Korrektur zur 12. Vorlesung (Onsager Reziprozität)

$$\begin{pmatrix} \vec{J} \\ \vec{J}_Q \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} \dfrac{\vec{E}_{el.ch}}{T} \\ -\dfrac{\nabla T}{T^2} \end{pmatrix}$$

$$K_{ij} = K_{ji}$$

$$T\left(\frac{\partial s}{\partial t} + \nabla \cdot \vec{J}_s\right) = -\vec{J}_Q \cdot \frac{\nabla T}{T} + \vec{J} \cdot \big(\underbrace{\vec{E} - \tfrac{1}{e}\nabla\mu}_{\vec{E}_{el.ch}}\big)$$

Onsager:

$$j_a = \sum_b K_{ab} X_b$$

$$\frac{\partial s}{\partial t} + \nabla \cdot \vec{j}_s = \sum_a j_a X_a$$

$$\frac{\partial s}{\partial t} + \nabla \cdot \vec{J}_s = -\vec{J}_Q \cdot \frac{\nabla T}{T^2} + \vec{J} \cdot \frac{\left(\vec{E} - \tfrac{1}{e}\nabla\mu\right)}{T}$$

$$X_E = \frac{\vec{E}_{el.ch}}{T} \qquad X_T = -\frac{\nabla T}{T^2}$$

$$\Rightarrow \quad \sigma = K_{11}/T \qquad P_E = K_{21}/K_{11}$$

$$\kappa = K_{22}/T^2 \qquad S = \frac{K_{12}}{T K_{11}}$$

Lokales Gleichgewicht (Chapman-Enskog Entwicklung)

Verschwindende Entropieproduktion für ein lokales Gleichgewicht:

$$f_k^{l.e.}(\vec{r}) = \frac{1}{e^{\beta(\vec{r})(\varepsilon_k - \mu(\vec{r}))} + 1}$$

wenn Energie + Teilchenzahl erhält.

Entwickeln die Verteilungsfunktion in der Nähe des lokalen Gleichgewichts.

$$f_k(\vec{r}) = f_k^{l.e.}(\vec{r}) + \delta f_k(\vec{r})$$

Boltzmann-Gleichung

$$\frac{\partial f_k}{\partial t} - e\vec{E}\cdot\nabla_k f_k + \vec{v}_k\cdot\nabla_r f = -\mathcal{C}[f]$$

Einsetzen

$$-e\vec{E}\cdot\nabla_k f_k^{l.e.} + \vec{v}_k\cdot\nabla_r f^{l.e.} = -\mathcal{C}[f] + e\vec{E}\cdot\cancel{\nabla_k\delta f_k - \vec{v}\cdot\nabla_r\delta f_k}$$

$$\nabla_k f_k^{l.e.}(\vec{r}) = \frac{\partial f^{l.e.}}{\partial\varepsilon_k}\,\vec{v}_k$$

$$\nabla_r f_k^{l.e.}(\vec{r}) = -\frac{\partial f^{l.e.}}{\partial\varepsilon_k}\left(\nabla_r\mu + \frac{\varepsilon_k-\mu}{T}\nabla_r T\right)$$

nochmal einsetzen

$$-\frac{\partial f^{l.e.}}{\partial\varepsilon_k}\,\vec{v}_k\left(\vec{E}_{el.ch.} + \frac{\varepsilon_k-\mu}{T}\nabla T\right) = -\frac{\overbrace{f - f^{l.e.}}^{\delta f_k}}{\tau}$$

$$\delta f_k = \tau\,\frac{\partial f^{l.e.}}{\partial\varepsilon_k}\,\vec{v}_k\cdot\left(e\vec{E} + \frac{\varepsilon_k-\mu}{T}\nabla T\right)$$

$$= \tau\,\frac{\partial f^{(0)}}{\partial\varepsilon_k}\,\vec{v}_k\cdot\left(e\vec{E} + \frac{\varepsilon_k-\mu}{T}\nabla T\right)$$

$$f_k^{(0)} = \frac{1}{e^{\beta(\varepsilon_k-\mu)}+1}$$

Aus der bestimmten Verteilungsfunktion folgen die Ladungs + Wärmeströme:

$$\vec{j} = -e \int_k \vec{v}_k \, \delta f_k(\vec{r})$$

$$= -e \int_k \tau \frac{\partial f_0}{\partial \varepsilon_k} \vec{v}_k \cdot \left(e\vec{E} + \frac{\varepsilon_k-\mu}{T} \nabla T \right) \vec{v}_k$$

$$\vec{j}_q = \int_k (\varepsilon_k-\mu) \vec{v}_k \, \delta f_k(\vec{r})$$

$$= \int_k \tau \frac{\partial f_0}{\partial \varepsilon_k} (\varepsilon_k-\mu) \vec{v}_k \cdot \left(e\vec{E} + \frac{\varepsilon_k-\mu}{T} \nabla T \right) \vec{v}_k$$

$$\begin{pmatrix} \vec{j} \\ \vec{j}_q \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} \frac{\vec{E}_{el.ch}}{T} \\ -\frac{\nabla T}{T^2} \end{pmatrix}$$

$$K_{11}^{\alpha\beta} = -e^2 T \int_k \tau \frac{\partial f_0}{\partial \varepsilon_k} v_k^\alpha v_k^\beta \qquad K_{ij}^{\alpha\beta} = \delta_{\alpha\beta} K_{ij}$$

$$K_{22}^{\alpha\beta} = -T \int_k \tau \frac{\partial f_0}{\partial \varepsilon_k} (\varepsilon_k - \mu)^2 v_k^\alpha v_k^\beta$$

$$K_{12}^{\alpha\beta} = T e \int_k \frac{\partial f_0}{\partial \varepsilon_k} (\varepsilon_k - \mu) v_k^\alpha v_k^\beta \qquad K_{21}^{\alpha\beta} = K_{12}^{\alpha\beta}$$

$$\int \frac{(v_k^x)^2 + (v_k^y)^2 + (v_k^z)^2}{3} \frac{\partial f}{\partial \varepsilon} \quad \ldots$$

$$K_{11} = -e^2 T \frac{v_F^2}{3} \tau \int d\varepsilon \frac{\partial f}{\partial \varepsilon} \rho(\varepsilon) = T\sigma$$

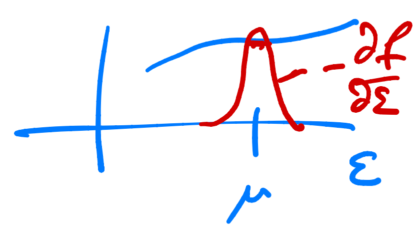
$$K_{22} = -T \frac{v_F^2}{3} \tau \int d\varepsilon \frac{\partial f}{\partial \varepsilon} \rho(\varepsilon) (\varepsilon - \mu)^2 = T^2 \kappa$$

$$K_{12} = -e T \frac{v_F^2}{3} \tau \int d\varepsilon \frac{\partial f}{\partial \varepsilon} \rho(\varepsilon) (\varepsilon - \mu) = S \sigma T^2$$

elektrische Leitfähigkeit $\left(-\frac{\partial f}{\partial \varepsilon} \overset{T \to 0}{=} \delta(\varepsilon - \mu) \right)$

$$\sigma = e^2 \frac{v_F^2}{3} \tau \rho_F \int d\varepsilon \frac{\partial f}{\partial \varepsilon}$$

$$= e^2 \frac{v_F^2}{3} \tau \rho_F \quad \left(= \frac{n e^2}{m} \tau \right)$$



$$\kappa = \frac{1}{T}\frac{v_F^2}{3}\tau\, g_F \int_{-\infty}^{\infty} d\varepsilon \left(-\frac{\partial f}{\partial \varepsilon}\right)(\varepsilon-\mu)^2$$

•) $\varepsilon - \mu = \varepsilon'$

$$f(\varepsilon) = \frac{1}{e^{\beta\varepsilon}+1}$$

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} = \frac{e^{\beta\varepsilon}}{(e^{\beta\varepsilon}+1)}\beta$$

•) $x = \beta\varepsilon$

$$\kappa = \frac{v_F^2}{3}\tau\, g_F\, T k_B^2 \int_{-\infty}^{\infty} dx \frac{x^2 e^x}{(e^x+1)^2}$$

$$= \frac{v_F^2}{3}\tau\, g_F\, T k_B^2 \frac{\pi^2}{3}$$

Zusammenhang zwischen der thermischen + elektrischen Leitfähigkeit

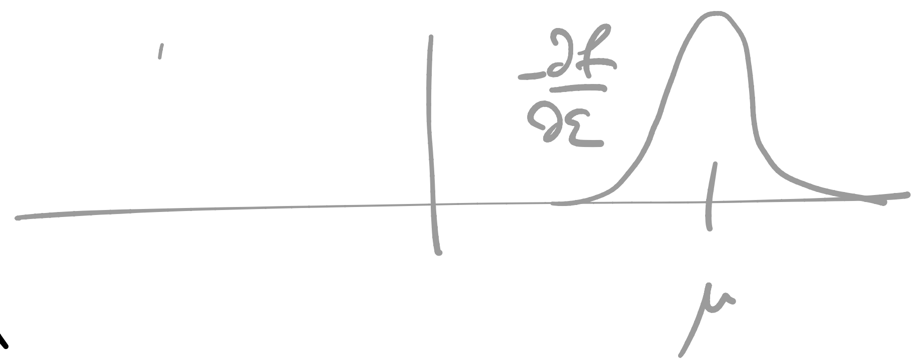
$$\frac{\kappa}{\sigma} = T\,L. \qquad L = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

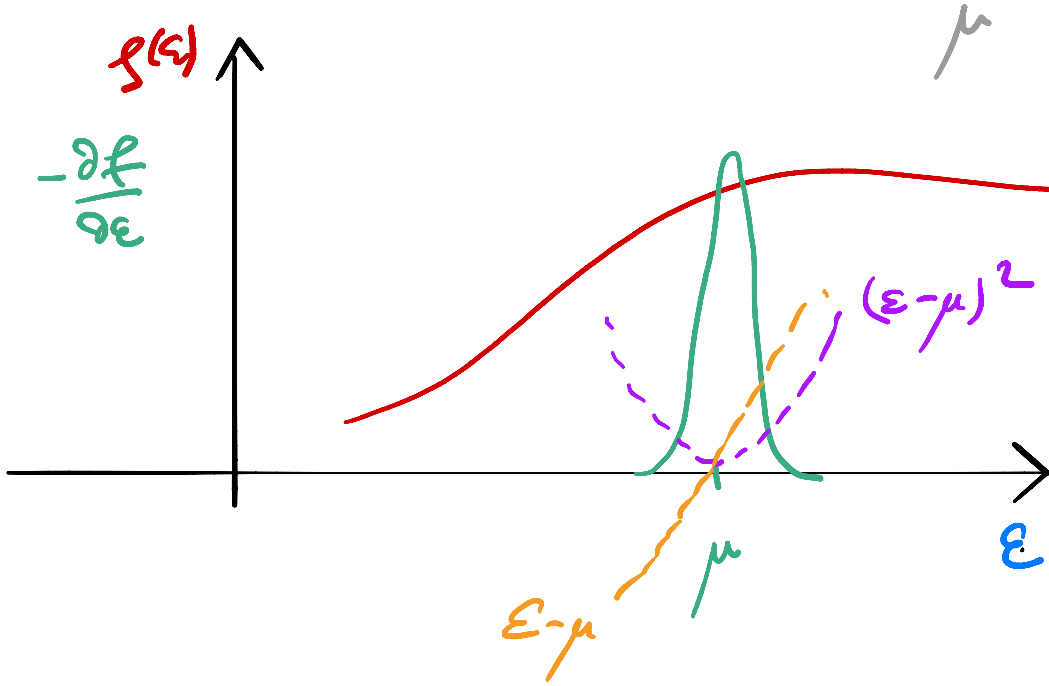
Wiedemann-Franz'sches Gesetz!

thermoelektrische Effekte :

$$S = \frac{-e \frac{v_F^2}{3} \tau \int d\varepsilon \, \frac{\partial f}{\partial \varepsilon} \, g(\varepsilon)(\varepsilon-\mu)}{\sigma T}$$

$$g_F \int_{-\infty}^{\infty} d\varepsilon \, \frac{\partial f}{\partial \varepsilon} (\varepsilon-\mu) = 0$$





Im Allgemeinen ist die Zustandsdichte nicht konstant

$$g(\varepsilon \simeq \mu) = g_F + g_F'(\varepsilon-\mu)$$

$$g_F' = \left.\frac{\partial g(\varepsilon)}{\partial \varepsilon}\right|_\mu$$

$$S = \frac{-e \frac{v_F^2}{3} \tau}{\sigma T} \int d\varepsilon \, \frac{\partial f}{\partial \varepsilon} \rho(\varepsilon) (\varepsilon - \mu)$$

$$= - e \frac{v_F^2}{3\sigma} \frac{\tau}{T} \rho_F' \underbrace{\int d\varepsilon \, \frac{\partial f}{\partial \varepsilon} (\varepsilon - \mu)^2}_{-\frac{\pi^2}{3}(k_B T)^2}$$

$$= + e \frac{v_F^2}{3\sigma} \frac{\tau}{T} \rho_F' \cdot \frac{\pi^2}{3} (k_B T) = \frac{\rho_F'}{e \rho_F} k_B^2 T$$

$$\frac{\rho_F'}{\rho_F} \sim \frac{1}{E_F}$$

Größenordnung $\qquad |S| \sim \frac{k_B}{e} \frac{k_B T}{E_F}$