Karlsruher Institut für Technologie
Institut für Theorie der Kondensierten Materie

Übungen zur Theoretischen Physik F SS 12

Prof. Dr. Jörg Schmalian
Dr. Igor Gornyi

Blatt 10: 30 Punkte + 10 Bonuspunkte
Besprechung 29.06.2012

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1. Landau theory of phase transitions: liquid crystals

(2 + 5 + 3 + 5 = 15 Punkte)

In liquid crystals, materials consisting of rod-shaped molecules which undergo an orientational order without entering a solid state, the free energy density as a function of the order parameter is given by

$$f(\varphi) = \frac{a(T)}{2}\varphi^2 - \frac{b}{3}\varphi^3 + \frac{c}{4}\varphi^4, \tag{1}$$

where, in distinction to the ferromagnetic case, a cubic term cannot be excluded from the expansion. Here $a(T) = a_0(T - T_0)$, $b > 0$, $c > 0$, and $\varphi = \langle 3\cos^2\theta - 1\rangle$ is a measure for the orientational order of the molecules which are oriented with an angle θ relative to some preferred axis.

(a) What is the order of the transition (first or second)?

(b) Find the transition temperature.

(c) Determine the value of φ at the transition temperature.

(d) Calculate the latent heat of the transition. How would the result change if $b = 0$?

The physics of liquid crystals is exciting but you do not need to know this stuff in order to solve the problem properly.

2. Coupled order parameters

(5 + 5 = 10 Punkte)

A system with two coupled order parameters, φ and χ, is characterized by the free energy density

$$f(\varphi, \chi) = \frac{a(T)}{2}\varphi^2 + \frac{c}{4}\varphi^4 + \frac{\tilde{a}}{2}\chi^2 - g\varphi^2\chi, \tag{2}$$

where $a(T) = a_0(T - T_0)$ and c, $\tilde{a}$ and g are positive constants. An example for this type of behavior is the coupling of lattice deformations χ to the magnetization φ. Without the coupling (i.e., for $g = 0$) finite deformation $\chi \neq 0$ is not allowed, that is, the lattice deformation is assumed to be driven by the onset of magnetic order.

(a) How is the ordering of φ and χ affected by the coupling term?

(b) What happens if the sign of g is changed?

3. Landau theory of phase transitions: φ^6-term

$(1 + 2 + 2 = 5$ Punkte)

Consider the Landau expansion of the free energy density

$$f(\varphi) = \frac{a(T)}{2}\varphi^2 + \frac{c}{4}\varphi^4 + \frac{g}{6}\varphi^6, \tag{3}$$

where we have included an additional term proportional to φ^6. We assume $g > 0$ to keep the theory well defined and $a(T) = a_0(T - T_0)$.

(a) Analyze the extrema of the free energy density Eq. (3).

(b) Consider the case $c > 0$ and demonstrate that the behavior of the order parameter close to the transition is unchanged by the additional term $\propto \varphi^6$. Show also that $T_c = T_0$.

(c) Consider now the behavior for $c < 0$. What is the value of the transition temperature? What is the order of the transition (first or second)? How does the order parameter behave as a function of temperature?

4. Bonus exercise: droplet nucleation

(10 Punkte)

Consider the Landau theory with an additional cubic term (like in nematic liquid crystals in exercise 1):

$$f[\varphi] = \frac{1}{2}(\nabla\varphi)^2 + \frac{a(T)}{2}\varphi^2 - \frac{b}{3}\varphi^3 + \frac{c}{4}\varphi^4 \tag{4}$$

Assume that the system is only slightly above its transition temperature. The ordered state is therefore still metastable (has a local minimum). Consider a droplet of size L and interface width l_0, with the stable solution inside and the metastable solution outside. Determine the critical nucleation radius and the energy of a nucleating droplet.