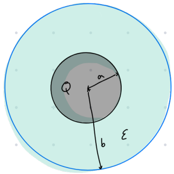
uctno / Leandra Prochnow

① a)



$\boxed{\vec{\nabla}\cdot\vec{D} = \rho = \delta(\rho - a)\, Q}$ Dimensionsfehler!

$\Longleftrightarrow \boxed{\int_V \vec{\nabla}\cdot\vec{D}\, dV = \oint_{\partial V} \vec{D}\, d\vec{f} = \int_V \rho\, dV = Q}, \quad r > a$

Aufgrund der Kugelsymmetrie gilt $\vec{D} \parallel d\vec{f} = df\, \hat{e}_r$

Das stimmt hingegen.

$$= \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta\, D\, d\theta\, d\varphi = 2\pi r^2 D \left[-\cos\theta\right]_0^{\pi} = 4\pi r^2 D$$

$$\Rightarrow \vec{D}(r) = \begin{cases} 0 & , \ r \le a \ \text{(Metallische Kugel)} \\ \dfrac{Q}{4\pi r^2}\hat{e}_r & , \ r > a \end{cases}$$

+10

b)

$$\vec{E}(r) = \frac{\vec{D}(r)}{\varepsilon(r)} = \begin{cases} 0 & , \ r \le a \\ \dfrac{Q}{4\pi\varepsilon r^2}\hat{e}_r & , \ a < r \le b \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{e}_r & , \ r > b \end{cases} \qquad \Bigg| \qquad \varepsilon(r) = \begin{cases} \varepsilon_0 , & r \le a \\ \varepsilon , & a < r \le b \\ \varepsilon_0 , & r > b \end{cases}$$

+4

$$\phi(r) = -\int_\infty^r \vec{E}(r)\, d\vec{l} = \begin{cases} -\displaystyle\int_\infty^b \frac{Q}{4\pi\varepsilon_0 r^2} dr - \int_b^a \frac{Q}{4\pi\varepsilon r^2} dr - \int_a^r 0 = \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_0 b} + \frac{1}{a\varepsilon} - \frac{1}{b\varepsilon}\right) & , \ r \le a \\ -\displaystyle\int_\infty^b \frac{Q}{4\pi\varepsilon_0 r^2} dr - \int_b^r \frac{Q}{4\pi\varepsilon r^2} dr = \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_0 b} + \frac{1}{r\varepsilon} - \frac{1}{b\varepsilon}\right) & , \ a < r \le b \\ -\displaystyle\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2} dr = \lim_{h\to\infty}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]_h^r = \frac{Q}{4\pi\varepsilon_0 r} & , \ r > b \end{cases}$$

+2 +2 +2

mit $\phi(\infty) = 0$

c) im Anhang +5

d) $\vec{P} = \varepsilon_0 \chi_e \vec{E}$ mit $\varepsilon = \varepsilon_0(1 + \chi_e) \Longleftrightarrow \chi_e = \frac{\varepsilon}{\varepsilon_0} - 1$

Innere Grenzfläche:

$$\sigma_a = \hat{n}\vec{P} = (-\hat{e}_r)\vec{P} = -\varepsilon_0 \chi_e \frac{Q}{4\pi\varepsilon a^2}$$

$$= -\varepsilon_0\left(\frac{\varepsilon}{\varepsilon_0} - 1\right)\frac{Q}{4\pi\varepsilon a^2} = -\frac{Q}{4\pi\cancel{\varepsilon} a^2} + \frac{Q\varepsilon_0}{4\pi\varepsilon a^2}$$

Äußere Grenzfläche:

$$\sigma_b = \hat{n}\vec{P} = \vec{P}\hat{e}_r = \varepsilon_0 \chi_e \frac{Q}{4\pi\varepsilon b^2}$$

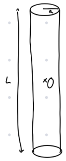
$$= -\frac{Q}{4\pi\cancel{\varepsilon} b^2} + \frac{Q\varepsilon_0}{4\pi\varepsilon b^2}$$

Bezogen auf die dielektrische Hohlkugel ist $\hat{n}$ an der Innenseite $(-\hat{e}_r)$ und an der Außenseite $\hat{e}_r$.
Somit ergeben sich Polarisationsladungsdichten durch die das elektrische Feld der Metallkugel im Dielektrikum abschwächt wird.

+8−2 RF

33 P / 35 P

2.



$$\vec M(\vec r) = M\,\Theta\left(\left(\tfrac{L}{2}\right)^2 - z^2\right)\Theta(R-\rho)\,\hat e_z$$

a) $$\vec j_M(\vec r) = \vec\nabla\times\vec M(\vec r) = \vec\nabla\times\left(M\,\Theta\left(\left(\tfrac{L}{2}\right)^2 - z^2\right)\Theta(R-\rho)\,\hat e_z\right)$$

$$= \left(\frac{1}{\rho}\partial_\varphi(\vec M\cdot\hat e_z) - \partial_z(\cancel{\vec M\hat e_\varphi})\right)\hat e_\rho + \left(\partial_z(\cancel{\vec M\hat e_\rho}) - \partial_\rho(\vec M\hat e_z)\right)\hat e_\varphi + \frac{1}{\rho}\left(\partial_\rho(\rho(\cancel{\vec M\hat e_\varphi})) - \partial_\varphi(\cancel{\vec M\hat e_\rho})\right)\hat e_z$$

$$= -\partial_\rho\left(M\,\Theta\left(\left(\tfrac{L}{2}\right)^2 - z^2\right)\Theta(R-\rho)\right)\hat e_\varphi = + M\underbrace{\delta(R-\rho)}_{\text{nur auf Mantel-oberfläche}}\Theta\left(\left(\tfrac{L}{2}\right)^2 - z^2\right)\hat e_\varphi \quad \text{mit } \frac{d\,\Theta(x)}{dx} = \delta(x)$$

+4−1 Ableitungsfehler

b) $$\vec B(\vec r) = \frac{\mu_0}{4\pi}\int_V dV'\left(\vec j_M(\vec r\,')\times\frac{\vec r - \vec r\,'}{|\vec r - \vec r\,'|^3}\right)$$

$$\vec B(z\hat e_z) = -\frac{\mu_0}{4\pi}\int_V dV'\left(M\,\delta(R-\rho')\,\Theta\left(\left(\tfrac{L}{2}\right)^2 - z^2\right)\begin{pmatrix}-\sin\varphi\\ \cos\varphi\\ 0\end{pmatrix}\times\begin{pmatrix}-\rho'\cos\varphi\\ -\rho'\sin\varphi\\ z-z'\end{pmatrix}\cdot\frac{1}{\sqrt{\rho'^2+(z-z')^2}^{\,3}}\right)$$

$\begin{smallmatrix}2&3&-&3&2\\3&1&-&1&3\\1&2&-&2&1\end{smallmatrix}$

$$= -\frac{\mu_0}{4\pi}\int_V dV'\left(\frac{M\,\delta(R-\rho')\,\Theta\left(\left(\tfrac{L}{2}\right)^2 - z^2\right)}{\sqrt{\rho'^2+(z-z')^2}^{\,3}}\begin{pmatrix}\cos\varphi'(z-z')\\ \sin\varphi'(z-z')\\ \underbrace{\sin^2\varphi'\rho' + \cos^2\varphi'\rho'}_{=\rho'}\end{pmatrix}\right)$$

$$= -\frac{\mu_0}{4\pi}\int_{-\infty}^{z}\int_0^{2\pi}\int_0^{r}\frac{M\,\delta(R-\rho')\,\Theta\left(\left(\tfrac{L}{2}\right)^2 - z'^2\right)}{\sqrt{\rho'^2+(z-z')^2}^{\,3}}\rho'\begin{pmatrix}\cos\varphi'(z-z')\\ \sin\varphi'(z-z')\\ \rho'\end{pmatrix}d\rho'\,d\varphi'\,dz'$$

$$= -\frac{\mu_0}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}dz'\frac{M}{\sqrt{R^2+(z-z')^2}^{\,3}}R^2\left[\begin{pmatrix}\sin\varphi'\,(z-z')\\ -\cos\varphi'\,(z-z')\\ \varphi'\end{pmatrix}\right]_0^{2\pi}, \quad \left(\tfrac{L}{2}\right)^2 \geq z'^2$$

$$= -\frac{\mu_0}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}dz'\frac{M}{\sqrt{R^2+(z-z')^2}^{\,3}}R^2\begin{pmatrix}0\\ -(z-z')+(z-z')\\ 2\pi\end{pmatrix}$$

$$= -\frac{\mu_0}{2}\cancel{R^2}M\,\hat e_z\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dz'}{\cancel{R^3}\sqrt{1+\left(\frac{z-z'}{R}\right)^2}^{\,3}} \qquad \text{subst } \frac{z-z'}{R} = u \iff dz' = -du\,R$$

$$= \frac{\mu_0}{2}M\,\hat e_z\int_{\frac{z+\frac{L}{2}}{R}}^{\frac{z-\frac{L}{2}}{R}}du\,\frac{1}{\sqrt{u^2+1}^{\,3}} = \frac{\mu_0 M}{R\,2}\hat e_z\left[\frac{u}{\sqrt{u^2+1}}\right]_{\frac{z+\frac{L}{2}}{R}}^{\frac{z-\frac{L}{2}}{R}}$$

$$= \frac{\mu_0 M}{2}\hat e_z\left[\frac{z-\frac{L}{2}}{R\sqrt{\left(\frac{z-\frac{L}{2}}{R}\right)^2+1}} - \frac{z+\frac{L}{2}}{R\sqrt{\left(\frac{z+\frac{L}{2}}{R}\right)^2+1}}\right]$$

$$= \frac{\mu_0 M}{2}\hat e_z\left[\frac{z-\frac{L}{2}}{\sqrt{\left(z-\frac{L}{2}\right)^2+R^2}} - \frac{z+\frac{L}{2}}{\sqrt{\left(z+\frac{L}{2}\right)^2+R^2}}\right]$$

FF +12

Im Inneren des Magneten ist $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$:

$$\vec{H}(z\hat{e}_z) = \frac{\vec{B}(z\hat{e}_z)}{\mu_0} - \vec{M}(z\hat{e}_z) = \frac{\mu_0 M}{2}\hat{e}_z\left[\frac{z-\frac{L}{2}}{\sqrt{(z-\frac{L}{2})^2+R^2}} - \frac{z+\frac{L}{2}}{\sqrt{(z+\frac{L}{2})^2+R^2}}\right] - M\hat{e}_z$$

Für $|z| \leq \frac{L}{2}$ (im Inneren des Magneten) zeigt $\vec{H}$ in $-\hat{e}_z$ Richtung, also $\vec{B}$ entgegengerichtet.

Außerhalb des Magneten ist $\vec{M}=0$ und somit $\vec{H} = \frac{\vec{B}}{\mu_0}$,

also: $$\vec{H}(z\hat{e}_z) = \begin{cases} \frac{M}{2}\left[\frac{z-\frac{L}{2}}{\sqrt{(z-\frac{L}{2})^2+R^2}} - \frac{z+\frac{L}{2}}{\sqrt{(z+\frac{L}{2})^2+R^2}}\right]\hat{e}_z, & \text{außerhalb } (\rho>R,\ |z|>\frac{L}{2}) \\ M\left[\frac{z-\frac{L}{2}}{2\sqrt{(z-\frac{L}{2})^2+R^2}} - \frac{z+\frac{L}{2}}{2\sqrt{(z+\frac{L}{2})^2+R^2}} - 1\right]\hat{e}_z, & \text{innerhalb } (\rho\leq R,\ |z|\leq\frac{L}{2}) \end{cases}$$

FF +3

c) $$\vec{B}(z\hat{e}_z) = \frac{\mu_0 M}{2}\left[\frac{z/L - \frac{1}{2}}{\sqrt{(z/L-\frac{1}{2})^2 + R^2/L^2}} - \frac{z/L + \frac{1}{2}}{\sqrt{(z/L+\frac{1}{2})^2 + R^2/L^2}}\right]\hat{e}_z$$ Plots im Anhang!

−5 Plots entsprechen nicht dem Ergebnis der b)

d) $$\vec{B}^{(1)}(\vec{r}) = \frac{\mu_0}{4\pi}\left.\frac{3(\vec{m}\cdot\vec{r})\vec{r} - \vec{m}r^2}{r^5}\right|_{|\vec{r}|>0} + \frac{2}{3}\mu_0\vec{m}\,\delta(\vec{r})$$

$$\vec{B}^{(1)}(z\hat{e}_z) = \frac{\mu_0}{4\pi}\left.\frac{3(\vec{m}\cdot z\hat{e}_z)z\hat{e}_z - \vec{m}z^2}{z^5}\right|_{z>0} + \frac{2}{3}\mu_0\vec{m}\,\delta(z\hat{e}_z)$$

mit $\vec{m} = \frac{1}{2}\int d\vec{r}\ \vec{r}\times\vec{j}(\vec{r})$

$$= -\frac{1}{2}\int_{-\infty}^{\infty}\int_0^{2\pi}\int_0^{r} d\rho\, d\varphi\, dz\ \rho\begin{pmatrix}\rho\cos\varphi\\ \rho\sin\varphi\\ z\end{pmatrix}\times\begin{pmatrix}-\sin\varphi\\ \cos\varphi\\ 0\end{pmatrix} M\delta(R-\rho)\,\Theta\left((\tfrac{L}{2})^2 - z^2\right)$$

$$= -\frac{MR}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\begin{pmatrix}z\cos\varphi\\ -z\sin\varphi\\ R\varphi\end{pmatrix}\right]_0^{2\pi} = -\frac{MR}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\begin{pmatrix}0\\0\\R2\pi\end{pmatrix} = -M\pi R^2 L\,\hat{e}_z$$

FF +2

$$\Rightarrow \vec{B}(z\hat{e}_z) = \frac{\mu_0}{4\pi}\frac{3M\pi R^2 L z^2\hat{e}_z - M\pi R^2 L z^2\hat{e}_z}{z^5} - \frac{2}{3}\mu_0\pi M R^2 L\hat{e}_z\,\delta(z)$$

$$= \frac{\mu_0}{2}\left.\frac{MR^2L}{z^3}\hat{e}_z\right|_{z\neq 0} + \frac{2}{3}\mu_0\pi M R^2 L\hat{e}_z\,\delta(z)$$

VZF des Dipolmoments verschwindet unerklärt

$$\Rightarrow \vec{H}(z\hat{e}_z) = M\hat{e}_z\begin{cases}\left(\frac{R^2L}{2z^3}\right), & \rho>R,\ |z|>\frac{L}{2} \\ \left(\left.\frac{R^2L}{2z^3}\right|_{z\neq 0} + \frac{2}{3}\pi R^2 L\,\delta(z)\,\boxed{-1}\right), & \rho\leq R,\ |z|\leq\frac{L}{2}\end{cases}$$

mit Dipolnäherung inkompatibel −1

Plots im Anhang! −1

FF +1

Plots quantitativ richtig, qualitativ falsch +2

e) $\vec{B}(x\hat{e}_x) = \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3} \vec{j}(\vec{r}\,') \times \frac{x\hat{e}_x - \vec{r}\,'}{|x\hat{e}_x - \vec{r}\,'|^3} d^3r' = \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{2\pi} \frac{-M\Theta((\frac{L}{2})^2 - z^2)\delta(R-\rho)}{|x\hat{e}_x - z'\hat{e}_{z'} - \rho\hat{e}_{\rho'}|^3} \hat{e}_{\varphi'} \times (x\hat{e}_x - z'\hat{e}_{z'} - \rho'\hat{e}_{\rho'})\rho' \, d\varphi' d\rho' dz'$

$$= \frac{\mu_0}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi} MR \frac{x\cos\varphi'\hat{e}_z + z'\hat{e}_{\rho'} - R\hat{e}_z}{\sqrt{z'^2 + x^2 + R^2 - 2Rx\cos\varphi'}} d\varphi' dz'$$

$$\Rightarrow \vec{H}(x\hat{e}_x) = \frac{1}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi} MR \frac{x\cos\varphi'\hat{e}_z + z'\hat{e}_{\rho'} - R\hat{e}_z}{\sqrt{z'^2 + x^2 + R^2 - 2Rx\cos\varphi'}} d\varphi' dz' - M\Theta((\tfrac{L}{2})^2 - z^2)\Theta(R-\rho)\hat{e}_z$$

Randbedingung: $(\vec{H}_2 - \vec{H}_1)\hat{t} = \hat{t}(\vec{k} \times \hat{n}) \Leftrightarrow (\vec{H}_2(R\hat{e}_x) - \vec{H}_1(R\hat{e}_x))\hat{e}_z = \hat{e}_z(\vec{k} \times \hat{e}_x)$

mit der Flächenstromdichte $\vec{k}$

$$\frac{1}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi} MR \left.\frac{x\cos\varphi'\hat{e}_z + z'\hat{e}_{\rho'} - R\hat{e}_z}{\sqrt{z'^2 + x^2 + R^2 - 2Rx\cos\varphi'}}\right|_{x=R^+} d\varphi' dz' - M\Theta((\tfrac{L}{2})^2 - 0)\Theta(R-R)\hat{e}_z$$

$$- \frac{1}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi} MR \left.\frac{x\cos\varphi'\hat{e}_z + z'\hat{e}_{\rho'} - R\hat{e}_z}{\sqrt{z'^2 + x^2 + R^2 - 2Rx\cos\varphi'}}\right|_{x=R^-} d\varphi' dz' = M = \hat{e}_z(M\hat{e}_\varphi|_{\varphi=0} \times \hat{e}_x) = M\cos\varphi|_{\varphi=0} = M$$

Die Randbedingung für $\vec{H}$ ist erfüllt. +6

Stetigkeitsbedingung an **B** fehlt.
Rechnung unvollständig

30 P / 30 P

(3)

$$D_i(\vec{k},\omega) = \varepsilon_{ij}(\vec{k},\omega)\, E_j(\vec{k},\omega)$$

$$B_i(\vec{k},\omega) = \mu_{ij}(\vec{k},\omega)\, H_j(\vec{k},\omega)$$

$$\varepsilon_{ij}(\vec{k},\omega) = \varepsilon\,\delta_{ij} \qquad \mu(\vec{k},\omega) = \begin{pmatrix} \mu_1 & -i\mu_2 & 0 \\ i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_z \end{pmatrix}$$

a)
$$\boxed{\begin{array}{ll} 1.\ \vec{\nabla}\vec{D} = \rho = 0 & 3.\ \vec{\nabla}\times\vec{H} - \partial_t\vec{D} = \vec{j} = 0 \\ 2.\ \vec{\nabla}\vec{B} = 0 & 4.\ \vec{\nabla}\times\vec{E} + \partial_t\vec{B} = 0 \end{array}}$$

(1) $\vec{\nabla}\vec{D}(r,t) \to \vec{\nabla}\int \frac{1}{2\pi}\vec{D}(\vec{r},\omega)e^{-i\omega t}\,d\omega \longrightarrow \frac{1}{(2\pi)^4}\vec{\nabla}\iint \vec{D}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} = \frac{1}{(2\pi)^4}\iint \vec{D}(\vec{k},\omega)\, i\vec{k}\, e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} = 0$

(2) $\vec{\nabla}\vec{B}(\vec{r},t) \to \vec{\nabla}\int\frac{1}{2\pi}\vec{B}(\vec{r},\omega)e^{-i\omega t}\,d\omega \longrightarrow \vec{\nabla}\iint\frac{1}{(2\pi)^4}\vec{B}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} = \frac{1}{(2\pi)^4}\iint\vec{B}(\vec{k},\omega)\, i\vec{k}\, e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} = 0$

(3) $\vec{\nabla}\times\vec{H}(\vec{r},t) - \partial_t\vec{D}(\vec{r},t) \longrightarrow \vec{\nabla}\times\iint\frac{1}{(2\pi)^4}\vec{H}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} - \partial_t\iint\frac{1}{(2\pi)^4}\vec{D}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k}$

$= i\vec{k}\times\int\frac{1}{(2\pi)^4}\vec{H}(\vec{k},\vec{\omega})e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} + i\omega\iint\frac{1}{(2\pi)^4}\vec{D}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} = 0$

(4) $\vec{\nabla}\times\vec{E}(\vec{r},t) + \partial_t\vec{B}(\vec{r},t) = 0 \longrightarrow \vec{\nabla}\times\iint\frac{1}{(2\pi)^4}\vec{E}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} + \partial_t\iint\frac{1}{(2\pi)^4}\vec{B}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k}$

$= i\vec{k}\times\iint\frac{1}{(2\pi)^4}\vec{E}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} - i\omega\iint\frac{1}{(2\pi)^4}\vec{B}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} = 0$ +4

Rechnung unvollständig −1

b) Gauß'sche Gesetze:

$\frac{1}{(2\pi)^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\vec{D}(\vec{k},\omega)\, i\vec{k}\, e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} = 0 \iff \vec{k}\,\vec{D}(\vec{k},\omega) = 0 \Rightarrow \vec{k}\perp\vec{D}$

$\frac{1}{(2\pi)^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\vec{B}(\vec{k},\omega)\, i\vec{k}\, e^{i\vec{k}\vec{r}-i\omega t}\,d\omega\,d\vec{k} = 0 \iff \vec{k}\cdot\vec{B}(\vec{k},\omega) = 0 \Rightarrow \vec{k}\perp\vec{B}$ +1

Faraday'sches Gesetz:

$i\vec{k}\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{(2\pi)^4}\vec{E}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t}\,d^3r\,dt - i\omega\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{(2\pi)^4}\vec{B}(\vec{k},\omega)e^{i\vec{k}\vec{r}-i\omega t} = 0$

$\iff i\vec{k}\times\vec{E}(\vec{k},\omega) = i\omega\vec{B}(\vec{k},\omega) \iff \vec{B}\ \cancel{\perp}\ \parallel\ \vec{k}\times\vec{E}$

b) Faraday: $\vec{k}\times\vec{B} = \omega\vec{D} \Rightarrow \vec{E}\perp\vec{B}$ +1

Gauß: • $\vec{k}\cdot\vec{D} = 0 \Rightarrow \vec{k}\perp\vec{D}$, $D_i(\vec{k},\omega) = \varepsilon\delta_{ij}E_j(\vec{k},\omega)$, $\vec{D} = \varepsilon\vec{E} \Rightarrow \vec{k}\perp\vec{E}$ +1

• $\vec{k}\vec{B} = 0 \Rightarrow \vec{k}\perp\vec{B}$

$P_k = \mathbb{1} - \hat{k}\hat{k}^T$ mit $\hat{k} = \frac{\vec{k}}{|\vec{k}|}$

Es gilt für das dyadische Produkt $(x\otimes y) = (x\cdot y^T)$:

$(x\otimes y)\cdot z = x\otimes(y\cdot z) = (y\cdot z)x$ und damit folgt

$P_k\cdot\vec{B} = \vec{B} - (\hat{k}\vec{B})\hat{k} = \vec{B} - \Big(\frac{1}{|\vec{k}|}\underbrace{\vec{k}\cdot\vec{B}}_{=0}\Big)\cdot\hat{k} = \vec{B}$ // +2

c) $\vec{k}\times\vec{H} = -\omega\vec{D} \Leftrightarrow (\vec{k}\times\mu^{-1}\vec{B}) = -\omega\varepsilon\vec{E}$ (Ampère-Maxwell)

$\Leftrightarrow \varepsilon^{-1}(\vec{k}\times\mu^{-1}\vec{B}) = -\omega\vec{E}$

$\vec{k}\times$

$\Leftrightarrow \vec{k}\times\varepsilon^{-1}(\vec{k}\times\mu^{-1}\vec{B}) = -\vec{k}\times\omega\vec{E}$

$\vec{k}\times\vec{E} = + \omega\vec{B}$ folgt

$\Leftrightarrow \varepsilon^{-1}\vec{k}\times(\vec{k}\times\mu^{-1}\vec{B}) = - \omega^2\vec{B}$

$\boxed{\cdot P_k}$

$\Leftrightarrow P_k\vec{k}\times(\vec{k}\times\mu^{-1}\underbrace{P_k\vec{B}}_{=\vec{B}}) = -\varepsilon\omega^2\underbrace{P_k\vec{B}}_{=\vec{B}}$

falsch

$\Leftrightarrow P_k\Big[\boxed{\cancel{(\vec{k}\mu^{-1}P_k\vec{B})\vec{k}}} - (\vec{k}\vec{k})\mu^{-1}P_k\vec{B}\Big]$

BAC-CAB, $\vec{k}\vec{B} = 0$

$\Leftrightarrow P_k(-k^2)\mu^{-1}P_k\vec{B} = \varepsilon\omega^2\vec{B}$

$P_k\vec{B} = \vec{B}$

$\Leftrightarrow P_k\mu^{-1}P_k\vec{B} = \varepsilon\frac{\omega^2}{k^2}\vec{B}$ // −10

d) $P_k = \mathbb{1} - \frac{nn^T}{\cdot} = \begin{pmatrix} 1-1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\mu^{-1} = \frac{1}{\det\mu} \begin{pmatrix} \mu_1 \mu_z & i\mu_2\mu_z & 0 \\ -i\mu_2\mu_z & \mu_1\mu_z & 0 \\ 0 & 0 & \mu_1^2-\mu_2^2 \end{pmatrix} \quad \text{(Cramersche Regel)}$$

mit $\det\mu = \mu_1^2\mu_z - \mu_z\mu_2^2 = \mu_z(\mu_1^2 - \mu_2^2)$

$$\Rightarrow \mu^{-1} = \begin{pmatrix} \frac{\mu_1}{\mu_1^2-\mu_2^2} & \frac{i\mu_2}{\mu_1^2-\mu_2^2} & 0 \\ \frac{-i\mu_2}{\mu_1^2-\mu_2^2} & \frac{\mu_1}{\mu_1^2-\mu_2^2} & 0 \\ 0 & 0 & \frac{1}{\mu_-}^2 \end{pmatrix}$$

$$P_k\mu^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\mu_1}{\mu_1^2-\mu_2^2} & \frac{i\mu_2}{\mu_1^2-\mu_2^2} & 0 \\ \frac{-i\mu_2}{\mu_1^2-\mu_2^2} & \frac{\mu_1}{\mu_1^2-\mu_2^2} & 0 \\ 0 & 0 & \frac{1}{\mu_z} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ \frac{-i\mu_2}{\mu_1^2-\mu_2^2} & \frac{\mu_1}{\mu_1^2-\mu_2^2} & 0 \\ 0 & 0 & \frac{1}{\mu_z} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\mu_1}{\mu_1^2-\mu_2^2} & 0 \\ 0 & 0 & \frac{1}{\mu_z} \end{pmatrix} \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} = \varepsilon \frac{\omega^2}{k^2} \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix}$$

charakteristisches Polynom:

$$(-\lambda)\left(\frac{\mu_1}{\mu_1^2-\mu_2^2} - \lambda\right)\left(\frac{1}{\mu_z} - \lambda\right) = 0$$

$\Rightarrow \lambda_1 = \frac{1}{\mu_z} \qquad \lambda_2 = \frac{\mu_1}{\mu_1^2-\mu_2^2} \qquad \lambda_3 = 0 \leftarrow$ Triviale Lösung wird nicht weiter beachtet

$\omega_1^2 = \frac{\lambda_1}{\varepsilon} k_x^2 \Rightarrow \omega_1 = \pm \frac{k_x}{\sqrt{\mu_z \varepsilon}}$ // , $\omega_2^2 = \frac{\lambda_2}{\varepsilon} k_x^2 \Rightarrow \omega_2 = \pm k_x \sqrt{\frac{\mu_1}{(\mu_1^2-\mu_2^2)\varepsilon}}$ //

Die zugehörigen Eigenvektoren sind Lösungen von $(P_{ik}\mu^{-1} - \lambda_i \mathbb{1})\vec{B}_i = 0$

- $\vec{0} = \begin{pmatrix} (\mu_z^2 - \mu_1^2) B_1 \\ (-\mu_2\mu_z + \mu_z^2 - \mu_1^2) B_1 + (\mu_1\mu_z + \mu_z^2 - \mu_1^2) B_2 \\ 0 \cdot B_3 \end{pmatrix} \Rightarrow \vec{B}_1 = \pm \hat{e}_z$

- $\vec{0} = \begin{pmatrix} -\mu_z \mu_1 B_1 \\ (-\mu_z\mu_z - \mu_z\mu_1) B_1 - 0 \cdot B_2 \\ (\mu_1^2 - \mu_z^2 - \mu_z\mu_1) B_3 \end{pmatrix} \Rightarrow \vec{B}_2 = \pm \hat{e}_y$

$\Rightarrow$ 1. Lösung mit $\omega_1 = \pm \frac{k_x}{\sqrt{\varepsilon \mu_z}}$ und Polarisation $\hat{B} = \pm \hat{e}_z$

2. Lösung mit $\omega_2 = \pm k_x \sqrt{\frac{\mu_1}{\varepsilon(\mu_z^2 - \mu_1^2)}}$ mit Polarisation $\hat{B} = \pm \hat{e}_y$ +5

e) fehlt −10

14 P / 35 P

77 P / 100 P

Korrektur: Jason Spitzmesser

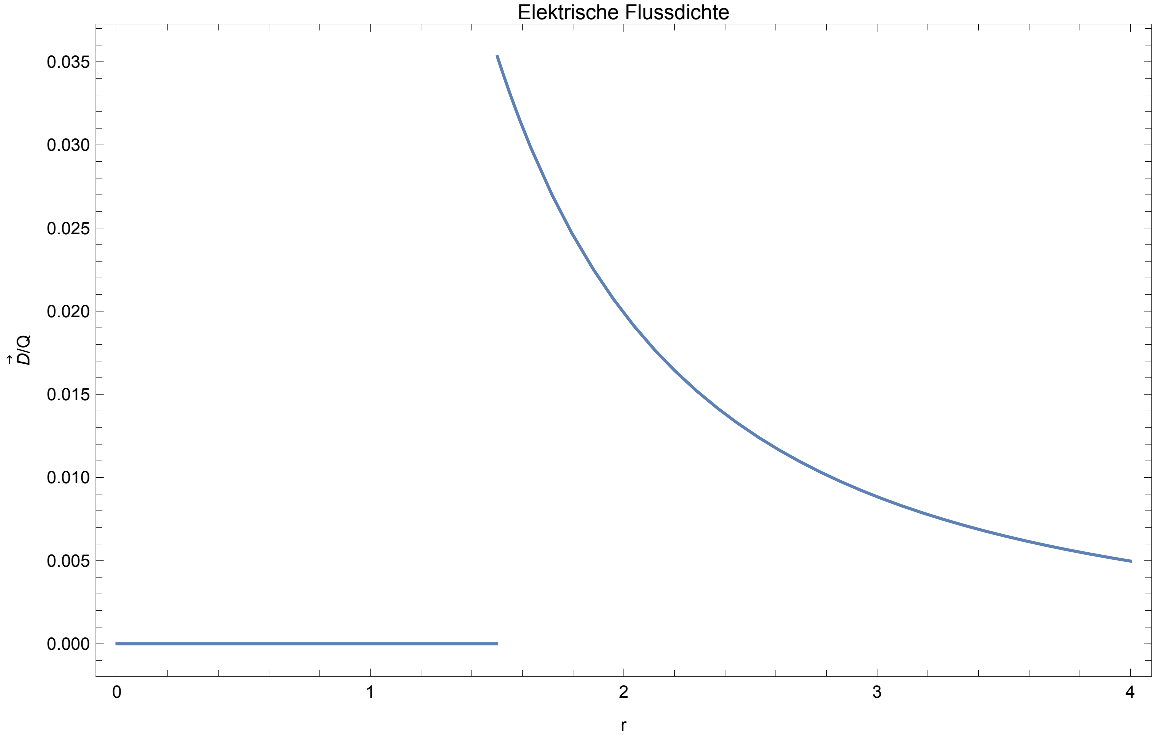
Theo C Blatt 11

Aufgabe 1 c)

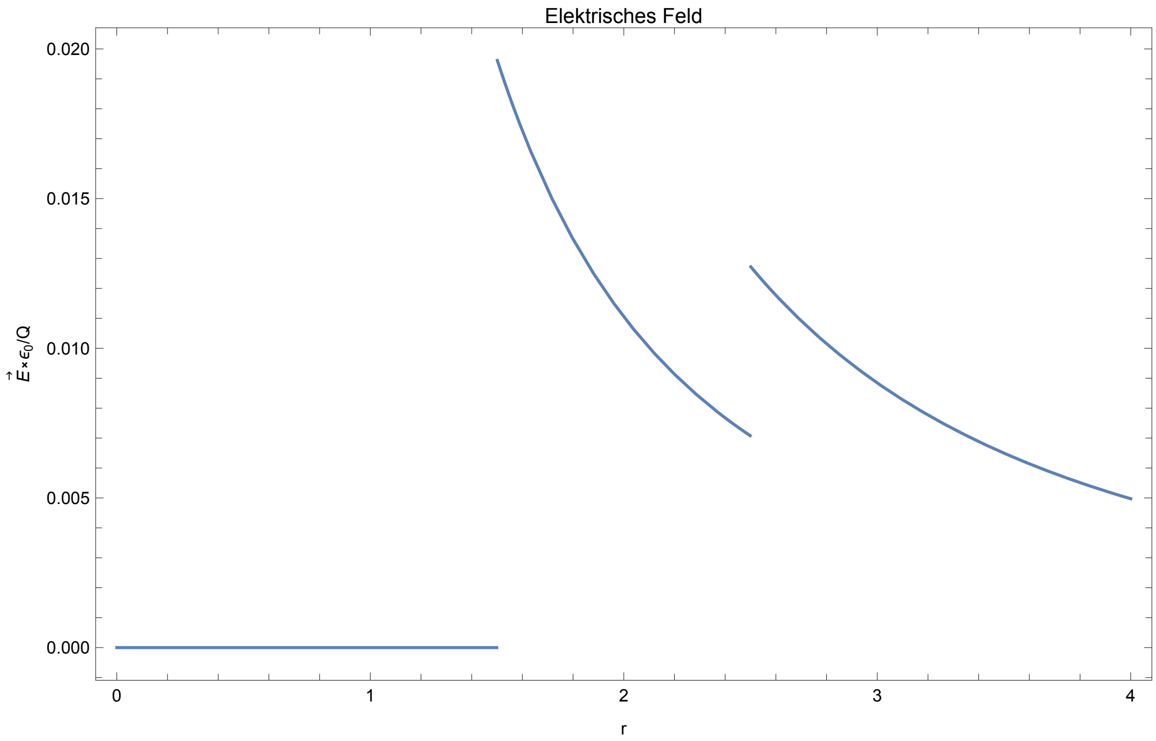
In[592]:=

```
(*Konstanten a,b und ϵ sind beispielhaft gewählt*)
a := 1.5
b := 2.5
ϵ := 1.8


DFeld[r_] := 1 / (4 * Pi) * 1 / r^2 * HeavisideTheta[r - a]
EFeld[r_] := 1 / (4 * Pi * ϵ) * 1 / r^2 * HeavisideTheta[r - a] * HeavisideTheta[b - r] +
   1 / (4 * Pi) * 1 / r^2 * HeavisideTheta[r - b]
ϕ[r_] := 1 / (4 * Pi) * ( (1 / b + 1 / (ϵ * a) - 1 / (ϵ * b)) * HeavisideTheta[a - r] +
     (1 / b + 1 / (ϵ * r) - 1 / (ϵ * b)) * HeavisideTheta[r - a] * HeavisideTheta[b - r]
     + 1 / r * HeavisideTheta[r - b])

Plot[DFeld[r], {r, 0, 4}, FrameLabel → {"r", "D⃗/Q"},
 PlotLabel → "Elektrische Flussdichte", ImageSize → 600, Frame → True]
Plot[EFeld[r], {r, 0, 4}, FrameLabel → {"r", "E⃗×ϵ0/Q"},
 PlotLabel → "Elektrisches Feld", ImageSize → 600, Frame → True]
Plot[ϕ[r], {r, 0, 4}, FrameLabel → {"r", "ϕ×ϵ0/Q"},
 PlotLabel → "Elektrisches Potential", ImageSize → 600, Frame → True]
```

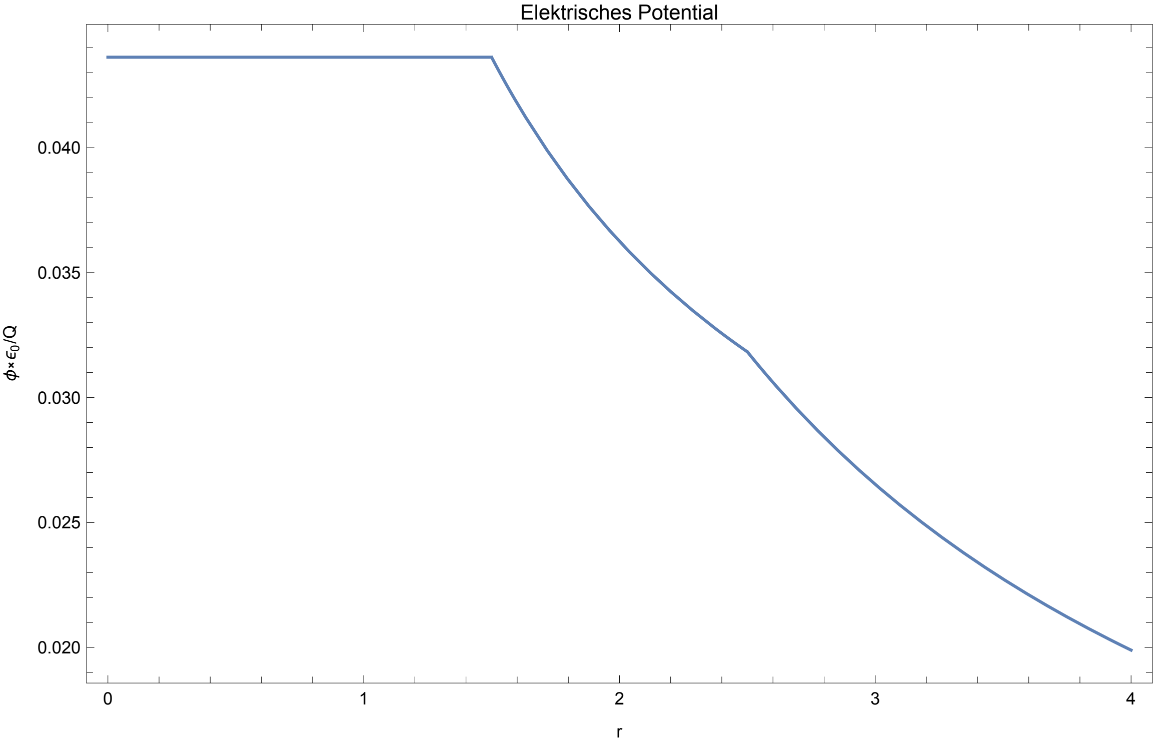
Out[598]=



Out[599]=



Out[600]=

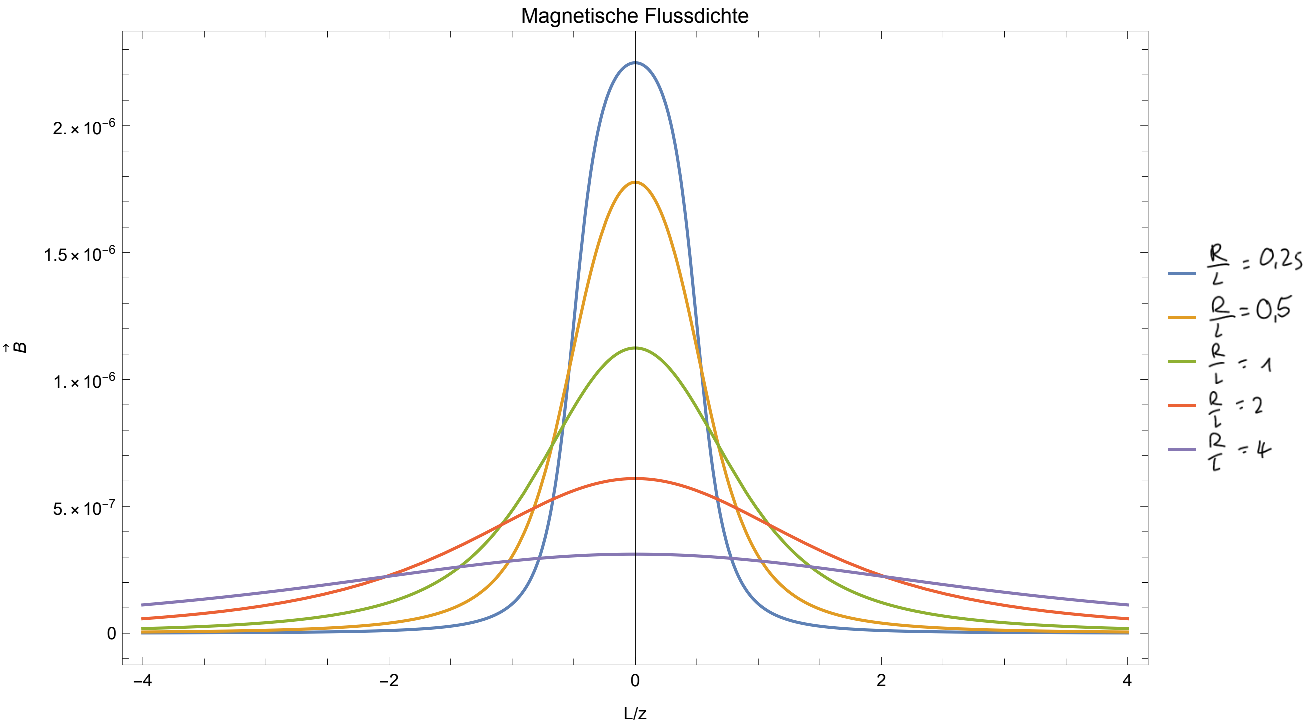

Elektrisches Potential
0.040
0.035
0.030
0.025
0.020
ϕ×ε0/Q
0
1
2
3
4
r

Aufgabe 2 c)

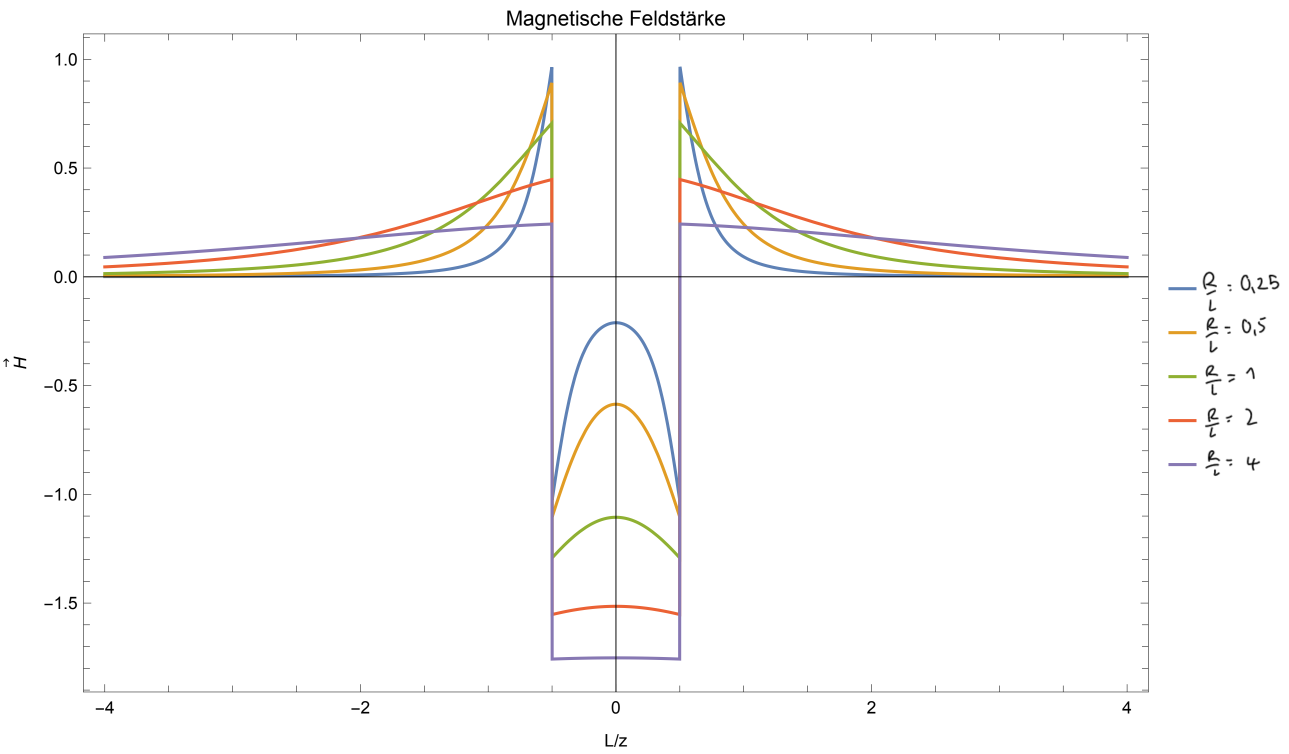
In[642]:=

```
(*Konstante M beispielhaft gewählt*)
M := 2
mu0 := 1.25663706127 * 10 ^ (-6)

Bfeld[zL_, RL_] := -M * mu0 / 2 *
  ((zL - 1 / 2) / ((zL - 1 / 2) ^ 2 + RL ^ 2) ^ 0.5 - (zL + 1 / 2) / ((zL + 1 / 2) ^ 2 + RL ^ 2) ^ 0.5)
Hfeld[zL_, RL_] := -M / 2 * ((zL - 1 / 2) / ((zL - 1 / 2) ^ 2 + RL ^ 2) ^ 0.5 -
     (zL + 1 / 2) / ((zL + 1 / 2) ^ 2 + RL ^ 2) ^ 0.5) * HeavisideTheta[zL ^ 2 - 1 / 4] -
  (M / 2 * ((zL - 1 / 2) / ((zL - 1 / 2) ^ 2 + RL ^ 2) ^ 0.5 -
        (zL + 1 / 2) / ((zL + 1 / 2) ^ 2 + RL ^ 2) ^ 0.5) + M) * HeavisideTheta[-zL ^ 2 + 1 / 4]


Plot[{Bfeld[zL, RL = 0.25], Bfeld[zL, RL = 0.5],
  Bfeld[zL, RL = 1], Bfeld[zL, RL = 2], Bfeld[zL, RL = 4]}, {zL, -4, 4},
 FrameLabel → {"L/z", "B⃗"}, PlotLabel → "Magnetische Flussdichte",
 ImageSize → 600, Frame → True, PlotLegends →
  {"R/L=0.25", "R/L=0.5", "R/L=1", "R/L=2", "R/L=4"}, PlotRange → All]
Plot[{Hfeld[zL, RL = 0.25], Hfeld[zL, RL = 0.5], Hfeld[zL, RL = 1],
  Hfeld[zL, RL = 2], Hfeld[zL, RL = 4]}, {zL, -4, 4}, FrameLabel → {"L/z", "H⃗"},
 PlotLabel → "Magnetische Feldstärke", ImageSize → 600, Frame → True,
 PlotLegends → {"R/L=0.25", "R/L=0.5", "R/L=1", "R/L=2", "R/L=4"},
 PlotRange → All]
```

Out[646]=



Out[647]=



Aufgabe 2 d)

In[706]:=

```
Hfeld[zL_, RL_] := -M / 2 * ((zL - 1 / 2) / ((zL - 1 / 2) ^ 2 + RL ^ 2) ^ 0.5 -
      (zL + 1 / 2) / ((zL + 1 / 2) ^ 2 + RL ^ 2) ^ 0.5) * HeavisideTheta[zL ^ 2 - 1 / 4] -
   (M / 2 * ((zL - 1 / 2) / ((zL - 1 / 2) ^ 2 + RL ^ 2) ^ 0.5 -
         (zL + 1 / 2) / ((zL + 1 / 2) ^ 2 + RL ^ 2) ^ 0.5) + M) * HeavisideTheta[-zL ^ 2 + 1 / 4]

HfeldNähr[zL_, RL_] := (1 / 2 * RL ^ 2 * 1 / zL ^ 3) * HeavisideTheta[zL ^ 2 - 1 / 4] +
   (1 / 2 * RL ^ 2 * 1 / zL ^ 3 - 1) * HeavisideTheta[-zL ^ 2 + 1 / 4]

LogLogPlot[{HfeldNähr[zL, RL = 0.25], HfeldNähr[zL, RL = 0.5],
  HfeldNähr[zL, RL = 1], HfeldNähr[zL, RL = 2], HfeldNähr[zL, RL = 4],
  Hfeld[zL, RL = 0.25], Hfeld[zL, RL = 0.5], Hfeld[zL, RL = 1],
  Hfeld[zL, RL = 2], Hfeld[zL, RL = 4]}, {zL, 0.1, 100},
 FrameLabel → {"L/z", "H⃗"}, PlotLabel → "Magnetische Feldstärke",
 ImageSize → 600, Frame → True, PlotLegends →
  {"Dipolnäherung: R/L=0.25", "Dipolnäherung: R/L=0.5", "Dipolnäherung: R/L=1",
   "Dipolnäherung: R/L=2", "Dipolnäherung: R/L=4", "Exakt: R/L=0.25",
   "Exakt: R/L=0.5", "Exakt: R/L=1", "Exakt: R/L=2", "Exakt: R/L=4"}]
```

Out[708]=