19/30

Insgesamt: 38/100

1

a) 8/10

$\Lambda^\mu{}_\nu g_{\mu\rho} \Lambda^\rho{}_\sigma = g_{\nu\sigma} \quad | \times g^{\alpha\nu}$ ~~$\times g^{\alpha\nu}$~~ Λ^T

$(\Lambda_\nu{}^\mu) = g_{\nu\rho} g^{\mu\sigma} \Lambda^\rho{}_\sigma$

$\Leftrightarrow g^{\alpha\nu} \Lambda^\mu{}_\nu g_{\mu\rho} \Lambda^\rho{}_\sigma = g^{\alpha\nu} g_{\nu\sigma} \quad | \; g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$

du multiplizierst von links!

$\Leftrightarrow (g^{\alpha\nu} \Lambda^\mu{}_\nu g_{\mu\rho}) \Lambda^\rho{}_\sigma = \delta^\alpha_\sigma \; (= I)$

$\Rightarrow (\Lambda^\alpha{}_\rho)^{-1} = g^{\alpha\nu} \Lambda^\mu{}_\nu g_{\mu\rho}$ (✓) bzw. aber nur wenn $\Lambda^\mu{}_\nu$ aus Λ^T!

$(\Lambda^\mu{}_\nu)^{-1} = g^{\mu\nu} \Lambda^\mu{}_\nu g_{\mu\nu}$? des macht kein Sinn

b) 1/10

$$(\Lambda(\beta,\theta)^\mu{}_\nu) = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{pmatrix} \text{ mit } \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$(\Lambda^\mu{}_\nu)^{-1} = g^{\mu\nu} \Lambda^\mu{}_\nu g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

hier muss die transponierte Matrix stehen! siehe a)

$$= \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ \gamma\beta & -\gamma & 0 & 0 \\ 0 & 0 & -\cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & \cos\theta - \sin\theta & \\ 0 & 0 & +\sin\theta & \cos\theta \end{pmatrix} \text{ ff}$$

$$(\Lambda^\mu{}_\nu)^{-1} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{pmatrix} \doteq \begin{pmatrix} \gamma & -\gamma\beta' & 0 & 0 \\ -\gamma\beta' & \gamma & 0 & 0 \\ 0 & 0 & \cos\theta' & \sin\theta' \\ 0 & 0 & -\sin\theta' & \cos\theta' \end{pmatrix} = (\Lambda(\beta',\theta')^\mu{}_\nu)$$

✓ 1

$\Rightarrow \beta' = -\beta, \quad \theta' = \theta$ ff

c) $\Lambda(\beta_1,\theta_1)^{\mu}{}_{\nu}\,\Lambda(\beta_2,\theta_2)^{\nu}{}_{\rho} = \Lambda(f(\beta_1,\beta_2), g(\theta_1,\theta_2))^{\mu}{}_{\rho}$

10/10

$$\begin{pmatrix} \gamma_1 & -\gamma_1\beta_1 & 0 & 0 \\ -\gamma_1\beta_1 & \gamma_1 & 0 & 0 \\ 0 & 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & 0 & -\sin\theta_1 & \cos\theta_1 \end{pmatrix} \cdot \begin{pmatrix} \gamma_2 & -\gamma_2\beta_2 & 0 & 0 \\ -\gamma_2\beta_2 & \gamma_2 & 0 & 0 \\ 0 & 0 & \cos\theta_2 & \sin\theta_2 \\ 0 & 0 & -\sin\theta_2 & \cos\theta_2 \end{pmatrix}$$

$$= \begin{pmatrix} \gamma_1\gamma_2 + \gamma_1\beta_1\gamma_2\beta_2 & -\gamma_1\gamma_2\beta_2 - \gamma_1\gamma_2\beta_1 & 0 & 0 \\ -\gamma_1\beta_1\gamma_2 - \gamma_1\gamma_2\beta_2 & \gamma_1\beta_1\gamma_2\beta_2 + \gamma_1\gamma_2 & 0 & 0 \\ 0 & 0 & \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2 \\ 0 & 0 & -\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 \end{pmatrix}$$

$$= \begin{pmatrix} \gamma_1\gamma_2(1+\beta_1\beta_2) & -\gamma_1\gamma_2(\beta_1+\beta_2) & 0 & 0 \\ -\gamma_1\gamma_2(\beta_1+\beta_2) & \gamma_1\gamma_2(1+\beta_1\beta_2) & 0 & 0 \\ 0 & 0 & \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) \\ 0 & 0 & -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{pmatrix}$$

$\Rightarrow \quad g(\theta_1,\theta_2) = \theta_1 + \theta_2$

(1) $\gamma = \gamma_1\gamma_2(1+\beta_1\beta_2) = \dfrac{(1+\beta_1\beta_2)}{\sqrt{1-\beta_1^2}\ \sqrt{1-\beta_2^2}}$

(2) $-\gamma\beta = -\gamma_1\gamma_2(\beta_1+\beta_2) = -\dfrac{\beta_1+\beta_2}{\sqrt{1-\beta_1^2}\ \sqrt{1-\beta_2^2}}$

(1)=(2) $\frac{1}{\gamma}$

$\Leftrightarrow \ -\dfrac{(1+\beta_1\beta_2)}{\sqrt{1-\beta_1^2}\ \sqrt{1-\beta_2^2}} = -\dfrac{\beta_1+\beta_2}{\beta\sqrt{1-\beta_1^2}\ \sqrt{1-\beta_2^2}}$

$\Leftrightarrow \ -(1+\beta_1\beta_2) = -\dfrac{\beta_1+\beta_1}{\beta}$

$\Leftrightarrow \quad \beta = \dfrac{\beta_1+\beta_2}{1+\beta_1\beta_2} \quad \Rightarrow \quad f(\beta_1,\beta_2) = \dfrac{\beta_1+\beta_2}{1+\beta_1\beta_2}$

② 0/45 $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ $\qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$

$(A^\mu) = (\frac{1}{c}\phi, \vec{A})$, $\vec{E} = -\vec{\nabla}\phi - \partial_t \vec{A}$, $\vec{B} = \vec{\nabla}\times\vec{A}$

a) 0/10

$$F_{\mu\nu} = g_{\mu\alpha}\partial^\alpha g_{\nu\alpha} A^\beta - g_{\nu\alpha}\partial^\beta g_{\mu\alpha} A^\alpha = g_{\mu\alpha} F^{\alpha\beta} g_{\nu\beta} \Rightarrow g_{\mu\alpha} g_{\nu\beta} (F^{\alpha\beta}) = g_{\mu\alpha} g_{\nu\beta} \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & -B_z & B_y \\ -\frac{E_y}{c} & B_z & 0 & -B_x \\ -\frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}$$

du sollst es über die Beziehung von A^μ und $\vec{E}$ bzw. $\vec{B}$ machen.

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & \frac{E_z}{c} & -\frac{E_y}{c} \\ B_y & -\frac{E_z}{c} & 0 & \frac{E_x}{c} \\ B_z & \frac{E_y}{c} & \frac{E_x}{c} & 0 \end{pmatrix}$$

rest fehlt!

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③ 19/25

$$\begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \quad (1)$$

a) 5/5 $\vec{j} = I\hat{e}_z\, \delta(x)\delta(y)$

$(j^\mu) = (c\rho, \vec{j})$

$$j'^\mu = \Lambda^\mu{}_\nu j^\nu = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ I\delta(x)\delta(y) \end{pmatrix} = \begin{pmatrix} -\gamma\beta I\delta(x)\delta(y) \\ 0 \\ 0 \\ \gamma I\delta(x)\delta(y) \end{pmatrix} ✓$$

$$\Rightarrow \rho' = -\frac{\gamma\beta I\delta(x)\delta(y)}{c} = -\frac{v\, I\,\delta(x)\delta(y)}{\sqrt{1-(\frac{v}{c})^2}\, c^2} ✓$$

Durch die Längenkontraktion sind die Abstände zw. "fließenden" Elektronen kleiner. Die Abstände gebundener Ladungen erfahren geringere Kontraktion und der Draht ist nicht mehr neutral geladen. ✓

b) $\vec\nabla \vec E' = \frac{\rho'}{\varepsilon_0} \iff \int_{\partial V} \vec E \, d\vec f = -\frac{1}{\varepsilon_0}\int_V d\vec r \, \frac{v\, I\, \delta(x)\delta(y)}{\sqrt{1-(\frac{v}{c})^2}\, c^2}$

Integriert wird über einen Zylinder um die z-Achse mit Länge L und Radius r.

10/10

$$\iff 2\pi r L E = -\frac{1}{\varepsilon_0} \frac{v I L}{\sqrt{1-(\frac{v}{c})^2}\, c^2}$$

$$\Rightarrow \vec E(\vec r) = -\frac{1}{2\pi\varepsilon_0} \frac{v I}{r c^2 \sqrt{1-(\frac{v}{c})^2}} \hat e_\rho = \frac{1}{2\pi\varepsilon_0} \gamma\beta \frac{1}{rc} \hat e_\rho$$

$\vec\nabla \times \vec B' = \mu_0 \vec j' \iff \int_{\partial F} \vec B \, d\vec s = \mu_0 \gamma I \int_{\partial V} \delta(x)\delta(y)\, d\vec f = \mu_0 \gamma I$ Integration über Rand des Zylinders

$$\Rightarrow \vec B(\vec r) = \frac{\mu_0 \gamma I}{2\pi r} \hat e_\varphi$$

c) $\vec F' = q(\vec E' + \vec v_q' \times \vec B')$ mit $\vec v_q' = \frac{v_q - v}{1+\frac{v v_q}{c^2}} \hat e_z$

4/10

$$= q\left(\frac{1}{2\pi\varepsilon_0}\gamma\beta\frac{1}{rc}\hat e_\rho + \frac{v_q - v}{1+\frac{v v_q}{c^2}} \hat e_z \times \frac{\mu_0 \gamma I}{2\pi r}\hat e_\varphi\right)$$

$$= q\left(\frac{1}{2\pi\varepsilon_0}\gamma\beta\frac{I}{rc}\hat e_\rho - \frac{v_q - v}{1+\frac{v v_q}{c^2}} \frac{\mu_0 \gamma I}{2\pi r}\hat e_\rho\right)$$

$r = d$

$$= \frac{q}{2\pi d}\left(\frac{I\gamma\beta}{\varepsilon_0 c} - \frac{v_q - v}{1+\frac{v v_q}{c^2}} \mu_0 \gamma I\right)\hat e_\rho$$

$$= \frac{I\gamma\beta}{\varepsilon_0 c} - \frac{v_q - v}{c^2 + v v_q} c^2 \mu_0 \gamma I$$

$$= I\gamma\left(\frac{v}{\varepsilon_0 c^2} - \frac{v_q - v}{c^2 + v v_q} c^2 \mu_0\right) \quad \text{mit } \mu_0\varepsilon_0 = \frac{1}{c^2}$$

$$= \frac{I\gamma}{\varepsilon_0}\left(\frac{v}{c^2} - \frac{v_q - v}{c^2 + v v_q}\right)$$

2

$v = 0$: $\vec F'_{v=0} = -q v_q \frac{\mu_0 I}{2\pi d}\hat e_\rho$

2

$v = v_q$: $\vec F'_{v=v_q} = \frac{q}{2\pi r} \frac{v_q I}{\varepsilon_0 c^2 \sqrt{1-(\frac{v_q}{c})^2}} \hat e_\rho$ f

Keine Kraft wirkt $\iff \vec F' = \vec 0$

$$\iff \frac{q}{2\pi r}\left(\frac{I\gamma\beta}{\varepsilon_0 c} - \frac{v_q - v}{1+\frac{v v_q}{c^2}}\mu_0\gamma I\right)\hat e_\rho = \vec 0$$

$$\frac{v}{c^2} - \frac{v_q - v}{c^2 + v v_q} \overset{!}{=} 0$$

$$\iff \frac{v}{\varepsilon_0 c^2} = \frac{(v_q - v)c^2}{c^2 + v v_q}\mu_0$$

$$\frac{v c^2 + v^2 v_q}{\varepsilon_0 c^2} = \mu_0 c^2 v_q - v c^2 \mu_0$$

$$v\left(\varepsilon_0 + \frac{v_q}{\varepsilon_0 c^2} + \frac{c^2}{\mu_0}\right) = \mu_0 c^2 v_q$$

$$v = \frac{\mu_0 c^2 v_q}{\varepsilon_0 + \frac{v_q}{\varepsilon_0 c^2} + \frac{c^2}{\mu_0}} \quad ?$$