Theoretische Physik D - Quantenmechanik SS 2005

Übungsblatt 4 - Lösung

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Aufgabe 1

(i) Berechnung von Φ

$$\begin{aligned}\Psi_I(x) &= e^{ikx} + Ae^{-ikx} \\ \Psi_{II}(x) &= Be^{-\delta x} \\ k^2 &= \frac{2mE}{\hbar^2} \qquad \delta^2 = \frac{2m}{\hbar^2}(V_0 - E)\end{aligned}$$

Anschlussbedingungen:

$$\Psi_I(0) = \Psi_{II}(0) \tag{1}$$
$$\Psi'_I(0) = \Psi'_{II}(0) \tag{2}$$

aus (1) folgt: $1 + A = B$

aus (2) folgt: $ik - ikA = -\delta B$

$$\begin{aligned}ik(1-A) &= -\delta(1+A) \\ \Rightarrow A &= \frac{k - i\delta}{k + i\delta} = e^{-2i\alpha} \qquad \alpha = \arctan\frac{\delta}{k}\end{aligned}$$

Umformung erfolgt mit den Hinweisen vom Übungsblatt 2, Aufgabe 2(i).

$$\begin{aligned}\Psi_R(x) &= e^{-2i\alpha}e^{-ikx} \\ \Rightarrow \qquad \Phi &= \alpha = \arctan\frac{\delta}{k} = \underline{\underline{\arctan\sqrt{\left(\frac{2mV_0}{\hbar^2 k^2} - 1\right)}}}\end{aligned}$$

(ii) Berechnung von δt

$$
\begin{aligned}
\Psi(x,t) &= \int_0^{\delta} \mathrm{d}k \; g(k) e^{i(kx-\omega t)} \\
\omega &= \frac{E}{\hbar} = \frac{k^2\hbar}{2m} \qquad \delta' = \sqrt{\frac{2mV_0}{\hbar}} \\
\Psi_R(x,t) &= \int_0^{\delta} \mathrm{d}k \; g(k) e^{-i(kx+\omega t)} = \int_0^{\delta} \mathrm{d}k \; e^{-i(kx+\omega t+2\Phi)} \\
x_M &= -\left.\frac{\mathrm{d}x}{\mathrm{d}k}\right|_{k=k_0} \qquad \alpha = \omega t + 2\Phi \\
x_M &= -\frac{\mathrm{d}\omega}{\mathrm{d}k} t - 2\left.\frac{\mathrm{d}\Phi}{\mathrm{d}k}\right|_{k=k_0} \\
-\left.\frac{\mathrm{d}\omega}{\mathrm{d}k}\right|_{k=k_0} &= -\frac{\hbar k_0}{m} \\
\left.\frac{\mathrm{d}\Phi}{\mathrm{d}k}\right|_{k=k_0} &= \frac{\mathrm{d}\arctan\sqrt{\frac{2mV_0}{\hbar^2k^2}-1}}{\mathrm{d}k} \\
\left.\frac{\mathrm{d}\Phi}{\mathrm{d}k}\right|_{k=k_0} &= \frac{1}{1+\left(\frac{2mV_0}{\hbar^2k^2}\right)-1} \cdot \frac{1}{2\sqrt{\frac{2mV_0}{\hbar^2k^2}-1}} \cdot \frac{2mV_0}{\hbar^2} \cdot \frac{-2}{k^3} \\
\left.\frac{\mathrm{d}\Phi}{\mathrm{d}k}\right|_{k=k_0} &= \frac{-1}{k\sqrt{\frac{2mV_0}{\hbar^2k^2}-1}} \qquad \Rightarrow \qquad x_M = -\frac{\hbar k_0 t}{m} + \frac{2}{\sqrt{\frac{2mV_0}{\hbar^2}-k_0^2}}
\end{aligned}
$$

Es folgt ein Vergleich des quantenmechanischen Weges mit dem klassischen Weg $x_M = x_{Kl}$.

$$
\begin{aligned}
x_{Kl} &= v_0 t \qquad v_0 = v = -\left.\frac{\mathrm{d}\omega}{\mathrm{d}k}\right|_{k=k_0} = -\frac{k_0\hbar}{m} \\
v_0 t_{Kl} &= -\frac{\hbar k_0}{m} t_{QM} + \frac{2}{\sqrt{\frac{2mV_0}{\hbar^2}-k_0^2}} \\
t_{QM} &= t_{Kl} + \underline{\underline{\frac{2m}{\hbar k_0\sqrt{\frac{2mV_0}{\hbar^2}-k_0^2}}}} = t_{Kl} + \delta t
\end{aligned}
$$

Aufgabe 2

(i)

$$
\begin{aligned}
a_n &= \frac{1}{\sqrt{\pi}}\left[\int\limits_{-\pi}^{0} \mathrm{d}x\ f(x)\cos nx + \int\limits_{0}^{+\pi} \mathrm{d}x\ f(x)\cos nx\right] \\
\text{Substituion:} &\quad x = -x' \qquad \mathrm{d}x = -\mathrm{d}x' \\
a_n &= \frac{1}{\sqrt{\pi}}\left[-\int\limits_{+\pi}^{0} \mathrm{d}x'\ f(-x')\cos -nx' + \int\limits_{0}^{+\pi} \mathrm{d}x\ f(x)\cos nx\right] \\
a_n &= \frac{1}{\sqrt{\pi}}\left[\int\limits_{0}^{+\pi} \mathrm{d}x'\ f(-x')\cos nx' + \int\limits_{0}^{+\pi} \mathrm{d}x\ f(x)\cos nx\right] \\
a_n &= \frac{1}{\sqrt{\pi}}\left[\int\limits_{0}^{+\pi} \mathrm{d}x'\ \ -f(x')\cos nx' + \int\limits_{0}^{+\pi} \mathrm{d}x\ f(x)\cos nx\right] \\
a_n &= 0
\end{aligned}
$$

$$
\begin{aligned}
b_n &= \frac{1}{\sqrt{\pi}}\left[\int\limits_{-\pi}^{0} \mathrm{d}x\ f(x)\sin nx + \int\limits_{0}^{\pi} \mathrm{d}x f(x)\sin nx\right] \\
\text{Substituion:} &\quad x = -x' \qquad \mathrm{d}x = -\mathrm{d}x' \\
b_n &= \frac{1}{\sqrt{\pi}}\left[-\int\limits_{+\pi}^{0} \mathrm{d}x'\ f(-x')\sin -nx' + \int\limits_{0}^{+\pi} \mathrm{d}x f(x)\sin nx\right] \\
b_n &= \frac{1}{\sqrt{\pi}}\left[\int\limits_{0}^{+\pi} \mathrm{d}x'\ f(x') - \sin nx' + \int\limits_{0}^{+\pi} \mathrm{d}x f(x)\sin nx\right] \\
b_n &= 0
\end{aligned}
$$

(ii)

$$
\begin{aligned}
f(x) &= \begin{cases} -\sin x & -\pi \le x \le 0 \\ 0 & ansonsten \end{cases} \\
a_0 &= -\frac{1}{\sqrt{2\pi}}\int\limits_{-\pi}^{0} \mathrm{d}x \sin x = \frac{1}{\sqrt{2\pi}}\cos x\Big|_{-\pi}^{0} = \sqrt{\frac{2}{\pi}}
\end{aligned}
$$

Für $n \neq 1$ gilt für b_n:

$$b_n = \frac{1}{\sqrt{\pi}} \int_{-\pi}^{0} \mathrm{d}x\ (-1) \sin x \sin nx = \frac{1}{\sqrt{\pi}} \int_{-\pi}^{0} \mathrm{d}x\ (-1) \frac{e^{ix} - e^{-ix}}{2i} \cdot \frac{e^{inx} - e^{-inx}}{2i}$$

$$b_n = \frac{1}{\sqrt{\pi}} \int_{-\pi}^{0} \mathrm{d}x\ \frac{1}{4} \left(e^{ix(1+n)} - e^{ix(1-n)} - e^{-ix(1-n)} + e^{-ix(1+n)}\right)$$

$$b_n = \frac{1}{\sqrt{\pi}} \left[\frac{1}{4}\left(\frac{1}{(n+1)i}\left(e^{ix(1+n)} - e^{-ix(1+n)}\right) - \frac{1}{(1-n)i}\left(e^{ix(1-n)} - e^{-ix(1-n)}\right)\right)\right]_{-\pi}^{0}$$

$$b_n = \frac{1}{\sqrt{\pi}} \left[\frac{\sin(1+n)x}{2(1+n)} - \frac{\sin(1-n)x}{2(1-n)}\right]_{-\pi}^{0} = 0$$

Für den Fall $n = 1$ ergibt sich b_1:

$$b_1 = -\frac{1}{\sqrt{\pi}} \int_{-\pi}^{0} \mathrm{d}x\ \sin^2 x = -\frac{1}{\sqrt{\pi}} \underbrace{\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_{-\pi}^{0}}_{\text{Bronstein}} = -\frac{\pi}{2\sqrt{\pi}} = -\frac{\sqrt{\pi}}{2}$$

Analoge Vorgehensweise der Lösung des Integrals für a_n oder durch Bronstein:

$$a_n = -\frac{1}{\sqrt{\pi}} \int_{0}^{-\pi} \mathrm{d}x\ \sin x \cos nx = -\frac{1}{\sqrt{\pi}} \underbrace{\left[-\frac{\cos(1+n)x}{2(1+n)} - \frac{\cos(1-n)x}{2(1-n)}\right]_{-\pi}^{0}}_{\text{Bronstein}}$$

$$a_n = \begin{cases} \frac{1}{\sqrt{\pi}}\left(\frac{1}{1+n} + \frac{1}{1-n}\right) = \frac{1}{\sqrt{\pi}}\frac{2}{1-n^2} & n = 2m+1; m \in \mathbb{N} \\ 0 & \text{ansonsten} \end{cases}$$

(iii)

$$f(x) = \begin{cases} 1 & -\pi/2 < x < \pi/2 \\ 0 & \text{ansonsten} \end{cases}$$

$$a_0 = \frac{1}{2\sqrt{\pi}} \int_{-\pi/2}^{\pi/2} \mathrm{d}x = \sqrt{\frac{\pi}{2}} \qquad \underbrace{b_n = 0}_{f(x)\text{ ist eine ungerade Funktion}}$$

$$
a_n = \frac{1}{\sqrt{\pi}} \int\limits_{-\pi/2}^{\pi/2} \mathrm{d}x \ \cos nx = \frac{1}{\sqrt{\pi}} \left[\frac{\sin nx}{n} \right]_{-\pi/2}^{\pi/2}
$$

$$
a_n = \begin{cases} 0 & n = 2, 4, 6, \ldots \\ \frac{2}{\sqrt{\pi} n} & n = 1, 5, 9, \ldots \\ \frac{-2}{\sqrt{\pi} n} & n = 3, 7, 11, \ldots \end{cases}
$$

Aufgabe 3

(i)

Gegeben:

$$
\begin{aligned}
|\Psi_1\rangle &= \frac{1}{\sqrt{2}} |1\rangle + \frac{i}{2} |2\rangle + \frac{-i}{\sqrt{2}} |3\rangle \\
|\Psi_2\rangle &= \frac{1}{\sqrt{3}} |1\rangle + \frac{-i}{\sqrt{3}} |3\rangle
\end{aligned}
$$

Gesucht ist $|\Psi_3\rangle$. Betrachtung von $|\Psi_2\rangle$ und der Bedingung $\langle\Psi_3|\Psi_2\rangle = 0$ führt zu (z.B.):

$$
\langle\Psi_3| = \frac{-i}{\sqrt{3}} \langle 1| + \alpha \langle 2| + \frac{1}{\sqrt{3}} \langle 3|
$$

Für die Bestimmung von α wird die zweite Bedinung $\langle\Psi_3|\Psi_1\rangle = 0$ überprüft. Daraus folgt:

$$
\begin{aligned}
\langle\Psi_3| &= \frac{-i}{\sqrt{3}} \langle 1| + \frac{2\sqrt{2}}{\sqrt{3}} \langle 2| + \frac{1}{\sqrt{3}} \langle 3| \\
\Rightarrow \qquad |\Psi_3\rangle &= \frac{i}{\sqrt{3}} |1\rangle + \frac{2\sqrt{2}}{\sqrt{3}} |2\rangle + \frac{1}{\sqrt{3}} |3\rangle
\end{aligned}
$$

Normierung der Vektoren $|\Psi_1\rangle$, $|\Psi_2\rangle$ und $|\Psi_3\rangle$:

$$
\left|\widehat{\Psi_1}\right\rangle = \frac{|\Psi_1\rangle}{\langle\Psi_1|\Psi_1\rangle} = \frac{|\Psi_1\rangle}{\sqrt{\frac{1}{2} + \frac{1}{4} + \frac{1}{2}}} = \frac{2}{\sqrt{5}} |\Psi_1\rangle
$$

$$
\begin{aligned}
\left|\widehat{\Psi_2}\right\rangle &= \frac{|\Psi_2\rangle}{\langle\Psi_2|\Psi_2\rangle} = \frac{|\Psi_2\rangle}{\sqrt{\frac{1}{3}+\frac{1}{3}}} = \frac{\sqrt{3}}{\sqrt{2}}\,|\Psi_2\rangle \\
\left|\widehat{\Psi_3}\right\rangle &= \frac{|\Psi_3\rangle}{\langle\Psi_3|\Psi_3\rangle} = \frac{|\Psi_3\rangle}{\sqrt{\frac{1}{3}+\frac{8}{3}+\frac{1}{3}}} = \frac{\sqrt{3}}{\sqrt{10}}\,|\Psi_3\rangle
\end{aligned}
$$

$$
\begin{aligned}
\left|\widehat{\Psi_1}\right\rangle &= \frac{1}{\sqrt{5}}(\sqrt{2}\,|1\rangle + i\,|2\rangle - \sqrt{2}i\,|3\rangle) \\
\left|\widehat{\Psi_2}\right\rangle &= \frac{1}{\sqrt{2}}(|1\rangle + i\,|3\rangle) \\
\left|\widehat{\Psi_3}\right\rangle &= \frac{1}{\sqrt{10}}(i\,|1\rangle + 2\sqrt{2}\,|2\rangle + |3\rangle)
\end{aligned}
$$

(ii)

$$
\begin{aligned}
P_{\Psi_1} &= |\Psi_1\rangle\langle\Psi_1| = \frac{1}{5}(\sqrt{2}, i, -\sqrt{2}i)\begin{pmatrix}\sqrt{2}\\ -i\\ \sqrt{2}i\end{pmatrix} \\
P_{\Psi_1} &= \frac{1}{5}\begin{pmatrix} 2 & \sqrt{2}i & -2i \\ -\sqrt{2}i & 1 & -\sqrt{2} \\ 2i & -\sqrt{2} & 2 \end{pmatrix} = P_{\Psi_1}^{\dagger} \;\Rightarrow\; \text{hermitesch!!}
\end{aligned}
$$

$$
\begin{aligned}
P_{\Psi_2} &= |\Psi_2\rangle\langle\Psi_2| = \frac{1}{2}(1, 0, i)\begin{pmatrix}1\\ 0\\ -i\end{pmatrix} \\
P_{\Psi_2} &= \frac{1}{2}\begin{pmatrix} 1 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 1 \end{pmatrix} = P_{\Psi_2}^{\dagger} \;\Rightarrow\; \text{hermitesch!!}
\end{aligned}
$$

$$
\begin{aligned}
P_{\Psi_3} &= |\Psi_3\rangle\langle\Psi_3| = \frac{1}{10}(i, 2\sqrt{2}, 1)\begin{pmatrix}-i\\ 2\sqrt{2}\\ 1\end{pmatrix} \\
P_{\Psi_3} &= \frac{1}{10}\begin{pmatrix} 1 & -2\sqrt{2}i & -i \\ 2\sqrt{2}i & 8 & 2\sqrt{2} \\ i & 2\sqrt{2} & 1 \end{pmatrix} = P_{\Psi_3}^{\dagger} \;\Rightarrow\; \text{hermitesch!!}
\end{aligned}
$$

$$\sum_{i=1}^{3} P_{\Psi_i} \;=\; \text{Einheitsmatrix} \Rightarrow \text{Vollständigkeit ist bewiesen.}$$

Aufgabe 4

(i)

$$\begin{aligned} L_y^\dagger = (L_y^*)^T &= -\frac{\hbar}{2i}\begin{pmatrix} 0 & -\sqrt{2} & 0 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix} \\ &= \frac{\hbar}{2i}\begin{pmatrix} 0 & \sqrt{2} & 0 \\ -\sqrt{2} & 0 & \sqrt{2} \\ 0 & -\sqrt{2} & 0 \end{pmatrix} = L_y \;\Rightarrow \text{hermitesch!!} \end{aligned}$$

Bestimmung der Eigenwerte:

$$\begin{aligned} \mathrm{Det}(L_y - \lambda\delta_{ij}) &= 0 \\ \begin{vmatrix} -\lambda & \sqrt{2}\frac{\hbar}{2i} & 0 \\ -\sqrt{2}\frac{\hbar}{2i} & -\lambda & \sqrt{2}\frac{\hbar}{2i} \\ 0 & -\sqrt{2}\frac{\hbar}{2i} & -\lambda \end{vmatrix} &= \underbrace{-\lambda\left(\lambda^2 - \hbar^2\frac{1}{2}\right) - \sqrt{2}\frac{\hbar}{2i}\left(\sqrt{2}\frac{\hbar}{2i}\lambda \;-0\right)}_{\text{Entwicklung nach der ersten Zeile}} \\ &= -\lambda(\lambda^2 - \hbar^2) \\ &\Rightarrow \quad \lambda_1 = 0 \qquad \lambda_{2,3} = \pm\hbar \end{aligned}$$

Bestimmung der Eigenvektoren:

$\lambda_1 = 0$:

$$\begin{pmatrix} 0 & \frac{\hbar}{2i}\sqrt{2} & 0 \\ -\frac{\hbar}{2i}\sqrt{2} & 0 & \frac{\hbar}{2i}\sqrt{2} \\ 0 & -\frac{\hbar}{2i}\sqrt{2} & 0 \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \to \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \qquad \vec{e_1} = \frac{1}{\sqrt{2}}(1,0,1)^T$$

$\lambda_1 = \hbar$:

$$\begin{pmatrix} -\hbar & \frac{\hbar}{2i}\sqrt{2} & 0 \\ -\frac{\hbar}{2i}\sqrt{2} & -\hbar & \frac{\hbar}{2i}\sqrt{2} \\ 0 & -\frac{\hbar}{2i}\sqrt{2} & -\hbar \end{pmatrix} \to \begin{pmatrix} -2i & \sqrt{2} & 0 \\ -\sqrt{2} & -2i & \sqrt{2} \\ 0 & -\sqrt{2} & -2i \end{pmatrix} \to \begin{pmatrix} -2i & \sqrt{2} & 0 \\ -2i & 2\sqrt{2} & 2i \\ 0 & -\sqrt{2} & -2i \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} -2i & \sqrt{2} & 0 \\ 0 & \sqrt{2} & 2i \\ 0 & -\sqrt{2} & -2i \end{pmatrix} \rightarrow \begin{pmatrix} -2i & \sqrt{2} & 0 \\ 0 & \sqrt{2} & 2i \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \frac{1}{\sqrt{2}}i & 0 \\ 0 & -\frac{1}{\sqrt{2}}i & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \qquad \vec{e_2} = \frac{1}{2}(1, i\sqrt{2}, -1)^T$$

$\lambda_1 = -\hbar$:

$$\begin{pmatrix} \hbar & \frac{\hbar}{2i}\sqrt{2} & 0 \\ -\frac{\hbar}{2i}\sqrt{2} & \hbar & \frac{\hbar}{2i}\sqrt{2} \\ 0 & -\frac{\hbar}{2i}\sqrt{2} & \hbar \end{pmatrix} \rightarrow \begin{pmatrix} 2i & \sqrt{2} & 0 \\ -\sqrt{2} & 2i & \sqrt{2} \\ 0 & -\sqrt{2} & 2i \end{pmatrix} \rightarrow \begin{pmatrix} 2i & \sqrt{2} & 0 \\ 2i & 2\sqrt{2} & -2i \\ 0 & -\sqrt{2} & 2i \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 2i & \sqrt{2} & 0 \\ 0 & \sqrt{2} & -2i \\ 0 & -\sqrt{2} & 2i \end{pmatrix} \rightarrow \begin{pmatrix} 2i & \sqrt{2} & 0 \\ 0 & \sqrt{2} & -2i \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -\frac{1}{\sqrt{2}}i & 0 \\ 0 & \frac{1}{\sqrt{2}}i & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \qquad \vec{e_3} = \frac{1}{2}(1, -i\sqrt{2}, -1)^T$$

(ii)

Berechnung der Projektoren:

$$\begin{aligned} P_1 &= \vec{e_1}\vec{e_1}^\dagger = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}(1,0,1) = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \\ P_2 &= \vec{e_2}\vec{e_2}^\dagger = \frac{1}{4}\begin{pmatrix} 1 \\ i\sqrt{2} \\ -1 \end{pmatrix}(1,-i\sqrt{2},-1) = \frac{1}{4}\begin{pmatrix} 1 & -i\sqrt{2} & -1 \\ i\sqrt{2} & 2 & -i\sqrt{2} \\ -1 & i\sqrt{2} & 1 \end{pmatrix} \\ P_3 &= \vec{e_3}\vec{e_3}^\dagger = \frac{1}{4}\begin{pmatrix} 1 \\ -i\sqrt{2} \\ -1 \end{pmatrix}(1,i\sqrt{2},-1) = \frac{1}{4}\begin{pmatrix} 1 & i\sqrt{2} & -1 \\ -i\sqrt{2} & 2 & i\sqrt{2} \\ -1 & -i\sqrt{2} & 1 \end{pmatrix} \end{aligned}$$

Vollständigkeit:

$$P_1 + P_2 + P_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Orthogonalität:

$$
\begin{aligned}
P_1 \cdot P_2 &= \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \frac{1}{4}\begin{pmatrix} 1 & -i\sqrt{2} & -1 \\ i\sqrt{2} & 2 & -i\sqrt{2} \\ -1 & i\sqrt{2} & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
P_2 \cdot P_3 &= \frac{1}{4}\begin{pmatrix} 1 & -i\sqrt{2} & -1 \\ i\sqrt{2} & 2 & -i\sqrt{2} \\ -1 & i\sqrt{2} & 1 \end{pmatrix} \frac{1}{4}\begin{pmatrix} 1 & i\sqrt{2} & -1 \\ -i\sqrt{2} & 2 & i\sqrt{2} \\ -1 & -i\sqrt{2} & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
P_1 \cdot P_3 &= \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \frac{1}{4}\begin{pmatrix} 1 & i\sqrt{2} & -1 \\ -i\sqrt{2} & 2 & i\sqrt{2} \\ -1 & -i\sqrt{2} & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\end{aligned}
$$