
Den Plots ist zu entnehmen, dass der Fehler durch die Näherung des E-Felds für große $|\vec{r}|$ gut vernachlässigbar ist. Außerdem ist der Fehler in der Nähe der y-Achse geringer als der Fehler in Nähe zur x-Achse.

6/10

E-Feld aus Superposition

In[∘]:=
```
r = {x, y};
a = {1, 0};
Show[
 VectorPlot[1/(4 π) ((r - a/2)/Norm[r - a/2]^3 - (r + a/2)/Norm[r + a/2]^3), {x, -2, 2},
  {y, -2, 2}, FrameLabel → {{"y", None}, {"x", "Superposition"}},
  PlotLegends → Automatic],
 Graphics[{Red, Disk[-a/2, 0.05], Blue, Disk[a/2, 0.05]}]
]
```

Out[∘]=

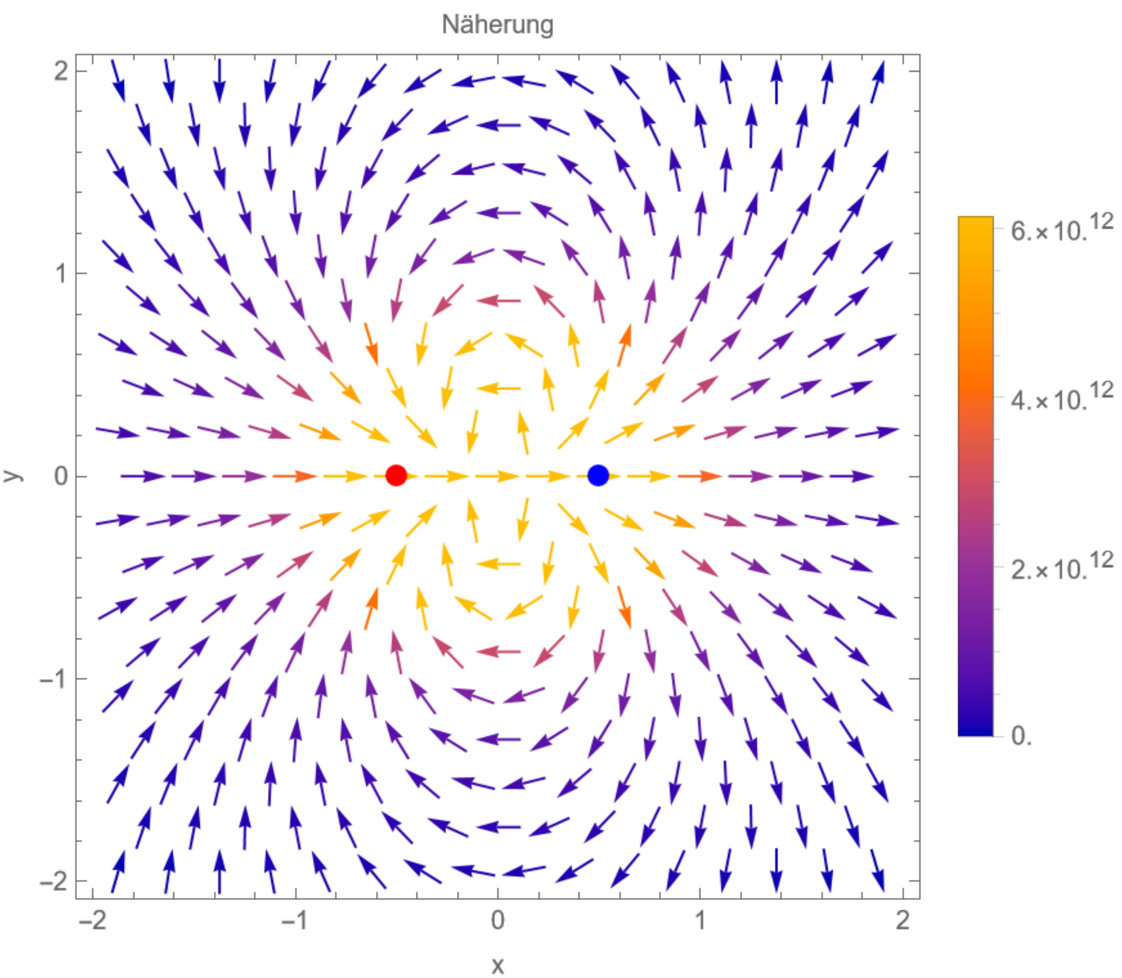


2

E-Feld aus Näherung

In[•]:=
```
r = {x, y};
a = {1, 0};
Show[
 VectorPlot[1/(4 π) (3 r (r.a)/Norm[r]^5 - a/Norm[r]^3), {x, -2, 2},
  {y, -2, 2}, FrameLabel → {{"y", None}, {"x", "Näherung"}},
  PlotLegends → Automatic],
 Graphics[{Red, Disk[-a/2, 0.05], Blue, Disk[a/2, 0.05]}]
]
```

Out[•]=

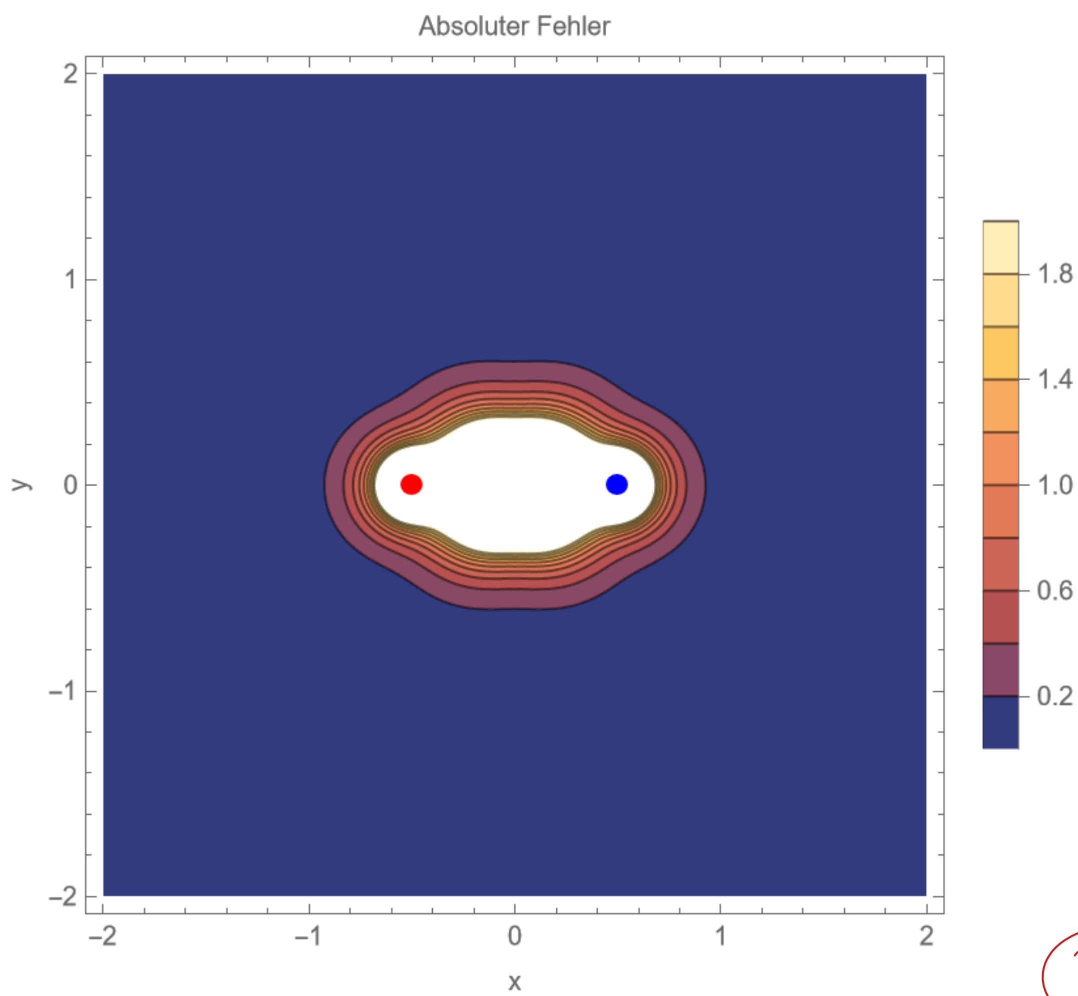


②

Absoluter Fehler

In[•]:=
```
r = {x, y};
a = {1, 0};
dipolExact = 1/(4 π) ((r - a/2)/Norm[r - a/2]^3 - (r + a/2)/Norm[r + a/2]^3);
dipolApprox = 1/(4 π) (3 r (r.a)/Norm[r]^5 - a/Norm[r]^3);
Show[
 ContourPlot[Norm[dipolApprox - dipolExact], {x, -2, 2},
  {y, -2, 2},
  FrameLabel → {{"y", None}, {"x", "Absoluter Fehler"}},
  PlotLegends → Automatic, PlotRange → {All, All, {0, 2}},
  PlotPoints → 50],
 Graphics[{Red, Disk[-a/2, 0.05], Blue, Disk[a/2, 0.05]}]
]
```

Out[•]=

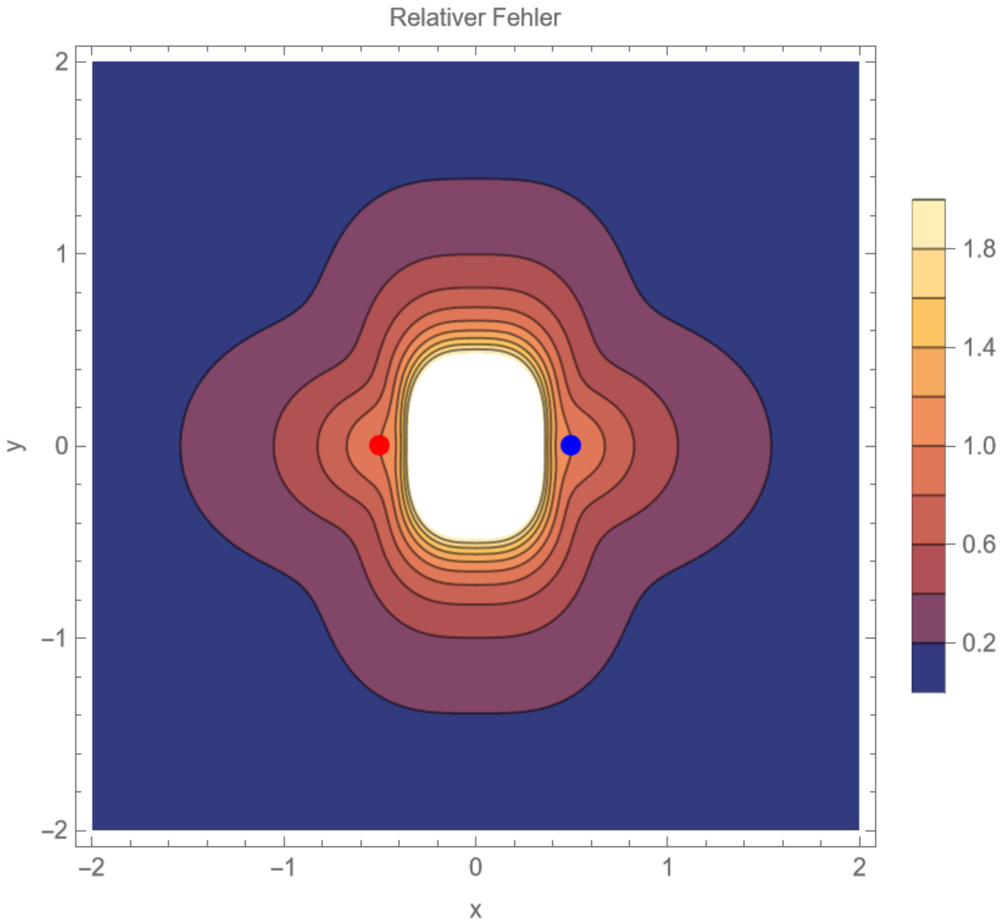


2

Relativer Fehler

```
In[•]:= r = {x, y};
a = {1, 0};
dipolExact = 1/(4 π) ((r - a/2)/Norm[r - a/2]^3 - (r + a/2)/Norm[r + a/2]^3);
dipolApprox = 1/(4 π) (3 r (r.a)/Norm[r]^5 - a/Norm[r]^3);
Show[
 ContourPlot[Norm[dipolApprox - dipolExact]/Norm[dipolExact], {x, -2, 2},
  {y, -2, 2},
  FrameLabel → {{"y", None}, {"x", "Relativer Fehler"}},
  PlotLegends → Automatic, PlotRange → {All, All, {0, 2}},
  PlotPoints → 50],
 Graphics[{Red, Disk[-a/2, 0.05], Blue, Disk[a/2, 0.05]}]
]
```

Out[•]=

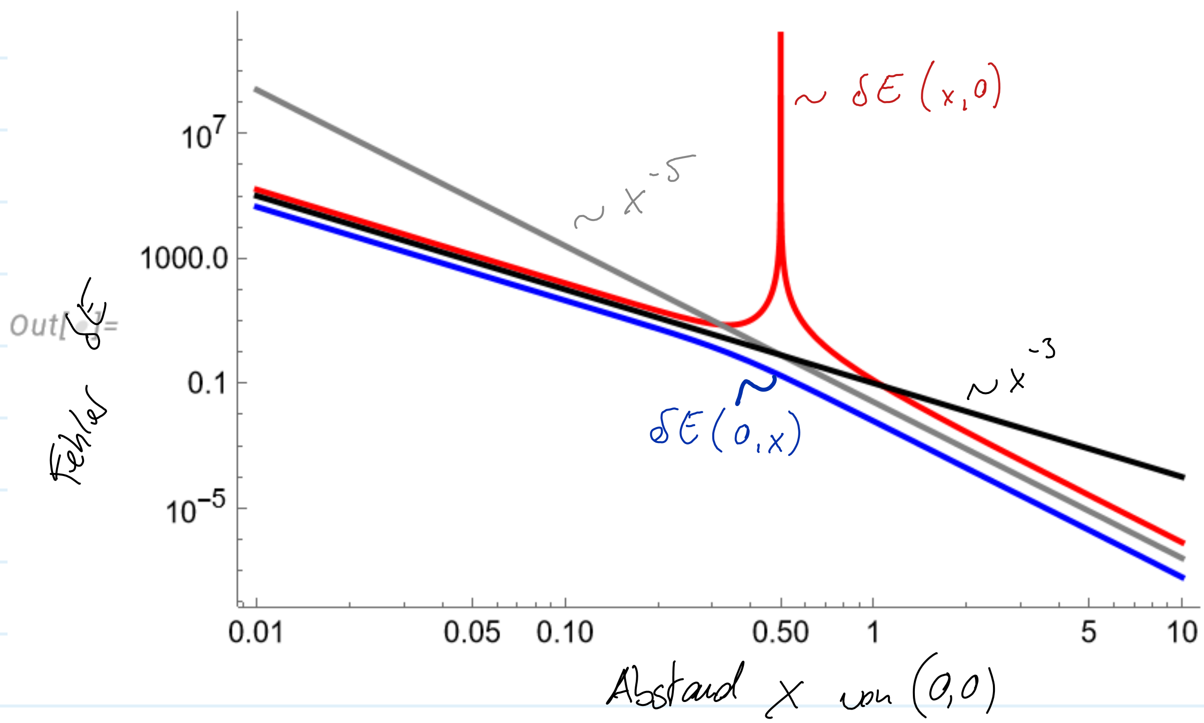


2

Konvergenz

In[•]:=
```
r = {x, y};
a = {1, 0};
dipolExact = 1/(4 π) ((r - a/2)/Norm[r - a/2]^3 - (r + a/2)/Norm[r + a/2]^3);
dipolApprox = 1/(4 π) (3 r (r.a)/Norm[r]^5 - a/Norm[r]^3);
LogLogPlot[{
   Norm[dipolApprox - dipolExact] /. y → 0,
   Norm[dipolApprox - dipolExact] /. x → 0 /. y → x,
   0.025 x^-5,
   0.1 x^-3
  }, {x, 0.01, 10}, PlotStyle → {Red, Blue, Gray, Black}]
```

Out[•]=



$\Rightarrow$ Für x bzw $y \to 0$ Konvergenz mit r^{-3}.

Für x bzw $y \to \infty$ Konvergenz mit r^{-5}.

2