Karlsruher Institut für Technologie

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Übungen zur Theoretischen Physik F   SS 12

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Blatt 3

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1. Ideales Gas mit f Freiheitsgraden pro Molekül: (4+4+2=10 Punkte)

Für ein ideales Gas aus N Teilchen (Molekülen) mit f Freiheitsgraden pro Molekül lauten die Zustandsgleichungen

$$U = \frac{f}{2} NkT, \quad pV = NkT.$$

(a) Betrachten Sie eine *adiabatische* Zustandsänderung bei konstanter Teilchenzahl, und zeigen Sie über den 1. Hauptsatz, dass gilt:

$$pV^{(f+2)/f} = \text{const.}, \quad VT^{f/2} = \text{const.}$$

(b) Berechnen Sie die Entropie

$$S(U,V,N) = S_0 \frac{N}{N_0} + Nk \left[\frac{f}{2} \ln\left(\frac{U}{U_0}\right) + \ln\left(\frac{V}{V_0}\right) - \frac{f+2}{2} \ln\left(\frac{N}{N_0}\right) \right]$$

wobei S_0, U_0, V_0, N_0 Integrationskonstanten sind.

Hinweis: Zeigen Sie zunächst:

$$\mathrm{d}s = \frac{1}{T}\,\mathrm{d}u + \frac{p}{T}\,\mathrm{d}v \quad \text{mit} \quad s = S/N, u = U/N, v = V/N.$$

(c) Warum verletzt das ideale Gas den 3. Hauptsatz der Thermodynamik?

2. Velocity and interparticle distance distributions in an ideal gas: $(4+4+4=12$ Punkte)

Consider an ideal gas.

(a) Find the probability distribution function, $f(v)$, for the velocity, such that $f(v)\,dv$ is the probability that the magnitude of the velocity of a particle is between v and $v+dv$.

(b) Use $f(v)$ to find the most probable velocity v^* from

$$\left.\frac{\partial f}{\partial v}\right|_{v=v^*} = 0,$$

the mean velocity, $\langle v \rangle$, and the mean square velocity, $\langle v^2 \rangle$.

(c) Consider now an ideal gas in a spherical container of radius R. Calculate the mean distance between the particles

$$\langle r\rangle = \int_0^\infty dr f(r)\, r \tag{1}$$

where

$$f(r) = \langle \delta (r - |\mathbf{r}_i - \mathbf{r}_j|)\rangle \text{ with } i \neq j \tag{2}$$

is the density of probability that two particles are separated by a distance r.

3. An ideal gas in the field of the Earth: $(4 + 4 = 8$ Punkte)

Consider an ideal gas in the gravitational field of the Earth with the Hamilton function

$$H = \sum_{i=1}^{N} \left(\frac{\mathbf{p}_i^2}{2m} + gmz_i \right). \tag{3}$$

The surface of the Earth is assumed to be flat at $z = 0$ (i.e. only $z > 0$ is allowed). The gas is confined to a cylinder with arbitrary height and radius R.

(a) Calculate the partition function and free energy of the gas, assuming that temperature is independent of z.

(b) Find the density

$$\rho(\mathbf{r}) = \langle \delta(\mathbf{r} - \mathbf{r}_j)\rangle = \frac{\prod_{i=1}^{N} \int d^3p_i d^3r_i \delta(\mathbf{r} - \mathbf{r}_j) \exp(-\beta H)}{\prod_{i=1}^{N} \int d^3p_i d^3r_i \exp(-\beta H)} \tag{4}$$

and pressure $p(z)$ as function of the height.

4. Logarithmic spectrum: (5+5=10 Punkte)

Consider a system of N noninteracting particles, each of which has an energy spectrum

$$E_n = \Delta \log(n), \quad n = 1, ..., \infty. \tag{5}$$

(a) Calculate the partition function of the problem for $k_B T < \Delta$. Analyze the leading temperature dependence of the entropy and specific heat for $k_B T$ close to Δ and discuss the result.

(b) Discuss what happens for $k_B T > \Delta$.