Übungen zu Moderne Theoretische Physik III(TP)

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Exercise Sheet 7 | SS-2024 | Due date: 23.07.24

QFT for simple processes (100 Points)

Exercise 7.1: *(20 points)* Consider a theory of two scalar fields ϕ and χ defined by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - g\chi\phi^2. \tag{1}$$

(a) *(8 points)* Identify the interaction term in the above Lagrangian and list all processes of the type $ab \to ab$, $a, b \in \{\phi, \chi\}$, that are possible in this theory.

(b) *(12 points)* Draw Feynman diagrams for each of these processes and provide mathematical expressions for corresponding transition amplitudes.

Exercise 7.2: *(35 points)* Consider a theory of a charged scalar field ϕ interacting with two different gauge fields A_μ and B_μ. The interaction part of the Lagrangian reads

$$\mathcal{L} = \frac{1}{2}D_\mu\phi D^\mu\phi, \tag{2}$$

where covariant derivative $D_\mu\phi = (\partial_\mu - ig_A A_\mu - ig_B B_\mu)\,\phi$.

(a) *(10 points)* Add missing kinetic terms for the gauge fields to the Lagrangian, and show how different terms in the Lagrangian transform under gauge transformation. Consider two fields A and B as gauge fields of two independent $U(1)$ groups.

(b) *(5 points)* Identify interaction terms in the Lagrangian Eq. (2)and draw interaction vertices.

(c) *(5 points)* Consider collision of two scalar particles $\phi + \phi \to X$. Identify all two-particle final states X that are allowed by the Lagrangian in Eq. (2). Draw diagrams for all allowed processes that are possible.

(d) *(5 points)* Consider the following processes $A\phi \to A\phi$ and $B\phi \to B\phi$ and draw Feynman diagrams that describe them.

(e) *(10 points)* Working in the lowest order of perturbation theory, which practically means using the minimal number of interaction vertices, draw diagrams for all possible gauge boson scattering processes with a pair of gauge bosons in the final state.

Exercise 7.3: *(45 points)* The experiment revealed that the proton has a structure. At different energies, its components interact with different strengths. For example, at LHC energies, mostly gluons collide. Consider a simplified model with interaction Lagrangian

$$\mathcal{L}_I = i\bar{\psi}_t\gamma^\mu D_\mu\psi_t - y_t\bar{\psi}_t\psi_t H, \tag{3}$$

where ψ_t is a heavy fermion(top-quark) field and covariant derivative is defined by $D_\mu = \partial_\mu - ig_s A_\mu$ if we work with abelian field A_μ for simplicity.

(a) *(5 points)* In this model write down all terms in Lagrangian responsible for interaction.

(b) *(10 points)* Identify all interaction terms in Lagrangian and draw corresponding interaction vertices.

(c) *(10 points)* It was observed that Higgs H is produced in collisions of two gluons, which we describe with the fild A_μ: $gg \to H$. Explain how this process is possible with the Lagrangian given in Eq. (3). Draw possible Feynman diagrams that describe the production of Higgs bosons in the collision of two gluons.

(d) *(20 points)* We can check diagrams found in the previous exercise by considering the transition amplitude for the specific process. Consider the process of single Higgs production $g(p_1) + g(p_2) \to H(q)$ in the collision of two gluons and write down a general expression for the transition amplitude using the interaction part of the Lagrangian (3).

In the obtained expression, expand the exponent to the first non-trivial order for the considered process to happen. Verify that the number of integrations in the obtained expression equals the number of vertices in diagrams you have found previously.